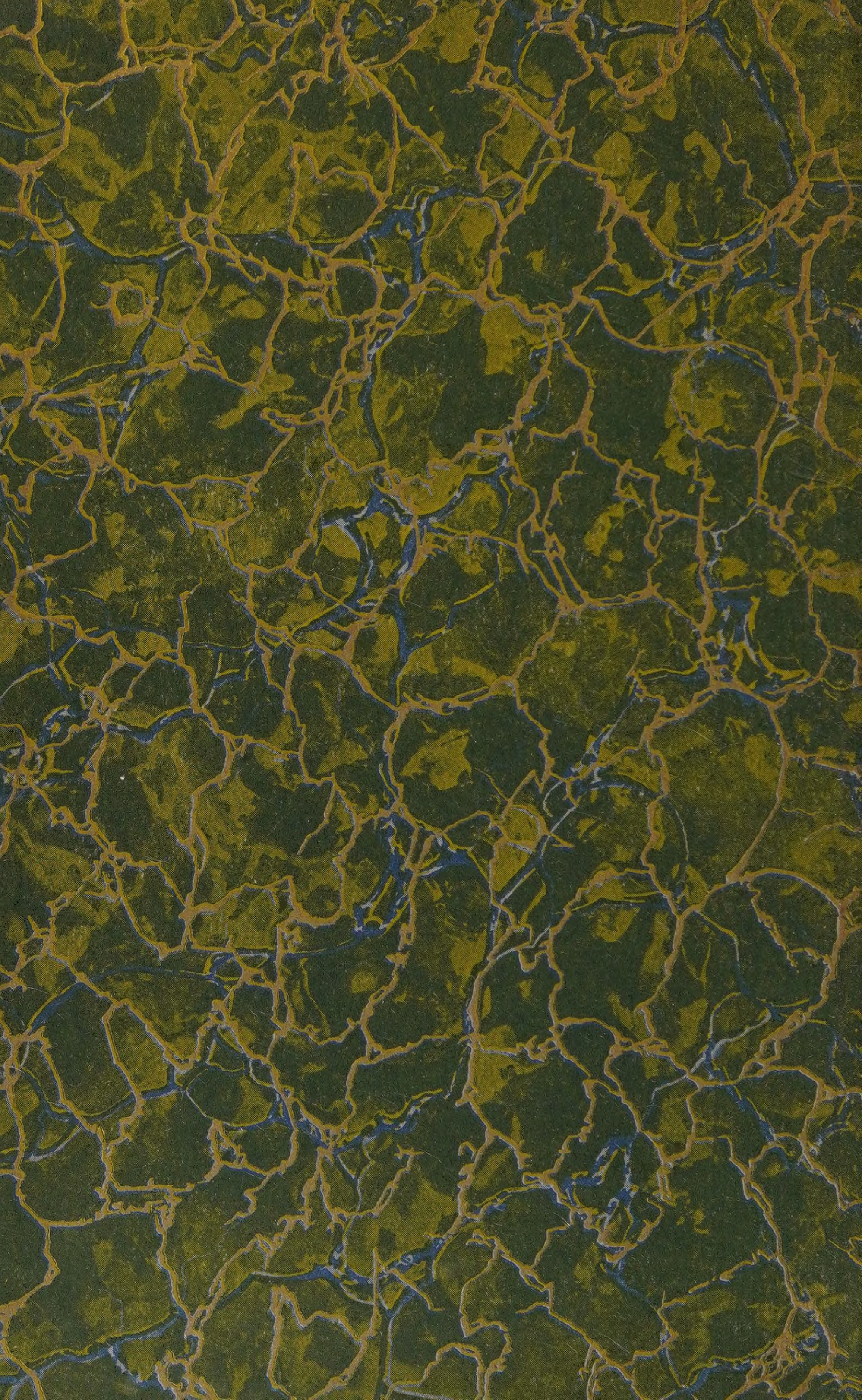

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Arithmetic, Penmanship, and Spelling

109 ILLUSTRATIONS

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ELEMENTS OF ARITHMETIC
FRACTIONS
DECIMALS
WEIGHTS AND MEASURES
RATIO AND PROPORTION
POWERS AND ROOTS
MENSURATION
COMMERCIAL CALCULATIONS
VERTICAL PENMANSHIP
PENMANSHIP
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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

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B

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ELEMENTS OF ARITHMETIC

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NOTATION AND NUMERATION

INTRODUCTION

1. Necessity for Calculations.—The worker in almost any branch of industry frequently meets with problems that require figuring. For example, the engineer may wish to figure the horsepower of an engine or the pressure that may be carried safely in a steam boiler. The blacksmith may have to find how long a piece of straight bar must be cut off, so that, when it is bent, it will form a ring of a certain size. The patternmaker may wish to know how to set his dividers so that they will space off a certain number of equal divisions on a circle. The foundryman may need to know the amount of metal required to pour a casting whose dimensions are given on a drawing, and so on in many other occupations. In each of these cases it is necessary to make calculations in order to obtain the desired information. Sometimes the calculations are short and simple, and at other times they are long and difficult.

2. Use of Arithmetic.—Before calculations of any kind can be made, something must be known about figures and numbers, because all calculations bring figures and numbers into use. The study of numbers, or the art of reckoning, is commonly called **arithmetic.** Thus, the various calculations that may be made really depend on an understanding of

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arithmetic. For this reason, it will be necessary to begin with a study of those arithmetical principles and processes that are to be used later in making engineering and commercial calculations.

3. Fundamental Processes.—The four fundamental processes of arithmetic are *addition, subtraction, multiplication,* and *division.* They are called fundamental processes because all operations in arithmetic are based on them. Every calculation that is made must use one or more of these processes.

4. Unit and Number.—A **unit** is *one,* or a single thing, as *one* inch, *one* dozen. A **number** is one or more units or things. It answers the question "How many?" For example, let the question be, "How many bolt holes are there in that cylinder head?" If the answer is "Eight holes," then *eight* is a number, because it tells how many. A number, however, may be either one or *more* than one, as *one* hour, *six* feet, *ten* dollars.

5. Concrete and Abstract Numbers.—If a number is applied to one particular kind of thing or measure, as three *horses,* five *dollars,* ten *pounds,* it is called a **concrete number.** If a number is not applied to any particular thing or measure, as *three, six, ten,* it is an **abstract number.**

6. Integer and Fraction.—Distinction is made between numbers that indicate one or more *whole* units and those numbers that represent a *portion* of a unit. A whole number is known as an **integer,** or an **integral number.** A number representing a portion, or part, of a unit is called a **fraction.**

7. Systems of Notation and Numeration.—Numbers are expressed by *words,* by *figures,* and by *letters.* **Notation** is the art of *expressing* numbers by figures or letters. **Numeration** is the art of *reading* numbers expressed by figures or letters. Two kinds of notation are in general use, the *Arabic* and the *Roman.*

ARABIC NOTATION

8. Meaning and Arrangement of Figures.—The **Arabic notation** is the method of expressing numbers by figures. This method employs ten characters, called **figures,** which are written and named as follows:

Figures	1	2	3	4	5	6	7	8	9	0
Names	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*	*naught*

The figure **naught** (0) is called also **cipher** and **zero,** and when standing alone means *nothing,* or *no value.* The other nine figures are called **digits,** and when standing alone each has *a definite value.* Ten is written 10.

9. Counting.—One of the first things learned in arithmetic is to *count.* **Counting** is done by naming the numbers successively in the order of their value. The method of counting above ten and up to one thousand, and the names of the various numbers, are given in the following list:

11	*eleven*	21	*twenty-one*	31	*thirty-one* and so	100	*one hundred*
12	*twelve*	22	*twenty-two*		on up to	200	*two hundred*
13	*thirteen*	23	*twenty-three*	40	*forty;* then	300	*three hundred*
14	*fourteen*	24	*twenty-four*	41	*forty-one* and so	400	*four hundred*
15	*fifteen*	25	*twenty-five*		on up to	500	*five hundred*
16	*sixteen*	26	*twenty-six*	50	*fifty;* then to	600	*six hundred*
17	*seventeen*	27	*twenty-seven*	60	*sixty*	700	*seven hundred*
18	*eighteen*	28	*twenty-eight*	70	*seventy*	800	*eight hundred*
19	*nineteen*	29	*twenty-nine*	80	*eighty*	900	*nine hundred*
20	*twenty*	30	*thirty*	90	*ninety*	1,000	*one thousand*

10. Ordinals and Cardinals.—Indicating by means of a number the position of a thing or unit in a row, or series, may also be considered as counting. For instance, *third* house, *seventh* month, etc. When numbers are used in this manner, they are known as **ordinals, or ordinal numbers.** Numbers that simply answer the question "How many?" are sometimes called **cardinals, or cardinal numbers.** For example, *one, two, three,* etc. are cardinals, and *first, second, third,* etc. are the corresponding ordinals. In general, an ordinal is formed by adding the letters *th* to a cardinal; as, four*th*, six*th*, ten*th*, fourteen*th*, etc. The ordinals of *one*, *two*, and *three*, are exceptions to this rule; in the ordinal *fifth* the spelling of five is slightly changed.

11. Simple and Relative Values.—The value represented by a figure depends on its position in relation to other figures; thus, figures may have *simple values* and *relative values*. The **simple value** of a figure is the value it has when it stands alone; for example, the figure 2 standing alone has a value that is one greater than 1 and one less than 3. But if a figure 1 is placed to the right of the 2, making 21, the first figure no longer has the value it had before. This new value that is given to it by placing another figure to the right of it is called its **relative value.** The difference between simple and relative values may be explained as follows:

If the figure 8 stands alone, thus.................. 8
it is simply *eight units*, or **eight.**

Place a 2 to the right of it; thus................... 82

The 2 is now *two units*, but the 8 has moved one place to the left, so that it is no longer *eight units*. Instead, it is *eight* **tens,** or *ten times* 8.

Now place a 5 to the right; thus.................... 825

The 8 is now moved another place to the left and its value is again increased ten times, or *ten times eight tens*, making 8 **hundreds.** At the same time the 2 is moved one place to the left and its value is increased to 2 **tens.**

Add a 6 to the right; thus........................ 8256

The 8 is now another place to the left and its value is increased ten times more, or to 8 **thousands.** The 2 is increased to 2 **hundreds,** and the 5 to 5 **tens.**

The last number of four figures is read *eight thousand two hundred fifty-six.*

12. Grouping of Figures.—In writing numbers that contain more than three figures, it is common to divide them into groups of three figures, counting from the *right*. This is called **pointing off,** because a comma (,) is used to point off, or mark, each group of three figures. The object of doing this is to enable the number to be read easily and accurately. The first group at the right is the units group, the next the thousands group, and the next the millions group. This method of pointing off and the naming of the groups is shown in the table at the end of this article.

The illustration given in Art. **11** shows that by moving a figure one place to the left in a number its value is made ten times as great as before. The last position at the right in a number is called the *units* place. Take the number 417,385,926 as an example. The figure 6 is in the units place and is simply *six*. The next place, occupied by the 2, is the *tens of units* place, so that the 2 in this position has a value of 2 tens of units, or *twenty*. The 9 is in the *hundreds of units* place, and its value is 9 hundreds of units, or *nine hundred*. This right-hand group of three figures, therefore, has a value of nine hundreds, two tens, and six units, or *nine hundred twenty-six*, as it would be read.

The next group of three figures, or 385, is at the left of 926; therefore, the figure 5 is in the fourth place from the right end, and its value is ten times as great as it would be in the third position. The third place is the hundreds place, and as the fourth place is ten times as great, it must be the *thousands* place, so that the figure 5 in this position represents *five thousands*. The figure 8 is in the *tens of thousands* place, and its value is 8 tens of thousands, or *eighty thousand*. The figure 3 is in the *hundreds of thousands* place, and its value is 3 hundreds of thousands, or *three hundred thousand*. The middle group of three figures, therefore, has a value of three hundred thousand, eighty thousand, and five thousand, which would be read *three hundred eighty-five thousand*.

The group of figures at the left, or 417, refer to millions, a *million* being ten times as great as a hundred thousand. The 7 is in the *millions* place and has a value of *seven million*. The 1 is in the *tens of millions* place and has a value of *ten million*. The 4 is in the *hundreds of millions* place and has a value of *four hundred million*. This group of figures, therefore, has a value of four hundred million, one ten million, and seven million, or *four hundred seventeen million*, to state it in the way in which it would be read.

The entire number 417,385,926, made up of the three groups of figures, would be read *four hundred seventeen million three hundred eighty-five thousand nine hundred twenty-six*. The following table shows the positions of the figures, the groups, and the name of each of the places, or positions:

Millions			*Thousands*			*Units*		
Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds of Units	Tens of Units	Units
4	1	7,	3	8	5,	9	2	6

13. Use of the Cipher.—The cipher, 0, has no value in itself, because it represents nothing, or zero, but it is useful in determining the position of other figures. Suppose, for example, that the number *two hundred five* is to be written. It would not be correct to write it 25, because that is *twenty-five*. The 2 must be in the hundreds place and the 5 in the units place, because *two hundred five* means *two hundreds* and *five units;* therefore, it is written by placing a cipher between the 2 and the 5, giving 205. The 2 is then in the hundreds place and the 5 in the units place, as required, and the cipher indicates that there are no *tens*. In the same way, *three thousand twenty-six* would be written 3026, and *six thousand four* would be written 6004. In the last case, two ciphers are needed, because the 6 must be in the thousands place, which is the fourth from the right. If the number to be written is *five thousand nine hundred eighty,* it could *not* be written 598 because that is only *five hundred ninety-eight*. The 5 must be in the fourth, or thousands, place, the 9 in the hundreds place, and the 8 in the tens place; consequently, a cipher is added at the right, giving 5980, which is the correct way of writing *five thousand nine hundred eighty.*

NUMERATION

14. Reading of Numbers.—When reading whole numbers the word *and* should not be used. For instance, the number 205 is read *two hundred five;* not two hundred *and* five.

In numbers that are pointed off, each group, or period, is read in its turn, as though it were a separate number, and then the name of the group is placed after it. For instance, the first group from the left in the number 987,765,432 is read nine hundred eighty-seven, as though standing alone, then *million,* the name of the group, is added. The second group is read seven hundred sixty-five thousand, and the last group is read simply as four hundred thirty-two, the word unit being omitted. In reading numbers, the final *s* in *thousands, millions,* etc. is omitted.

EXAMPLES FOR PRACTICE

Point off and read the following numbers:

(1) 31072; (2) 317020; (3) 1007; (4) 6051; (5) 28970093.

Write the following numbers, using figures:

(6) Seven thousand seventeen; (7) One thousand nine hundred fourteen; (8) Ten million eighty-two thousand thirty-six.

Ans. (1) 31,072 or thirty-one thousand seventy-two; (2) 317,020 or three hundred seventeen thousand twenty; (3) 1,007 or one thousand seven; (4) 6,051 or six thousand fifty-one; (5) 28,970,093 or twenty-eight million nine hundred seventy thousand ninety-three; (6) 7,017; (7) 1,914; (8) 10,082,036.

ROMAN NOTATION

15. Fundamental Letters and Their Combinations.—The method of expressing numbers by means of seven capital letters is known as **Roman notation.** This notation is generally used for numbering chapters in books, tables, rules, formulas, etc. The seven letters and their fixed values, when used singly, are as follows:

Letter	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

When numbers expressed in Roman notation are written by hand, the letters must be formed like the printed capitals, and not like ordinary capitals used in handwriting.

16. The letters used in Roman notation can be combined according to the following principles to represent any number:

I. *If a letter is written before one of greater value, their difference is the value represented;* as, IV, four; IX, nine; XC, ninety.

II. *If a letter is written after one of greater value, their sum is the value represented;* as, VI, six; XI, eleven.

III. *Repeating a letter repeats its value;* thus, II = 2, XX = 20, CC = 200, CCC = 300. The letters V, D, and L are never repeated; only I, X, C, and M are ever used more than once in any combination.

17. Some of the combinations in most common use and the values they represent are as follows:

II	two	IX	nine	XVI	sixteen
III	three	XI	eleven	XVII	seventeen
IV	four	XII	twelve	XVIII	eighteen
VI	six	XIII	thirteen	XIX	nineteen
VII	seven	XIV	fourteen	XX	twenty
VIII	eight	XV	fifteen	XXI	twenty-one

ADDITION

VERTICAL ADDITION

18. Definitions.—Addition is the process of *finding a number that is equal to two or more numbers taken together.* The number so obtained is called the **sum,** or the **total.**

19. The sign of addition is +. It is read *plus,* and means *more* or *and.* Thus, 5+6 is read 5 *plus* 6, and means that 5 and 6 are to be added. Or, 5+6 may be read 5 *and* 6.

The sign of equality is =. It is read *equals,* or *is equal to.* Thus, $5+6=11$ may be read, *5 plus 6 equals 11.*

In preparing his lessons, the student will have frequent occasion to use the equality sign, and he should therefore clearly understand its meaning. All on the right of the sign should always be equal to all on the left.

Numbers expressed in units of the same kind can be added, but numbers expressed in units of different kinds cannot be added. Thus, 6 dollars can be added to 7 dollars and the sum will be 13 dollars; but 6 dollars cannot be added to 7 feet.

20. Use of Table I.—Table I gives the sum of any two numbers from 1 to 12. This table should be carefully committed to memory. As 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 is 17, or $17+0=17$.

21. Method of Vertical Addition.—The term *vertical* means upright, in the direction of a plumb-line. Hence, for **vertical addition,** place the numbers to be added in a vertical row, one below another, taking care to place *units* under *units, tens* under *tens, hundreds* under *hundreds,* and so on. When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure of the one above it,* thus bringing units under units, tens under tens, etc. In a group of numbers arranged in this manner, each vertical row is said to form a *column.* From the fact that addition takes place up or down these vertical columns, this method of addition is called **vertical addition** to distinguish it from *horizontal addition,* explained later.

The number of columns in a group will depend on the number of figures in each horizontal line. To find the sum of the figures in these columns proceed as in the following rule:

Rule.—I. *When adding numbers, begin at the right, add each column separately, and write the sum, if it is only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, write down the right-hand figure of the sum under that*

TABLE I

ADDITION TABLE

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

column and add the remaining figure or figures to the next column.

22. Examples of Addition.—The application of the preceding rule is shown in the following examples:

Example 1.—Find the sum of 131, 222, 21, 2, and 413.

Solution.—After placing the numbers in the proper order, begin at the bottom of the units column and add in accordance with part I of the rule. Thus, 3 and 2 is 5; 5 and 1 is 6; 6 and 2 is 8; 8 and 1 is 9. Write 9 in the units place of the sum. Proceed in a like manner with the second and third columns, thus finding the number 8 for the tens place and 7 for the hundreds place in the sum. The sum is therefore 789.

$$\begin{array}{rrl} & 131 & \\ & 222 & \\ & 21 & \\ & 2 & \\ & 413 & \\ \cline{2-2} \textit{sum} & 789 & \text{Ans.} \end{array}$$

The result obtained in solving an example is called the **answer;** as shown in the preceding example, the word is written in the short form Ans.

Example 2.—Find the sum of 425, 36, 9,215, 4, and 907.

Solution.—Write the numbers as shown, and beginning with the bottom of the right-hand column, add in the following manner, which is shorter than that used in the preceding example, as the successive sums are not repeated. Thus, instead of 7 and 4 is 11; 11 and 5 is 16, etc., we have: 7 and 4 is 11 and 5 is 16 and 6 is 22 and 5 is 27. In accordance with part II of the rule, write 7 in the units place in the sum and carry 2 to the tens column. The term *carry* means in this case that a figure such as 2, is transferred from one column to the next one and added to it. Then, 2 and 1 is 3 and 3 is 6 and 2 is 8. Write 8 in the tens place in the sum, and add the next, or hundreds, column; 9 and 2 is 11 and 4 is 15. Write 5 in the hundreds place in the sum, carry 1 to the thousands place, and add to 9, making 10 to place in the sum; 0 is written in the thousands place and 1 in the tens of thousands place. The sum is therefore 10,587.

$$\begin{array}{rrl} & 425 & \\ & 36 & \\ & 9215 & \\ & 4 & \\ & 907 & \\ \cline{2-2} \textit{sum} & 10587 & \text{Ans.} \end{array}$$

23. Method of Checking Addition.—A good plan, especially when adding several large numbers, is to record the sum of each column plus the number carried, so that the addition can be readily tested, or *checked,* by adding again.

Example.—Add 7,329, 8,564, 9,238, 76,563, 6,417, 36,849, and 58,796.

Solution.—The columns are added as before explained, and each sum is recorded in some convenient place for reference. 46 is the sum of the numbers in the units column, 35 the sum of the numbers in the tens column plus the 4 tens carried, and so on. The final sum is given by writing the last sum, 20, and following it with the last figure of each of the other sums, 3, 7, 5, 6, giving the total sum 203,756.

	7329	
	8564	46
	9238	35
	76563	37
	6417	53
	36849	20
	58796	
sum	203756	Ans.

These records of the sums of separate columns are often useful in proving the correctness of the addition.

24. Proof.—*To prove addition, add each column from top to bottom. If the same result is obtained as by adding from bottom to top, the work is probably correct.*

25. Addition of Long Columns.—The following method of addition is useful when adding long columns:

Example.—Find the sum of 425, 36, 9,215, 4, and 907.

Solution.—Beginning with the bottom of the right-hand column, add as follows: 7 and 4 is 11 and 5 is 16 and 6 is 22 and 5 is 27. The first partial sum is 27 and is written as shown. The sum of the numbers in the second, or tens, column is 6 tens, or 60, which is the second partial sum. Write 60 underneath 27, as shown. The sum of the numbers in the third, or hundreds, column, is 1,500, which is the third partial sum. Write 1500 under the two preceding partial sums as shown. There is only one number in the fourth, or thousands, column, 9, which represents 9,000. Write 9000 as the fourth partial sum under the preceding sums. Adding the four partial sums, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

	425	
	36	
	9215	
	4	
	907	
first partial sum	27	
second partial sum	60	
third partial sum	1500	
fourth partial sum	9000	
sum	10587	Ans.

26. Mental Addition.—In adding a column containing, for instance, the figures 5, 6, 1, 9, 7, 5, 2, 4, 8, 9 it is permissible, before one becomes familiar with the process of addition, to proceed as follows: 5 and 6 is 11; 11 and 1 is 12; 12 and 9 is 21; and so forth, until all the figures in the column are included in the sum.

After one becomes more experienced the addition should be performed, mentally, in such a manner that only the successive sums are recorded in the mind, as the eye momentarily rests on one figure after another. Thus, the addition of the preceding figures would be: 5, 11, 12, 21, 28, 33, 35, 39, 47, 56.

This abbreviated method of adding may appear more difficult in the beginning, but it will pay to persevere until one becomes fully familiar with it. Adding aloud should be avoided under all circumstances.

EXAMPLES FOR PRACTICE

1. Find the sums of the following numbers:

	Ans.
(*a*) 104+203+613+214.	(*a*) 1,134
(*b*) 1,875+3,143+5,826+10,832.	(*b*) 21,676
(*c*) 4,865+2,145+8,173+40,084.	(*c*) 55,267
(*d*) 14,204+8,173+1,065+10,042.	(*d*) 33,484

2. Four castings have the following weights: 3,265, 1,092, 748, and 2,587 pounds (abbreviated lb.). What is their combined weight? Ans. 7,692 lb.

3. The monthly output of a shop manufacturing a line of small tools was as follows: January, 8,502; February, 8,748; March, 9,215; April, 9,770; May, 10,269; June, 12,184. What was the total output in the six months? Ans. 58,688

4. The number of pounds of coal burned in a power plant each day during a week was as follows: Monday, 1,800; Tuesday, 1,655; Wednesday, 1,725; Thursday, 1,690; Friday, 1,648; and Saturday, 1,020. How much coal was burned during the week? Ans. 9,538 lb.

5. A piece of land has three sides, which are respectively 375 feet (abbreviated ft.), 980 feet, and 760 feet long. What is the length of the fence that will be needed to enclose it? Ans. 2,115 ft.

6. During the first week of the month a mill received supplies valued at 3,475 dollars; the supplies furnished during the second, third, and fourth weeks were worth 2,950, 4,380, and 4,895 dollars, respectively. What was the total value of the supplies received for the month? Ans. 15,700 dollars

7. One day's report of a small creamery operating four stations shows the weight, in pounds, of milk and cream received, as follows:

	Sta. 1	Sta. 2	Sta. 3	Sta. 4
Milk	2,833	2,718	3,054	2,967
Cream	1,376	1,271	1,515	1,334

How many pounds of milk and how many pounds of cream were received by the creamery on that day?

Ans. 11,572 lb. of milk
5,496 lb. of cream

8. On a three-day trip a ship covers 360 miles on the first day, 362 miles on the second day, and 359 miles on the third day; what was the total distance covered? Ans. 1,081 miles

9. A pump operated 2 hours and 45 minutes to empty a tank filled with water. The meter readings showed that 4,200 gallons (abbreviated gal.) were removed during the first hour, 5,420 during the second hour, and 3,600 during the last 45 minutes. How many gallons of water did the tank contain originally? Ans. 13,220 gal.

10. A heating, ventilating, and plumbing firm completes three contracts. The payment received on the first contract was 2,560 dollars, on the second 3,125 dollars, and on the third 2,850 dollars. How much was received in all? Ans. 8,535 dollars

HORIZONTAL ADDITION

27. Explanation of Method.—Very often examples are met with that require crosswise addition of numbers standing in different columns or on a line with one another, as well as the ordinary up-and-down addition of numbers arranged in vertical columns. Because this involves picking out from each number in succession the single figure desired, while retaining in mind the partial total already obtained, besides being a method that is unfamiliar, this kind of addition is likely to cause difficulty unless practice is given to it and the closest attention paid to accuracy.

To illustrate the process required, take the following numbers to be added as they stand, without arrangement in columns: $123+567+792+221+546=2,249$.

First take the right-hand figure of each number in turn. beginning at the right and ignoring all the other figures. Add as in ordinary addition, thus obtaining 7, 9, 16, 19. Place the 9 as the units figure in the sum and add the tens. Be very careful about picking out the tens figure of each number without losing sight of the partial total in mind. Thus, 1 (carried), 5, 7, 16, 22, 24. Put down 4 and carry 2. Pick out the hundreds and add them; 2 (carried), 7, 9, 16, 21, 22.

Care must be taken to keep in mind the kind of figure that is being added, whether tens, hundreds, etc., as it is easy to err and add the tens figure of one column, for instance, to the hundreds figure of the next, if care is not exercised. Practice will make it easier to choose the right figures.

The following example shows the usual grouping of numbers that require horizontal addition:

Example.—Add the following numbers crosswise, then add the results:

			Totals
29,680	56,318	73,267	159,265
9,297	89,219	54,298	152,814
76,351	34,876	47,695	158,922
2,987	73,187	47,187	123,361
29,864	69,785	39,284	138,933
37,279	11,567	36,684	85,530
59,812	71,091	29,345	160,248
67,677	64,597	55,641	187,915
45,328	99,873	67,298	212,499
87,875	62,144	76,541	226,560

Grand total, 1,606,047

28. Proof.—To prove results in horizontal addition, add each column vertically, then add the sums of the vertical columns. The result should be the same as the sum of the horizontal totals. In the example just given, the total of the first column is 446,150, that of the second 632,657, and that of the third 527,240. The sum of these three totals is 1,606,047, the same as that of the horizontal totals.

Since it is so easy to make mistakes in examples of this kind it is well worth while to verify results in every case; in fact, the student should accustom himself in his practice work to prove his results in this manner.

29. Practical Example.—The advantages derived from the ability to perform horizontal addition may be seen from the succeeding table, which is an example of a great number of similar ones required in government statistical work. The table is supposed to give, by days, the grain exports, in bushels, of a certain city for 1 week; it is desired to find the totals in both directions and the grand total. These totals represent the amount of grain exported each day, the total amount of each kind of grain exported during the week, and, finally, the total amount of grain exported during the week.

Grain Export of a City For 1 Week, in Bushels

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total
Corn	28,325	15,236	35,715	29,128	75,183	46,217	******
Wheat	35,719	41,719	50,108	32,546	59,275	81,126	******
Oats	12,136	9,237	18,265	7,268	6,950	17,230	******
Barley	18,230	15,738	21,375	15,928	19,263	13,637	******
Rye	5,275	6,829	7,201	11,325	7,825	13,261	******
Totals	*****	*****	*****	*****	*****	******	******

The student should find the totals, and prove that the results are correct by adding the totals in the right-hand column, and then adding the totals in the bottom row; the two results should be the same, or 757,270 bushels. The other results are: Corn, 229,804; wheat, 300,493; oats, 71,086; barley, 104,171; rye, 51,716; Mon., 99,685; Tues., 88,759; Wed., 132,664; Thurs., 96,195; Fri., 168,496; Sat., 171,471.

EXAMPLES FOR PRACTICE

1. Find the sum of each of the following columns, then add them crosswise and at last add the results of the horizontal addition:

(*a*)	(*b*)	(*c*)
4568	15431	7386
7391	29685	45371
7854	73648	13764
53469	34519	9887
13470	78234	64348
58143	7843	14627

Ans.: (*a*) 144,895; (*b*) 239,360; (*c*) 155,383; grand total, 539,638.

2. Add the following numbers crosswise, then add the results:

(*a*)	(*b*)	(*c*)	
49850	6542	62165	
17370	63834	16732	
68429	76343	85696	
23156	80931	71883	
21017	79883	50149	
67154	83578	31572	
64353	35647	76844	

Grand total, 1,133,128

SUBTRACTION

30. Definitions.—The process of **subtraction** is just the opposite of that of addition. Instead of combining two numbers to find their value taken together, to *subtract* is to take away a specified number of units from a given number and find out how many units remain. Thus, 9 and 7 taken together make 16. Now, if the operation is reversed to find how many units are left after taking 9 units from 16 units, the process is subtraction. As it is known that to add 9 to 7 gives 16, it is clear that to take the 9 away from 16 leaves 7.

In subtraction but two numbers can be used at a time, and the smaller number is taken from the larger in every case.

The number to be reduced is called the **minuend;** the one to be taken away, the **subtrahend;** the number left after the subtraction is performed, the **difference.** The sign of subtraction is $-$, read *minus.* $12-7$ is read *twelve minus seven,* and means that 7 is to be taken from 12.

31. Method of Subtraction.—The manner in which subtraction is carried out is illustrated in the following solutions:

Example.—From 7,568 take 3,425.

Solution.—The larger number is written above the smaller number, and a line is drawn below them. The remainder is placed below this line, thus:

minuend 7568
subtrahend 3425
difference 4143 Ans.

Explanation.—Begin at the right-hand, or units, column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

32. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are less than the figures directly under them in the subtrahend, proceed as in the following example:

Example 1.—From 8,453 take 844.

Solution.—

$$\begin{array}{rr} \textit{minuend} & 8453 \\ \textit{subtrahend} & 844 \\ \hline \textit{remainder} & 7609 \text{ Ans.} \end{array}$$

Explanation.—Begin at the right-hand, or units, column to subtract. As 4 cannot be taken from 3, it is necessary to take 1 from 5 in the tens column and prefix it to the 3 in the units column. The 1 taken from the tens column is equal to 10 units, which added to the 3 in the units column gives a sum of 10+3=13 units. 4 from 13=9, which is the first, or units, figure in the remainder.

As 1 was taken from 5, only 4 remains; then, 4 from 4=0, which is the second, or tens, figure. As 8 cannot be taken from 4, it is necessary to take 1 from 8 in the thousands column. Since 1 thousand=10 hundreds, it follows that 10 hundreds+4 hundreds=14 hundreds, and that 8 from 14 leaves 6 as the third, or hundreds, figure in the remainder.

As 1 was taken from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder, thus completing the answer.

The operation of taking 1 away from one of the figures in the minuend is performed by mentally placing 1 before the figure following the one from which it is taken. For example, the 1 taken from 5 is placed before 3, making it 13, from which 4 is subtracted. The 1 taken from 8 is placed before 4, making 14, from which 8 is subtracted.

Example 2.—Out of 306 pieces of work inspected, 14 were rejected as being too small. How many pieces were passed by the inspector?

Solution.—The number of pieces passed by the inspector is the difference between 306 and 14. The subtraction is as follows:

$$\begin{array}{r} 306 \\ 14 \\ \hline 292 \text{ Ans} \end{array}$$

Explanation.—The first step is to subtract 4 from 6, leaving 2 as a remainder, which is set down. Now, 1 cannot be taken from 0; so, 1 is taken from the 3 and brought over to add to the 0. But, the 1 taken from 3 has a value 10 times as great as it would have in the position of the cipher, according to Arts. **11** and **12**. Therefore, 10 is added to the cipher, making 10, and the 3 becomes decreased to 2. The minuend might then be written as in the following:

$$\begin{array}{rrr} & {\scriptstyle 10} & \\ 2 & 0 & 6 \\ & 1 & 4 \\ \hline 2 & 9 & 2 \end{array}$$

The remainder of the solution is easy; 1 from 10 leaves 9, and nothing from 2 leaves 2. Hence, 292 pieces were passed by the inspector. It is not customary to rewrite the minuend as here shown; instead, the taking of 1 from the 3 is done mentally.

Example 3.—From a stock of 20,000 small machine parts 8,763 were used. How many remained?

Solution.—To find how many remained, subtract the number used from the total stock, thus:

$$\begin{array}{rl} 20000 & \\ 8763 & \\ \hline 11237 & \text{Ans.} \end{array}$$

Explanation.—As 3 cannot be taken from 0 in the units column, the attempt is made to borrow 1 from the tens column, so as to obtain 10 units. But there are no tens, and on going, successively, to the hundreds column and to the thousands column, a cipher is found in each case. It is not until the ten-thousands place is reached that a number greater than zero is found—2 in this case. It is now necessary to take 1 ten-thousand, or 10 thousands, from the 2 and add it to the 0 thousands, giving 10+0=10 thousands; but it is necessary to continue the borrowing and to take 1 thousand, or 10 hundred, away from the 10 thousands and add it to the hundreds column, leaving a remainder of 10−1=9 thousands. Adding 10 hundreds to 0 hundreds gives a sum of 10 hundreds. But 1 hundred, or 10 tens, must be taken away from the hundreds to add to 0 tens, leaving 10−1=9 hundreds.

Adding 1 hundred, or 10 tens, to 0 tens, the sum is 10 tens. Finally, 1 ten, or 10 units, must be taken from the 10 tens to be added to the 0 unit, giving 10 units as the sum. The minuend might now be written as in the following:

$$\begin{array}{rrrrr} & & & & {\scriptstyle 10} \\ 1 & 9 & 9 & 9 & 0 \\ & 8 & 7 & 6 & 3 \\ \hline 1 & 1 & 2 & 3 & 7 \end{array}$$

Then, 3 from 10 leaves 7; 6 from 9 leaves 3; 7 from 9 leaves 2; 8 from 9 leaves 1; and nothing from 1 leaves 1. Usually the taking of

1 unit from the several places is done mentally, and is not written down as shown.

33. Rule for Subtraction.—Using the preceding explanations as a basis the following rule may be formed:

Rule.—*When subtracting, write the subtrahend under the minuend with units under units, tens under tens, etc., and draw a horizontal line beneath the subtrahend.*

Subtract units of the subtrahend from units of the minuend, tens from tens, etc., writing the remainders beneath the line in the order in which they are obtained.

If any figure in the minuend represents a smaller number than the figure beneath, borrow 1 from the preceding number and add 10 to the smaller number; then subtract the lower number from the sum.

34. Proof.—*To prove an example in subtraction, add the remainder to the subtrahend. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of example 3, Art. **32.**

$$\begin{array}{r} 8763 \\ 11237 \\ \hline 20000 \end{array}$$

35. General Remarks on Subtraction.—The student should practice finding the differences between small numbers, until he is able to name with ease and rapidity the difference between any two numbers less than ten. A special subtraction table is not required for this purpose, as the addition table can be referred to in case of necessity. For instance, according to the table the sum of 7 and 6 is 13. It follows, that if either one of the numbers 7 and 6 is subtracted from 13, the other number must be equal to the remainder. Thus, on subtracting 6 from 13, the remainder is 7. If 7 is subtracted from 13, then 6 is the remainder. In the same manner, it is found that 8 from 16 leaves 8, 9 from 13 leaves 4.

It will be a great help to the student if he at odd moments, while walking or working, will mentally reverse the addition table and use it as a subtraction table. For instance, instead of saying to himself, 6 and 5 is 11, let him ask, 6 from 11 is how much? 9 from 17 is what? etc., and in a short time the right answer will present itself without any mental effort whatever.

EXAMPLES FOR PRACTICE

1. From:

(*a*) 94,278 take 62,574.	Ans.	(*a*) 31,704
(*b*) 53,714 take 25,824.		(*b*) 27,890
(*c*) 71,832 take 58,109.		(*c*) 13,723
(*d*) 20,804 take 10,408.		(*d*) 10,396

2. A shop employing 3,214 hands was forced to lay off 736 of them. What number was left? Ans. 2,478

3. A casting weighing 2,785 pounds was poured from a ladle containing 4,210 pounds of metal. How many pounds remained in the ladle to be used for other castings? Ans. 1,425 lb.

4. If an electric meter registers 7,968 watt-hours at one reading, and 10,430 at the next reading, how many watt-hours were used by the customer between the two readings? Ans. 2,462 watt-hours

NOTE.—A watt-hour is the unit by which electric power is measured.

5. On January 1 a power plant had 19,860 tons of coal on hand. During the entire month 3,100 tons were consumed; how much coal was on hand February 1? Ans. 16,760 tons

6. The total weight of a mine car loaded with coal is 4,326 pounds. If the empty car weighs 1,564 pounds, what is the weight of the coal?
Ans. 2,762 lb.

7. In setting out the boundary line of a piece of land, it is found that at one point of the line the distance from the starting point is 385 feet. At another point of the same line the distance from the starting point is 1,065 feet. What is the distance between the two points? Ans. 680 ft.

8. On completing a piece of work it was found that out of a stock of 1,037 pounds of pipe fittings there remained a quantity of 259 pounds. How many pounds were used? Ans. 778 lb.

9. During a certain month, a textile mill purchased wool valued at 5,642 dollars and cotton costing 3,834 dollars. How much greater was the expenditure for wool than for cotton? Ans. 1,808 dollars

10. The distance between New York City and Liverpool, England, is 3,166 nautical miles. If a ship sailing for England has covered 1,275 miles of this distance, how far is it from Liverpool?

Ans. 1,891 miles

NOTE.—A nautical mile differs from a statute mile. The former contains 6,080 feet, the latter only 5,280 feet.

MULTIPLICATION

36. Definitions.—Multiplication is merely a shortened process of addition, used when the sum of a group of equal numbers is to be found. To **multiply** a number is to *add* it to itself a certain number of times. For instance, suppose the sum is required that results from using the number 4 three times in the process of addition. By the method of addition it is easy to see that 4+4+4=12. But, suppose it is desired to take the number 13,976 ninety-nine times. It would be a long, tedious process to find the desired result by addition, so the need of a shorter method is imperative. Multiplication meets this need. By its means it is possible to find the result of taking any number as many times as desired.

37. The number that is to be added to itself, or the number to be multiplied, is called the **multiplicand.**

The number that shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The sign of multiplication is ×. It is read *times* or *multiplied by*. Thus 9×6 is read *9 times 6*, or *9 multiplied by 6*.

38. The result obtained by multiplying is called the **product.** Thus, 40×8=320 is read *40 multiplied by 8 equals 320*, or *40 times 8 equals 320*. In this example, 40 is the multiplicand, 8 the multiplier, and 320 the product.

The product of several numbers is the same, whatever may be the order in which they are multiplied; thus, 4×3×2=24, 4×2×3=24, and 3×4×2=24, etc.

The two or more numbers that, when multiplied together produce a product, are said to be the **factors** of that product.

Thus, 2, 3, and 4 are factors of 24; 5, 6, and 3 are factors of 90; and so on.

39. Practicing Multiplication.—In Table II, the product of any two numbers (neither of which exceeds 12) may be found. The table should be carefully committed to memory. This is especially important, as successful future work depends largely on the student's ability to name promptly these products in any order. Study one section at a time until the product of any pair of numbers can be given at once; then go to the next section. Review the sections repeatedly.

Any number multiplied by 1 gives a product equal to the number itself, as indicated in the first line of each section of the table.

40. Rule for Multiplication.—To multiply numbers larger than those given in Table II proceed according to the following rule and examples:

Rule.—I. *When multiplying, write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right, and multiply the multiplicand by the figures of the multiplier, taken in succession, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will be the required product.*

41. To Multiply by a Number of One Figure Only. The application of the preceding rule to multiplication by one figure will be shown by the following example and solution:

Example.—Multiply 425 by 5.

Solution.—According to part I of the rule, the multiplier 5 is written under the figure 5 of the multiplicand. On looking in the multiplication table, it is seen that 5×5=25, of which the figure 5 is written in the product directly under 5 in the multiplier and 2 is *carried*, or added to the product of 5×2; thus, 5×2+2=12, of which the figure 2 is written in the product and 1 is carried, that is,

$$\begin{array}{rrl} \textit{multiplicand} & 425 & \\ \textit{multiplier} & 5 & \\ \cline{2-2} \textit{product} & 2125 & \text{Ans.} \end{array}$$

added to the product of 5×4. Thus, $5\times4=20$ and $20+1=21$, both figures being written in the product because all the figures of the multiplicand have been used. The work is now complete and the product is 2,125.

This result can also be obtained by adding 425 five times; thus,

$$\begin{array}{rr}
 & 425 \\
 & 425 \\
 & 425 \\
 & 425 \\
 & 425 \\ \cline{2-2}
\textit{sum} & 2125 \text{ Ans.}
\end{array}$$

42. To Multiply by a Number of Two or More Figures.—The following example and solution shows the application of the rule, Art. **40,** to cases in which the multiplier has two or more figures:

Example.—Multiply 475 by 234.

Solution.—Proceeding according to part II of the rule, $4\times5=20$, of which 0 is written in the first partial product and 2 is carried; $4\times7=28$ and $28+2=30$, of which 0 is written as the next figure of the first partial product and 3 is carried; $4\times4=16$ and $16+3=19$, which completes the first partial product, 1,900. Using the next figure of the multiplier, $3\times5=15$, of which 5 is written directly under 3 as the first figure of the second partial product and 1 is carried; $3\times7=21$ and $21+1=22$, of which 2 is written in the second partial product and 2 is carried; $3\times4=12$ and $12+2=14$, which completes the second partial product, 1,425. Using the last figure, 2, of the multiplier, $2\times5=10$, of which 0 is written directly under 2 as the first figure of the third partial product and 1 is carried; $2\times7=14$ and $14+1=15$, of which 5 is written as the second figure of the third partial product and 1 is carried, or added, to the product of 2×4, or 8, making 9, which figure completes the third partial product, 950. According to part III of the rule, the sum of the three partial products gives the total product, 111,150.

$$\begin{array}{rcccccc}
\textit{multiplicand} & & & & 4 & 7 & 5 \\
\textit{multiplier} & & & & 2 & 3 & 4 \\ \cline{5-7}
\textit{first partial product} & & & 1 & 9 & 0 & 0 \\
\textit{second partial product} & & 1 & 4 & 2 & 5 & \\
\textit{third partial product} & & 9 & 5 & 0 & & \\ \cline{2-7}
\textit{product} & 1 & 1 & 1 & 1 & 5 & 0 \text{ Ans.}
\end{array}$$

43. Proof.—*To prove the correctness of multiplication, review the work carefully, or multiply the multiplier by the multiplicand. If the same product is obtained, the work is probably correct.*

TABLE II

MULTIPLICATION TABLE

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3	4 times 1 is 4
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6	4 times 2 is 8
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9	4 times 3 is 12
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12	4 times 4 is 16
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15	4 times 5 is 20
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18	4 times 6 is 24
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21	4 times 7 is 28
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24	4 times 8 is 32
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27	4 times 9 is 36
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30	4 times 10 is 40
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33	4 times 11 is 44
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36	4 times 12 is 48
5 times 1 is 5	6 times 1 is 6	7 times 1 is 7	8 times 1 is 8
5 times 2 is 10	6 times 2 is 12	7 times 2 is 14	8 times 2 is 16
5 times 3 is 15	6 times 3 is 18	7 times 3 is 21	8 times 3 is 24
5 times 4 is 20	6 times 4 is 24	7 times 4 is 28	8 times 4 is 32
5 times 5 is 25	6 times 5 is 30	7 times 5 is 35	8 times 5 is 40
5 times 6 is 30	6 times 6 is 36	7 times 6 is 42	8 times 6 is 48
5 times 7 is 35	6 times 7 is 42	7 times 7 is 49	8 times 7 is 56
5 times 8 is 40	6 times 8 is 48	7 times 8 is 56	8 times 8 is 64
5 times 9 is 45	6 times 9 is 54	7 times 9 is 63	8 times 9 is 72
5 times 10 is 50	6 times 10 is 60	7 times 10 is 70	8 times 10 is 80
5 times 11 is 55	6 times 11 is 66	7 times 11 is 77	8 times 11 is 88
5 times 12 is 60	6 times 12 is 72	7 times 12 is 84	8 times 12 is 96
9 times 1 is 9	10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
9 times 2 is 18	10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
9 times 3 is 27	10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
9 times 4 is 36	10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
9 times 5 is 45	10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
9 times 6 is 54	10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
9 times 7 is 63	10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
9 times 8 is 72	10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
9 times 9 is 81	10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
9 times 10 is 90	10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
9 times 11 is 99	10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
9 times 12 is 108	10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

44. Ciphers as Multipliers.—When there is a cipher in the multiplier, multiply by it the same as with the other figures. Thus,

```
 (a)        (b)        (c)         (d)
  0          2         15          708
 ×0         ×0        × 0         ×  0
  0  Ans.    0  Ans.   00  Ans.    000  Ans.
```

```
  (e)
  3114
   203
  9342
 0000
6228
632142 Ans.
```

```
    (f)
   4008
    305
  20040
  0000
12024
1222440 Ans.
```

```
    (g)
   31264
    1002
   62528
  00000
 00000
31264
31326528 Ans.
```

45. When multiplying by a number containing one or more ciphers, the work may be shortened by simply writing the first cipher of a partial product in its proper place and the next partial product alongside it. For example, the solutions to examples (*e*) and (*g*) in the preceding article may be written as follows:

```
  3114             31264
   203              1002
  9342             62528
62280            3126400
632142 Ans.      31326528 Ans.
```

46. If there are ciphers at the right-hand end of the multiplier, they need not be used in the multiplication, but may be carried down to the product.

EXAMPLE.—Multiply 2,675 by 3,900.

SOLUTION.—

```
  2675
    3900
 24075
 8025
10432500 Ans.
```

EXPLANATION.—In a case like this, the multiplier is written so that the ciphers at its right extend to the right of the units figure in the multiplicand. These two ciphers are brought down vertically to the end of the product of 2,675 and 39.

47. Ciphers in the Multiplicand.—If there are ciphers at the end of the multiplicand, the procedure is similar in all respects to that explained in Art. **46;** thus, to multiply 4,907,600 by 487 proceed as follows:

```
    4907600
        487
  ---------
    343532
   392608
  196304
  ---------
  2390001200  Ans.
```

If both multiplicand and multiplier end in ciphers, place the right-hand digits, or figures that are not ciphers, under each other, as above, and add to the product a number of ciphers equal to the sum of the ciphers contained in the multiplicand and the multiplier on the right of their right-hand digits.

Example.—Multiply 590,000 by 420.

Solution.—

```
     590000
     420
  ---------
    118
   236
  ---------
   247800000  Ans.
```

Explanation.—In this case there are, in all, 5 ciphers on the right of the right-hand digits in the multiplicand and the multiplier. Therefore, there must be 5 ciphers on the right of the right-hand digit in the product.

EXAMPLES FOR PRACTICE

1. Find the product of the following:

	Ans.
(*a*) 61,483×6.	(*a*) 368,898
(*b*) 12,375×5.	(*b*) 61,875
(*c*) 4,836×47.	(*c*) 227,292
(*d*) 3,257×246.	(*d*) 801,222
(*e*) 2,875×302.	(*e*) 868,250
(*f*) 17,819×1,004.	(*f*) 17,890,276

2. A certain machine is capable of turning out 48 finished pieces of work in a day. At this rate, find the output of the machine in one year of 296 working days. Ans. 14,208

3. What is the total weight of cast iron in 649 pumps, if the amount of iron in each pump weighs 37 pounds? Ans. 24,013 pounds

4. A ranchman owns 945 head of cattle, valued at 117 dollars a head; how much are they all worth? Ans. 110,565 dollars

5. A certain mill contains 1,830 looms, each of which produces 175 yards (abbreviated yd.) of cloth per week; what is the total production? Ans. 320,250 yd.

6. A machine turns out 50 finished pieces of work in a day. What would be the output of the machine in one year of 306 working days? Ans. 15,300 pieces

7. In a month of 30 days, how many miles can be run by a ship that averages 432 miles per day? Ans. 12,960 miles

8. A water-tube boiler containing 144 tubes needs to be fitted with new tubes. Assuming the cost for each tube to be 9 dollars, what will be the total cost for the 144 tubes? Ans. 1,296 dollars

NOTE.—A *water-tube* boiler is one in which the water is *in* the tubes, which are surrounded by the hot gases of combustion. In a *fire-tube* boiler the water *surrounds* the tubes, and the hot gases pass through the interior of the latter.

9. If 40,635,000 acres of wheat produce, on an average, 13 bushels to the acre, what is the total yield? Ans. 528,255,000 bushels

10. A power station has on hand 38 barrels of lubricating oil; the average contents of a barrel is 50 gallons. What is the total quantity on hand? Ans. 1,900 gallons

DIVISION

48. Definitions.—Division is the process of finding how many times one number is contained in another; or, it is the process of separating a number into a given number of equal parts.

For instance, a brass rod, 32 inches long, is to be divided into short pieces, 2 inches in length. This is an example of finding how many times one number, in this case 2, is contained in another number, here 32. Or, suppose that 150 dollars is to be divided equally among 5 men; how much will each man receive? Here it is a question of separating a number, as 150, into a number of equal parts, corresponding to the number of men.

49. The **dividend** is the number to be divided, or to be separated into equal parts.

The **divisor** is the number by which the dividend is divided.

The **quotient** is the number showing how many times the dividend contains the divisor.

The **sign of division** is ÷. It is read *divided by*. Thus, 54÷9 denotes that 54 is to be divided by 9. Another way to write 54 divided by 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$.

In both of these cases 54 is the *dividend*, 9 is the *divisor*, and 6 is the *quotient*.

Division is the reverse of multiplication, as the latter process, instead of *separating* a number into a number of equal parts, or factors, *combines* a number of equal parts into a complete whole.

NOTE.—The student should study Table II again until when any product given therein and one of its factors are named he can immediately name the other factor. A good plan is to cover the second column of factors with a strip of paper or cardboard and recall them from memory. In carrying out these tests the dividends should not be taken in order, but selected at random; otherwise the answer will be known without testing one's ability in division.

50. Rule for Division.—To divide numbers larger than those given in Table II, proceed according to the following rule and examples:

Rule.—I. *Write the divisor at the left of the dividend, with a curved line between them.*

II. *Begin at the left-hand end of the dividend and find how many times the divisor is contained in the least number of figures that will contain it, and write the result for the first figure of the quotient, separating it from the dividend by a curved line.*

III. *Multiply the divisor by this quotient; subtract the product from the partial dividend used, and to the remainder annex the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there is a remainder after the last figure of the dividend has been used, draw a straight line after the quotient and write this remainder above the line and the divisor below.*

51. To Divide When the Divisor Consists of But One Figure.—Proceed as in the following example:

Example.—What is the quotient of 861÷7?

Solution.—The dividend and the divisor are written according to part I of the rule in Art. **50,** and a place is provided for the quotient at the right of the dividend. On application of part II of the rule, it is found that of the figures at the left-hand end of the dividend, the one figure 8 is sufficient to contain the divisor 7. As 7 is contained 1 time in the figure 8, the figure 1 is written as the first figure of the quotient. According to part III of the rule, 1×7=7, which product is written under 8 and subtracted, leaving 1, to which is annexed the next figure, 6, of the dividend, giving the new partial dividend 16. In this dividend 7 is contained 2 times, and 2 is written after 1 in the quotient. The product 2×7=14 is written under 16 and subtracted, leaving 2, to which is annexed the next figure, 1, of the dividend, giving the second partial dividend 21. In this dividend 7 is contained 3 times, and 3 is written after 2 in the quotient. 3×7=21, which is written under 21 and subtracted, leaving 0. All the figures of the dividend have now been used and the division is complete, the quotient being 123.

```
                 divisor  dividend  quotient
                        7)861(123  Ans.
                          7
                          -
partial dividend          16
                          14
                          --
     partial dividend      21
                           21
                           --
                            0
```

52. To Divide When the Divisor Consists of Two or More Figures.—The rule in Art. **50** applies also in this case, its application being illustrated by means of the following example:

Example.—Divide 2,702,839 by 63.

Solution.—Applying part II of the rule, Art. **50,** it is necessary to find the least number of figures of the dividend that will contain the divisor. As 63 is not contained in 2 nor in 27, the first three figures of the dividend, that is, 270, must be used. To determine how many times 63 is contained in 270, it is noted that 6 is contained in 27 4 times with a remainder, and 4 is tried as the first figure of the quotient. As the product 4×63=252 is found to be smaller than 270, it is retained and subtracted from the latter, giving the remainder 18. Annexing the next figure, 2, of the dividend to this remainder, the first partial dividend 182 is obtained. It is found that 63 is contained in 182 2 times with a remainder; so 2 is written as the second figure of the quotient. At first thought, it might seem that this figure should be 3, because 18÷6=3; but 3×63=189, a number larger than 182; so 3 is too large for the second figure of the quotient. 2×63=126, which is written under 182 and subtracted. To the remainder, 56, the next figure, 8, of the dividend is annexed, giving 568 as a new partial dividend. As 6 is contained in 56 9 times with a remainder, 63 may be assumed to be contained 9 times in 568, and 9 is accordingly written as the third figure of the quotient. 9×63=567, which is written under 568 and subtracted. To the remainder, 1, the next figure, 3, of the dividend is annexed, giving 13, but 63 is not contained in 13. According to part IV of the rule, Art. **50,** 0 is written in the quotient, and the next figure, 9, of the dividend is brought down, making 139 as a new partial dividend. 139÷63=2, with 13 remaining. Since this is the last remainder, all figures of the dividend having been used, 13 is written above a straight line after the quotient with the divisor underneath, according to part V of the rule.

divisor *dividend* *quotient*
63)2702839(42902$\frac{13}{63}$ Ans.
252
partial dividend 182
126
partial dividend 568
567
partial dividend 139
126
remainder 13

53. Long Division and Short Division.—When the process of division is carried out in full, as in the preceding examples, it is called **long division;** when shortened, or abridged, by performing some of the work mentally instead of writing it, the process is called **short division.** Short division is preferable when the divisor is not greater than 12, but should not be attempted before one is fully familiar with long division.

In short division, only the divisor, the dividend, and the quotient are written, the operations being performed mentally. The method followed will be explained by the succeeding example.

Example.—Divide 992 by 8.

Solution.—The divisor is placed to the left of the dividend, as in long division, but the quotient is placed under the dividend, and separated from it by a line, as shown. The mental operation is performed as follows: The divisor 8 is contained in 9 1 time with 1 remaining. This remainder is conceived to be written immediately before the next figure, 9, of the dividend, as shown by 1 in small type, and 8 is contained in 19 2 times with 3 remaining. This remainder is conceived to be written before the next figure, 2, of the dividend, and 8 is contained in 32 4 times with no remainder. This completes the division, giving the quotient 124.

divisor dividend
$8\,)\,9^{1}9^{3}2$
quotient 124 Ans.

54. Proof.—*To prove division, multiply the quotient by the divisor, and add the remainder, if there is any, to the product. The result will be the dividend.* Thus, the correctness of the work in the example, Art. **52,** can be proved, as shown, by multiplying 42,902 by 63 and adding 13. The result is equal to the dividend.

quotient	42902
divisor	63
	128706
	257412
	2702826
remainder	13
dividend	2702839

EXAMPLES FOR PRACTICE

1. Divide the following:

	Ans.
(*a*) 126,498 by 58.	(*a*) 2,181
(*b*) 3,207,594 by 767.	(*b*) 4,182
(*c*) 11,408,202 by 234.	(*c*) 48,753
(*d*) 2,100,315 by 581.	(*d*) 3,615

2. A lot of castings weigh 11,060 pounds. If they are alike, and one weighs 28 pounds, how many are there in the lot? Ans. 395

3. If the driving shaft of a machine makes 9,730 turns in 35 minutes, how often does it turn in 1 minute? Ans. 278 times

4. If sound, under certain conditions, travels at the rate of 1,118 feet per second (abbreviated sec.), what time will be required for it to travel 23,478 feet? Ans. 21 sec.

5. A steamship company pays 6,120 dollars monthly wages to 85 firemen, each receiving the same amount. How much does each fireman receive per month? Ans. 72 dollars

6. There are 5,280 feet in a mile. How many rails would it take to lay a double row of rails, each row 1 mile long, if the length of a rail is 30 feet? Ans. 352 rails

7. How many electric lamps at 30 cents each can be purchased for 90 cents? Ans. 3 lamps

8. If 16 carloads of coal weigh 46,336 pounds, what is the weight per carload, assuming each car to contain the same quantity? Ans. 2,896 lb.

9. A mill purchased cotton to the value of 12,600 dollars, at 42 dollars per bale; how many bales were purchased? Ans. 300 bales

10. If 30 water tanks of equal size contain 25,500 gallons, when all are filled, how many gallons does each tank contain? Ans. 850 gal.

COMBINATIONS OF ARITHMETICAL OPERATIONS

55. Order of Operations.—The various signs used in arithmetical operations, such as +, −, ×, and ÷, are known as **symbols.** Several numbers may be connected by various symbols, so as to constitute a row, or a series, as, for instance, $4+8-3\times2+10\div5$. Such a combination of numbers and signs is called an **expression.** This expression indicates that the product obtained by multiplying 3 by 2 is to be subtracted from the sum found by adding 8 to 4. To the resulting difference is to be added the quotient obtained by dividing 10 by 5.

To insure correct results it is necessary that the operations indicated by the symbols in this and similar expressions should be performed in the following order:

1. Multiplication.
2. Division.
3. Addition or subtraction.

Applying this method to the preceding series, the process of multiplication is performed first; then, division. Thus, $3\times2=6$, and $10\div5=2$. The series may now be written $4+8$

$-6+2$. Performing the operations indicated by the symbols from left to right the final result is 8.

56. Symbol of Aggregation.—The ordinary parenthesis, ( ), is used also in arithmetical operations. It is then called a **symbol of aggregation,** the word aggregation meaning combination.

A parenthesis indicates that the numbers enclosed within it are to be considered as a whole with respect to any preceding or following arithmetical symbol. It is important to note that the operations indicated by the sign or signs within the parenthesis should be performed first. Thus, in the expression $13\times(8-3)$ the numbers contained within the parenthesis are considered as a whole, and the process of subtraction, indicated by the symbol, is to be performed before multiplication takes place. That is, 3 is to be taken from 8 before multiplying by 13; thus, $13\times(8-3)=13\times5=65$.

When the parenthesis is not used, then the multiplication is to be performed before the subtraction, as explained in Art. **55.** Thus, $13\times8-3=104-3=101$.

Again, $2\times(8-3)$ means $2\times5=10$. On removing the parenthesis, thus, $2\times8-3$, multiplication precedes subtraction. Hence, $2\times8=16$, and $16-3=13$.

57. Examples.—The application of the preceding rules will be further illustrated by the following examples:

Example 1.—Find the value of the expression $4\times24-8+17$.

Solution.—Performing the operations indicated according to Art. **55,** $4\times24=96$; $96-8=88$; $88+17=105$. Ans.

Example 2.—Find the value of the expression $1{,}296\div12+160-22\times3$.

Solution.—Multiplication and division must be performed before subtraction can take place. Thus, $22\times3=66$; $1{,}296\div12=108$. The expression may now be written $108+160-66$. Performing the remaining operations from left to right: $108+160=268$; $268-66=202$. Ans.

Example 3.—Solve the following expression: $(26-4)\times(16+4)$.

Solution.—First perform the operations indicated within the parenthesis. Thus, $(26-4)=22$, and $(16+4)=20$; then, $22\times20=440$. Ans.

Example 4.—Solve the following expression: $(26-4)\times16+4$.

Solution.—First remove the parenthesis by performing the operation indicated within; thus, $(26-4)=22$. Next remove the multiplication sign by performing the required operation; thus, $22\times16=352$. Finally, $352+4=356$. Ans.

NOTE.—Comparison should be made between the answers obtained in the two preceding examples. The only difference in the examples is the omission of a parenthesis in example 4; yet, there is a difference in the results of $440-356=84$.

EXAMPLE 5.—Solve the following expression: $5\times4-21\div7-(15-5+4)$.

SOLUTION.—The parenthesis is removed first by carrying out the operation indicated by the signs within it. Thus, subtracting 5 from 15 leaves 10, to which is added 4, thus obtaining 14. The same result may be obtained by adding 4 to 15, thus obtaining 19, and subtracting 5. Next, the signs of multiplication and division must be removed by performing the required operations. Thus, $5\times4=20$; $21\div7=3$. The expression now has the following form: $20-3-14$. Performing the two subtractions, $20-3=17$, and $17-14=3$. Ans.

EXAMPLES FOR PRACTICE

Find the values of the following expressions:

(*a*)	$(8+5-1)\div4$.	Ans.	(*a*) 3
(*b*)	$5\times24-32$.		(*b*) 88
(*c*)	$5\times24\div15$.		(*c*) 8
(*d*)	$144-5\times24$.		(*d*) 24
(*e*)	$2{,}080+120-80\times4-1{,}670$.		(*e*) 210
(*f*)	$(90+60)\div(2\times5)$.		(*f*) 15

SIMPLE ARITHMETICAL EQUATIONS

58. Definitions.—An **equation** is an expression in which two equal *quantities* are connected by an equality sign ($=$). The term **quantity** in this case refers to a single number or to several numbers connected by arithmetical symbols. For example, the number 8 is a quantity, and so is $5+3$. These two quantities are equal, and if they are connected by an equality sign, as $5+3=8$, an equation is obtained. Examples of other equations are: $6+2+1=9$; $4+5+7=10+6$. They are all called **simple equations.**

59. Solution of Equations.—In an equation there is usually a missing number or quantity the value of which has to be found. The position of the missing quantity may be indicated by a question mark (?) or by a letter; for this purpose the letter x is generally used. Following are some examples of simple equations and the methods followed in finding the missing quantity:

EXAMPLE 1.—Find the value of the missing number belonging on the right side of the equality sign in the equation $5+2=?$

SOLUTION.—The sum of $5+2$ is equal to 7; therefore, 7 is the value of the missing number.

EXAMPLE 2.—What is the value of x in the equation $9+x=15$.

SOLUTION.—It is required to find a number, which added to 9 gives a sum equal to 15. If 9 and the unknown number make a sum of 15, it is evident that by subtracting 9 from 15 the difference will give the number required. Thus, $15-9=6$ and $x=6$. Ans.

EXAMPLE 3.—Find the value of x in the equation $5\times x=40$.

SOLUTION.—It is required to find a number, that multiplied by 5 gives a product equal to 40. If the latter product is obtained by multiplying a number by 5, it is evident that the quotient found by dividing 40 by 5 must be the number required. Thus, $40\div 5=8$, and $5\times 8=40$; therefore, $x=8$. Ans.

EXAMPLE 4.—Find the value of x in the equation $x\div 5=6$.

SOLUTION.—It is required to find a number which, when divided by 5 will give the quotient 6. It follows that the product of 5 and 6, which is 30, must be the value of x. Thus, $5\times 6=30$, and $30\div 5=6$. The value of x is 30. Ans.

60. Incorrect Use of the Equality Sign.—In showing the several steps of a calculation, the equality sign is sometimes used in an incorrect manner, because it is made to connect expressions that are not of equal value. For instance, it is incorrect to use an equality sign as in the following example:

$$200\times 4=800\div 25=32$$

The expression is apparently an equation, but on closer examination it is found that this is not the case. The product $200\times 4=800$, found on one side of the equality sign, is not equal to the quotient 32, obtained by dividing 800 by 25. Hence, the equality sign is out of place and is misleading. The two operations indicated should be written separately, as, for example:

$$200\times 4=800$$
$$800\div 25=32$$

EXAMPLES FOR PRACTICE

Find the value of x in the following equations:

(*a*)	$4 \times x = 12$	Ans.	(*a*) 3	(*e*)	$x - 5 = 12$	Ans.	(*e*) 17
(*b*)	$x + 12 = 16$		(*b*) 4	(*f*)	$16 \div x = 4$		(*f*) 4
(*c*)	$7 + x = 14$		(*c*) 7	(*g*)	$36 \div x = 3$		(*g*) 12
(*d*)	$9 - x = 7$		(*d*) 2	(*h*)	$x \div 8 = 5$		(*h*) 40

FRACTIONS

FUNDAMENTAL PROCESSES

REDUCTION OF FRACTIONS

INTRODUCTION

1. Fractions in General.—A **fraction** is one or more of the equal parts into which a whole thing, or unit, is divided. The unit may be anything, as a circle, a dollar, a mile, a plot of land, etc. In Fig. 1 (*a*) is shown a circle divided into two equal parts by a straight line, with one of the parts shaded. Each of these parts is a half of the whole circle; that is, each is a fraction of a circle. When the circle is divided into two equal parts, as shown, each part, or fraction, is *one-half*, which is written $\frac{1}{2}$.

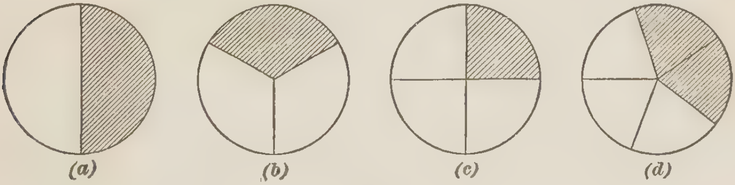

FIG. 1

Another circle is shown in (*b*), divided into three equal parts. Each of these equal parts is a third, or *one-third*, of the entire circle. This fraction is written $\frac{1}{3}$. The circle in (*c*) is divided into four equal parts, each of which is a fourth, or *one-fourth*, of the whole circle. The fraction *one-fourth* is written $\frac{1}{4}$. The circle in (*d*) is divided into five equal parts, each of which is

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one-fifth of the whole circle. This fraction is written $\frac{1}{5}$. Two of the equal parts in (*d*) are shaded; consequently, the part of the whole circle that is shaded is *two-fifths*, which is written $\frac{2}{5}$. Similarly, in (*b*), there are two parts not shaded, so that the part or fraction not shaded is *two-thirds*, written $\frac{2}{3}$. In (*c*), three of the equal parts are not shaded; consequently, the fraction of the circle that is not shaded is *three-fourths*, or $\frac{3}{4}$.

2. Numerator and Denominator.—From what has just been stated it must be plain that two numbers are needed to write a fraction. The numbers are written one above the other, with a line between them, and each has its own name. A fraction expressed by means of two numbers, one over the other, separated by a line is known as a **common fraction,** to distinguish it from a *decimal fraction*, which will be described in a succeeding Section. The number above the line is called the **numerator** of the fraction and the number below the line is called the **denominator** of the fraction. Every common fraction must have a numerator and a denominator. The denominator shows how many equal parts a thing is divided into, and the numerator shows how many of those equal parts are taken, or considered.

For example, in Fig. 1 (*a*) the shaded part of the circle is $\frac{1}{2}$ of the circle. In the fraction $\frac{1}{2}$, the numerator is 1 and the denominator is 2. The denominator 2 shows that the circle is divided into two equal parts, and the numerator 1 shows that one of those parts is considered. In (*d*), the fraction of the circle that is shaded is $\frac{2}{5}$. The denominator 5 of the fraction shows that the circle is divided into five equal parts, and the numerator 2 shows that the two parts that are shaded are considered. If the unshaded part had been considered, the numerator would have been 3, and the fraction would have been $\frac{3}{5}$, because three of the five equal parts are not shaded.

The numerator and the denominator of a fraction are called the **terms** of a fraction.

3. Effect of Denominator on Value of Fraction.—The larger the denominator of a fraction, the smaller is the fraction, the numerator being the same. This may easily be shown by referring to Fig. 1 (*b*) and (*d*). The fraction represented by one of the equal parts in (*b*) is $\frac{1}{3}$ and that represented by one of the equal parts in (*d*) is $\frac{1}{5}$. The denominator 5 is greater than the denominator 3, but the fraction $\frac{1}{5}$ is smaller than the fraction $\frac{1}{3}$. This can be seen by comparing one of the equal parts in (*d*) with one in (*b*). One of the parts in (*d*), or $\frac{1}{5}$ of the circle, is much smaller than one of the parts in (*b*), or $\frac{1}{3}$ of the circle. Hence, if the numerators of two fractions are equal, the fraction with the smaller denominator is the greater. Thus, of the two fractions $\frac{3}{10}$ and $\frac{3}{4}$, the latter is the greater. But if the denominators are equal, the one with the larger numerator is the greater. Take $\frac{3}{5}$ and $\frac{4}{5}$, for example; in this case the denominators are equal, and $\frac{4}{5}$ is greater than $\frac{3}{5}$ because 4 is greater than 3.

To illustrate the point more fully, it may be supposed that the circle represents some object, such as a cheese or a large cake weighing 60 pounds. Then,

One-half weighs $60 \div 2 = 30$ pounds
One-third weighs $60 \div 3 = 20$ pounds
One-fourth weighs $60 \div 4 = 15$ pounds
One-fifth weighs $60 \div 5 = 12$ pounds
One-sixth weighs $60 \div 6 = 10$ pounds
One-tenth weighs $60 \div 10 = 6$ pounds

Therefore, the larger the denominator of the fraction the smaller is its value. It was said before that $\frac{3}{10}$ is less than $\frac{3}{4}$. In the case of the object weighing 60 pounds, one-tenth weighs 6 pounds, and $\frac{3}{10}$, which is three times $\frac{1}{10}$, weighs $3 \times 6 = 18$ pounds; one-fourth weighs 15 pounds, and $\frac{3}{4}$ weighs $3 \times 15 = 45$ pounds. We thus see in another way that $\frac{3}{10}$ weighs less than $\frac{3}{4}$, or $\frac{3}{10}$ is less than $\frac{3}{4}$.

4. Division Expressed by a Fraction.—A fraction may also be used to express division; for example, $4 \div 5$ may be written $\frac{4}{5}$, which is a fraction; and similarly, $3 \div 16$ may be written $\frac{3}{16}$. The **value** of a fraction is the result obtained by dividing the numerator by the denominator.

5. Proper Fractions and Improper Fractions.—If the numerator of a fraction is less than the denominator, the fraction is called a **proper fraction**; thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{16}$, $\frac{3}{10}$ are proper fractions. If the numerator is equal to or greater than the denominator, the fraction is an **improper fraction;** thus, $\frac{2}{2}$, $\frac{5}{5}$, $\frac{16}{12}$, $\frac{12}{4}$, $\frac{28}{7}$ are improper fractions.

6. Prime Numbers and Composite Numbers.—A **prime number** is one that cannot be divided by any number except itself and 1, without a remainder; thus, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc. are prime numbers.

Any number not a prime number is called a **composite number,** and may be considered as the product of two or more prime numbers. Thus, 60 is a composite number, and is equal to the product of the prime numbers 2, 2, 3, and 5; or, $2\times2\times3\times5=60$.

7. Whole Numbers and Mixed Numbers.—A **whole number** is a number that does not contain a fraction. For example, 2, 36, 185, 4,063 are whole numbers. A **mixed number** is a number composed of a whole number and a fraction united. For example, $3\frac{3}{8}$ is a mixed number, being composed of a whole number 3 and a fraction $\frac{3}{8}$. This number is read *three and three-eighths*. It is equal to $3+\frac{3}{8}$, but for convenience the plus sign is omitted in writing it and it appears simply as $3\frac{3}{8}$. The mixed number $10\frac{5}{16}$ is read *ten and five-sixteenths*. A whole number is very frequently called an **integer.** It is also occasionally referred to as an *integral number*.

8. Fractions Smaller or Greater Than 1.—A fraction whose numerator and denominator are equal has a value of 1; thus, $\frac{4}{4}=1$, $\frac{2}{2}=1$, $\frac{8}{8}=1$. If the numerator is less than the denominator, the value of the fraction is less than 1; thus, the value of each of the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{16}$ is less than 1. If the numerator of the fraction is larger than the denominator, the value of the fraction is more than 1; thus, $\frac{12}{2}=6$, $\frac{10}{5}=2$, $\frac{3}{2}=1\frac{1}{2}$.

SIMPLE REDUCTION

9. Same Fraction in Different Forms.—The form of a fraction may be changed without changing its value; or, in other words, a fraction may be expressed in several ways. The form chosen depends on the number of equal parts into which a thing is divided. This may easily be understood by referring to Fig. 2, which shows three circles of equal size. It is supposed that one-half of each circle is to be taken, but that the circles

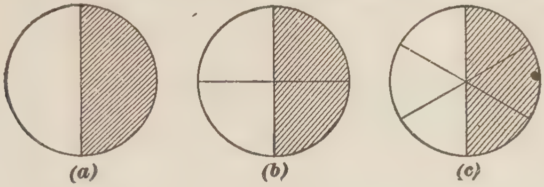

FIG. 2

are not divided into the same number of equal parts. The circle in (*a*) is divided into two equal parts, one of which is shaded; that is, the part shaded is $\frac{1}{2}$ of the whole circle. The circle in (*b*) is divided into four equal parts, two of which are shaded; that is, the shaded portion is $\frac{2}{4}$ of the whole. The circle in (*c*) is divided into six equal parts, three of which are shaded; that is, the shaded portion is $\frac{3}{6}$ of the whole. But, as the three circles are of the same size, and the same amount is shaded in each, it follows that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ must be equal to one another, because each one represents the same amount, or one-half of the circle.

10. That fractions of equal value may be expressed in different forms may be further illustrated by Fig. 3, which is an oblong divided into equal spaces. There are 4 squares in a row and there are 6 rows, making 24 squares in all. Hence, each square is 1 twenty-fourth of the whole. As there are 6 rows, one row is 1 sixth of the whole. From this it is seen that 4 twenty-fourths equals 1 sixth, or writing this in figures,

$$\frac{4}{24}=\frac{1}{6}$$

Since 4 twenty-fourths equals 1 sixth, 8 twenty-fourths equals 2 sixths, 12 twenty-fourths equals 3 sixths, and 20 twenty-fourths equals 5 sixths. It is also seen that 3 rows are equal to 3 sixths; but 3 rows are equal to 3×4 = 12 squares, which is equal to one-half of the oblong. It follows, then, that

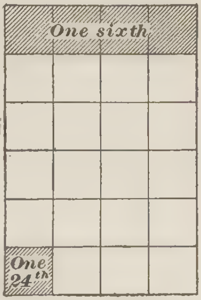

FIG. 3

$$\frac{3}{6}=\frac{1}{2}$$

In each strip running up and down the figure there are 6 squares; but, as there are 4 strips, each strip is 1 fourth of the figure; hence, 6 twenty-fourths equals 1 fourth, or

$$\frac{6}{24}=\frac{1}{4}$$

In two strips, or one-half of the figure, there are 12 squares; hence,

$$\frac{12}{24}=\frac{2}{4}=\frac{1}{2}$$

If the figure is divided into 12 equal parts there will be 2 squares in each part; then,

$$\frac{2}{24}=\frac{1}{12}$$

11. General Principle of Reducing Fractions.—If both terms of a fraction, that is, both numerator and denominator, are multiplied or divided by the same number, the value of the fraction is not changed. For example, suppose that in the fraction $\frac{1}{2}$, both terms are multiplied by 2. Then, $\frac{1\times 2}{2\times 2}=\frac{2}{4}$. The fraction $\frac{2}{4}$ has the same value as $\frac{1}{2}$, as was shown in the preceding article. Again, take $\frac{1}{2}$ and multiply both terms by 3. Then, $\frac{1\times 3}{2\times 3}=\frac{3}{6}$, which has the same value as $\frac{1}{2}$, according to the preceding article. Now take the fraction $\frac{3}{6}$ and divide both terms by 3. Then, $\frac{3\div 3}{6\div 3}=\frac{1}{2}$. But it was shown that $\frac{1}{2}=\frac{3}{6}$, in the preceding article; therefore, by dividing both terms by the same number, the value of the fraction is not changed. This process of changing the form of fractions without changing their values is called **reduction of fractions.**

12. Reducing a Fraction to Higher Terms.—A fraction is reduced to *higher terms* by *multiplying both terms of the fraction by the same number*. For example, $\frac{1}{6}$ is reduced to $\frac{4}{24}$ by multiplying both terms of the fraction by 4. The operation may be written as follows:

$$\frac{1}{6}=\frac{1\times 4}{6\times 4}=\frac{4}{24}$$

That these fractions are equal may be seen from Fig. 3, in which one row is equal to 1 sixth of the whole. Each row contains 4 twenty-fourths; hence, $\frac{1}{6}=\frac{4}{24}$.

As another example, let $\frac{2}{3}$ be changed to $\frac{16}{24}$ by multiplying each term by 8; thus,

$$\frac{2}{3}=\frac{2\times 8}{3\times 8}=\frac{16}{24}$$

If Fig. 3 is divided into three equal parts, and two of the parts are considered, there will be 16 squares in the parts considered, or $\frac{16}{24}$ of the whole figure. This shows that the result just obtained is correct.

13. Reducing a Fraction to Lower Terms.—A fraction is reduced to *lower terms* by *dividing both terms by the same number*. For example, $\frac{20}{24}$ is reduced to $\frac{5}{6}$ by dividing both terms by 4; thus,

$$\frac{20}{24}=\frac{20\div 4}{24\div 4}=\frac{5}{6}$$

In Art. **10** it was shown that 5 sixths was equal to 20 twenty-fourths, thus proving that the preceding reduction is correct.

14. A fraction is reduced to its *lowest terms* when both its numerator and its denominator cannot be divided by the *same* number without a remainder; for example, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, $\frac{8}{15}$ are fractions reduced to their lowest terms.

EXAMPLE 1.—Reduce $\frac{12}{16}$ to its lowest terms.

SOLUTION.—By trial find the greatest number that will divide 12 and 16 without a remainder. This number is 4.

$$\frac{12}{16}=\frac{12\div 4}{16\div 4}=\frac{3}{4}. \quad \text{Ans.}$$

EXAMPLE 2.—Reduce $\frac{8}{10}$ to its lowest terms.

SOLUTION.—The number that will divide 8 and 10 without a remainder is 2.

$$\frac{8}{10}=\frac{8\div 2}{10\div 2}=\frac{4}{5}. \quad \text{Ans.}$$

15. Reducing a Fraction to One of Equal Value and With a Given Denominator.—In practice, it is often required to change a given fraction to another one of equal value, but with a different denominator. In such cases the following rule applies:

Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the result.*

EXAMPLE 1.—Reduce $\frac{7}{8}$ to an equal fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will, in this case, make the product 96; this number is evidently $96\div 8=12$, because $8\times 12=96$. Hence,

$$\frac{7\times 12}{8\times 12}=\frac{84}{96}. \quad \text{Ans.}$$

Example 2.—Reduce $\frac{3}{4}$ to 100ths· that is, to a fraction having 100 for a denominator.

Solution.— $100 \div 4 = 25$; hence,

$$\frac{3\times 25}{4\times 25}=\frac{75}{100}. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Reduce the following:

(*a*)	$\frac{7}{16}$ to 128ths.	Ans.	(*a*) $\frac{56}{128}$
(*b*)	$\frac{24}{132}$ to its lowest terms.		(*b*) $\frac{2}{11}$
(*c*)	$\frac{64}{1000}$ to its lowest terms.		(*c*) $\frac{8}{125}$
(*d*)	$\frac{5}{7}$ to 49ths.		(*d*) $\frac{35}{49}$
(*e*)	$\frac{13}{16}$ to 10,000ths.		(*e*) $\frac{8125}{10000}$

16. Reducing a Whole Number or a Mixed Number to an Improper Fraction.—The process of reducing a whole number or a mixed number to an improper fraction is of importance as it is used to a great extent in the multiplication and division of fractions. The following rule applies:

Rule.—*To reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the denominator of the given fraction, and write the result over the denominator.*

Example 1.—Reduce 5 to an improper fraction having 4 as a denominator.

Solution.—There are 4 fourths in 1, because $\frac{4}{4}=1$, and in 5 there will be 5×4 fourths, or 20 fourths; that is, $5\times\frac{4}{4}=\frac{20}{4}$. Ans.

Example 2.—Reduce $8\frac{3}{4}$ to an improper fraction.

Solution.—According to the rule,

$$8\tfrac{3}{4}=\frac{8\times 4+3}{4}=\frac{32+3}{4}=\frac{35}{4}. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.
(*b*) $5\frac{3}{16}$.
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{4}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.

Ans. $\begin{cases} (a)\ \frac{33}{8} \\ (b)\ \frac{83}{16} \\ (c)\ \frac{102}{10} \\ (d)\ \frac{151}{4} \\ (e)\ \frac{254}{5} \\ (f)\ \frac{112}{16} \end{cases}$

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17. Reducing an Improper Fraction to a Whole or to a Mixed Number.—The result obtained in the multiplication or division of fractions is in many cases in the form of an improper fraction. Preferably, the answer should be in the form of a mixed number. An improper fraction is reduced to a mixed number by applying the following rule:

Rule.—*To reduce an improper fraction to a whole or a mixed number, divide the numerator by the denominator and write the result as in ordinary division.*

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.— 4 is contained in 21, 5 times with 1 as a remainder. The latter number is written as the numerator of a fraction with 4 as a denominator. This fraction is added to the whole number. Therefore, $5+\frac{1}{4}$, or $5\frac{1}{4}$, is the mixed number required. Ans.

EXAMPLES FOR PRACTICE

Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.
(*b*) $\frac{185}{3}$.
(*c*) $\frac{701}{6}$.
(*d*) $\frac{149}{3}$.
(*e*) $\frac{76}{19}$.
(*f*) $\frac{125}{25}$.

Ans. $\begin{cases} (a)\ 24\frac{1}{6} \\ (b)\ 61\frac{2}{3} \\ (c)\ 116\frac{5}{6} \\ (d)\ 49\frac{2}{3} \\ (e)\ 4 \\ (f)\ 5 \end{cases}$

PROCESSES PREPARATORY TO ADDITION AND SUBTRACTION OF FRACTIONS

FINDING LEAST COMMON DENOMINATOR

18. Adding Fractions With Different Denominators. Fractions cannot be added unless they have the same denominator. If the denominators are not the same the fractions must be changed into such forms that they have the same denominator. For instance, $\frac{3}{4}$ may be added to $\frac{1}{4}$, but $\frac{3}{4}$ cannot be added to $\frac{7}{8}$ as the fractions now stand, because the denominators are different; fourths cannot be added to eighths. The reason for this may be seen from the following example:

Suppose that an apple is divided into 4 equal parts, and that 2 of these are each divided into 2 equal parts. It is evident that there are 2 one-fourths and 4 one-eighths. If these parts are added, the sum is 6. But what does this sum represent? It cannot be fourths, for 6 fourths is equal to $\frac{6}{4}=1\frac{1}{2}$, and only 1 apple was divided; neither can it be eighths; for $\frac{6}{8}=\frac{3}{4}$ is less than 1 apple. By reducing the quarters to eighths, there are $\frac{2}{4}=\frac{4}{8}$; on adding these to the other 4 eighths, there are $4+4=8$ eighths. This result must be correct, since $\frac{8}{8}=1$. Another way would be to combine the eighths into quarters. Thus, $\frac{4}{8}=\frac{2}{4}$; then, adding the quarters, there are $2+2=4$ quarters. This result is also correct, since $\frac{4}{4}=1$.

19. Common Denominator.—Several fractions are said to have a **common denominator** when all their denominators are the same, as $\frac{1}{11}$, $\frac{4}{11}$, $\frac{7}{11}$, $\frac{10}{11}$. Two or more fractions having different denominators can be reduced to others having a common denominator by reducing each fraction to higher or lower terms. For example, $\frac{1}{2}$ and $\frac{1}{3}$ can each be reduced to sixths; thus, $\frac{1}{2}=\frac{3}{6}$, because $\frac{1\times3}{2\times3}=\frac{3}{6}$; also $\frac{1}{3}=\frac{2}{6}$, because $\frac{1\times2}{3\times2}=\frac{2}{6}$. Likewise, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$ can each be reduced to twelfths; thus, $\frac{1}{2}=\frac{6}{12}$, $\frac{1}{3}=\frac{4}{12}$, and $\frac{3}{4}=\frac{9}{12}$.

20. Least Common Denominator.—Fractions with different denominators may have as a common denominator any number that will contain each of the denominators *without a* *remainder;* for example, the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ can be reduced to twelfths, twenty-fourths, thirty-sixths, or to fractions having as a common denominator any number that will contain 2, 3, and 4 without a remainder. But the **least common denominator** is the *least* number that may be divided by each denominator of the given fractions without a remainder. The following example will illustrate the meaning of this definition:

Let it be supposed that the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are to be reduced to fractions that have the same denominator. As 2, 3, and 4 can be divided into 12 without a remainder, 12 is taken as the common denominator. Thus, $12 \div 2 = 6$, then $\frac{1 \times 6}{2 \times 6} = \frac{6}{12}$; similarly, $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$, and the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ therefore become $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$. The number 24 could also be used as a common denominator, because it can be divided by 2, 3, or 4 without a remainder. The three fractions would then become $\frac{12}{24}$, $\frac{16}{24}$, and $\frac{18}{24}$, and these values would be the same as $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$. But the point to be noticed is that although 12 and 24 can both be used as common denominators for these three fractions, 12 is the *least* common denominator, because it is the *smallest* number that can be divided by 2, 3, and 4 without leaving a remainder. A series of fractions may have a great many different common denominators, but they can have only *one* least common denominator. In adding or subtracting, fractions are generally reduced to the *least* common denominator. The methods by which fractions are reduced to the least common denominator will be explained in detail further on.

21. Finding the Least Common Denominator by Inspection.—The least common denominator of several fractions may be found either by inspection or by calculation. The method of finding it by inspection will be explained by means of an example taken from practice. For this purpose the foot rule, generally used in English-speaking countries for taking measurements of length, will be considered.

A foot rule is divided into equal parts called *inches*, and each inch is further divided into equal parts called fractions of an inch. A part of a rule showing a common way of dividing the inch is illustrated in Fig. 4. The long line *a* at the middle of the inch divides it into halves. At the middle of the halves are shorter lines *b* and *c* that divide the half inches into halves, making quarter inches, and the quarter inches are still further divided by the lines *d*, *e*, *f*, and *g* into eighths. Finally, the shortest lines divide the eighths, so that there are sixteen equal divisions in 1 inch, and these are called sixteenths. Suppose that the rule is used to measure the length of a block *h*. The end of the rule is put in line with the end of the block, and the other end of the block comes just to the line *a*, which is the half-inch mark; therefore, the block is said to be $\frac{1}{2}$ inch long.

22. Now, suppose that the length of the block *h*, Fig. 4, is to be found in quarters, eighths, or sixteenths of an inch. By looking at the rule, it is seen that there are just two quarter inches between the line *a* and the end of the rule; so the length of the block is two quarter inches or two-fourths of an inch, which is written $\frac{2}{4}$ inch. This is exactly equal to $\frac{1}{2}$ inch, because $\frac{1\times2}{2\times2}=\frac{2}{4}$. If the length of the block is expressed in eighths of an inch, it is $\frac{4}{8}$ inch long, because there are four eighth-inch divisions between the line *a* and the end of the rule. Also, the block is $\frac{8}{16}$ inch long, because there are eight sixteenth-inch divisions from the line *a* to the end of the rule. This simply shows that $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}$. Sometimes rules are divided into thirty-seconds and sixty-fourths of an inch.

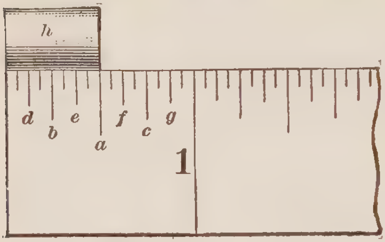

Fig. 4

If a rule similar to Fig. 4 is also divided into thirty-seconds and sixty-fourths of an inch, the fractions considered will be halves, fourths, eighths, sixteenths, thirty-seconds, and sixty-fourths. When a common denominator is to be found for several such fractions, all that is necessary is to take the greatest denominator in the group. For example, in the fractions $\frac{5}{16}$, $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{1}{2}$ the largest denominator is 16, and this is chosen as the least common denominator. Then, $\frac{3}{4}=\frac{12}{16}$, $\frac{7}{8}=\frac{14}{16}$, and $\frac{1}{2}=\frac{8}{16}$. Again, suppose that the fractions are $\frac{27}{64}$, $\frac{3}{8}$, $\frac{7}{32}$, and $\frac{1}{4}$. The largest of these denominators is 64, which is taken as the least common denominator, and then $\frac{3}{8}=\frac{24}{64}$, $\frac{7}{32}=\frac{14}{64}$, and $\frac{1}{4}=\frac{16}{64}$. It is therefore a very simple matter to choose the least common denominator for fractions of an inch as found by a rule divided in the manner explained. This method of finding the least common denominator by merely looking at the denominators and finding the common denominator mentally at once, or after a few trials, is called finding the least common denominator by **inspection.**

23. Finding the Least Common Denominator by Calculation.—It is not always possible to find the least common denominator by inspection as in the preceding article. For this reason, the following method is given, showing how to find by calculation the least common denominator of *any* set of fractions. It is important that this method should be thoroughly understood. The method will be explained by applying it directly to the following examples:

EXAMPLE 1.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—The denominators of the fractions are placed in a line, as in short division, and separated by commas. In this case they are 4, 3, 9, and 16, as shown. The denominators are now divided by some prime number other than 1 that will be contained in at least two of the numbers without a remainder. The number 2 is contained in 4 and 16; hence, dividing by 2 and placing the quotients in the second line, under the numbers divided, the quotients 2 and 8 are obtained. The numbers 3 and 9, which will not contain the divisor without a remainder, are transferred to the second line, as shown.

$$\begin{array}{r|rrrr} 2 & 4, & 3, & 9, & 16 \\ \hline 2 & 2, & 3, & 9, & 8 \\ \hline 3 & 1, & 3, & 9, & 4 \\ \hline & 1, & 1, & 3, & 4 \end{array}$$

$2\times2\times3\times3\times4=144$, the least common denominator. Ans.

Next, another prime number is found that will be contained in at least two of the numbers in the second line. The divisor 2 is again selected, as being contained in the numbers 2 and 8. The quotients 1 and 4 are written in the third line, as well as the numbers not divisible by 2, in this case 3 and 9. The divisor 3 is chosen for the third line, giving the quotients 1 and 3. In the fourth line there will then appear the numbers 1, 1, 3, 4, as shown.

These operations are repeated until the last line contains no two numbers that can be divided by the same prime number. In this case the numbers 3 and 4 remain, which cannot both be divided by any one prime number. Finally, the divisors and the remaining numbers in the last line are multiplied together, the product being the least common denominator. In the example the product is $2\times2\times3\times1\times1\times3\times4=144$, which is the least common denominator. Ordinarily the prime numbers 1 are omitted as being without effect on the final result.

Example 2.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

Solution.—

$$\begin{array}{r|rrr} 3 & 9, & 12, & 18 \\ \hline 3 & 3, & 4, & 6 \\ \hline 2 & 1, & 4, & 2 \\ \hline & 1, & 2, & 1 \end{array}$$

$3\times3\times2\times2=36$. Ans.

24. Reducing Fractions to Equivalent Fractions Having the Least Common Denominator.

—The preceding article explained the method of finding the least common denominator of several fractions. There remains yet to be shown how a number of given fractions may be reduced to equivalent fractions having the least common denominator. Reductions of this kind are performed in accordance with the following rule:

Rule.—*To reduce fractions to the least common denominator, divide the least common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

Example 1.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having the least common denominator.

Solution.—The least common denominator is first found as follows:

$$\begin{array}{r|rrr} 2 & 3, & 4, & 2 \\ \hline & 3, & 2, & 1 \end{array}$$

Then, $2\times3\times2=12$, the least common denominator

In this example, each of the three fractions must be changed to a form in which it will have 12 for its denominator. In the first fraction, $\frac{2}{3}$, the denominator is 3. By the rule, the least common denominator is to be divided by the denominator of the fraction; or, $12\div3=4$. Both terms of the fraction are then to be multiplied by this quotient, 4; that is, $\frac{2\times4}{3\times4}=\frac{8}{12}$. The fraction to be considered next is $\frac{3}{4}$, whose denominator is 4.

Following the same method, 12÷4=3, and $\frac{3\times 3}{4\times 3}=\frac{9}{12}$. The third fraction, $\frac{1}{2}$, has 2 for a denominator. Then, 12÷2=6, and $\frac{1\times 6}{2\times 6}=\frac{6}{12}$. Hence, the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$, when reduced to the least common denominator, become $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{6}{12}$. Ans.

Example 2.—Reduce $\frac{13}{16}$, $\frac{5}{7}$, and $\frac{11}{12}$ to the least common denominator.

Solution.—The least common denominator is found first, as follows:

$$\begin{array}{r|lll} 2 & 16, & 7, & 12 \\ \hline 2 & 8, & 7, & 6 \\ \hline & 4, & 7, & 3 \end{array}$$

Least common denominator = 2×2×4×7×3 = 336

In the case of the fraction $\frac{13}{16}$, 336÷16=21, and $\frac{13\times 21}{16\times 21}=\frac{273}{336}$. In the case of $\frac{5}{7}$, 336÷7=48, and $\frac{5\times 48}{7\times 48}=\frac{240}{336}$. Finally, in the case of $\frac{11}{12}$, 336÷12 =28, and $\frac{11\times 28}{12\times 28}=\frac{308}{336}$. Therefore, the three fractions reduced to the least common denominator are $\frac{273}{336}$, $\frac{240}{336}$, and $\frac{308}{336}$. Ans.

EXAMPLES FOR PRACTICE

Reduce to fractions having the least common denominator:

		Ans.
(*a*)	$\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.	(*a*) $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$
(*b*)	$\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.	(*b*) $\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$
(*c*)	$\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.	(*c*) $\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$
(*d*)	$\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.	(*d*) $\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$
(*e*)	$\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.	(*e*) $\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$
(*f*)	$\frac{7}{15}$, $\frac{17}{30}$, $\frac{7}{10}$.	(*f*) $\frac{14}{30}$, $\frac{17}{30}$, $\frac{21}{30}$

ADDITION OF FRACTIONS

25. Rule for Addition of Fractions.—The preceding rules and exercises referring to finding the least common denominator are to be considered as preparatory processes that must be performed before addition of fractions can take place. The reasons for providing fractions with the same denominator before adding them have been given in Art. 18, but it may be well to emphasize the point again at this place. For instance, if a workman measures off the thicknesses of three pieces of material as $\frac{1}{16}$ inch, $\frac{5}{16}$ inch, and $\frac{7}{16}$ inch, respectively, he knows that the total thickness is $\frac{13}{16}$ inch. This result is obtained by adding the numerators, thus, 1+5+7=13, and placing that sum over the common denominator, 16, giving $\frac{13}{16}$. But if the thicknesses measured off are, for instance, $\frac{1}{4}$ inch, $\frac{5}{8}$ inch, and $\frac{7}{16}$ inch, the total thickness cannot be found by adding the numerators, because the fractions have different denominators. Before the fractions can be added they must be changed to forms in which they have the same denominator, that is, a common denominator. After they have been reduced to this form the process of addition is a very simple one. The following rule applies to the addition of common fractions:

Rule.—I. *To add fractions, first reduce them to a common denominator. Then add the numerators, and write their sum over the common denominator. If the result is an improper fraction, reduce it to a mixed number and reduce the fractional part to its lowest terms.*

II. *To add whole and mixed numbers, add the whole numbers and the fractions separately, and then add the results.*

The application of this rule will be shown in the following examples:

Example 1.—Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{12}$.

Solution.— $\frac{1}{2}+\frac{3}{4}+\frac{5}{12}=\frac{6}{12}+\frac{9}{12}+\frac{5}{12}=\dfrac{6+9+5}{12}=\frac{20}{12}=1\frac{8}{12}=1\frac{2}{3}$. Ans.

Part I of the rule applies. The least common denominator is readily seen to be 12, the new numerators are 6, 9, and 5, and their sum is 20. The rest of the work is reduction, as previously explained. The sum of the fractions is $\frac{20}{12}$, but as the latter is an improper fraction, it must, according to part I of the rule, be reduced to a mixed number, that is, $1\frac{8}{12}$. The fractional part, $\frac{8}{12}$, should, according to the same part of the rule, be reduced to its lowest terms. It is seen that it may be so reduced by dividing numerator and denominator by 4, the reduced fraction being $\frac{2}{3}$. The final form of the mixed number is $1\frac{2}{3}$, as shown.

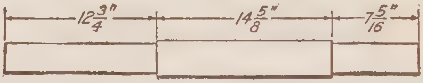

Fig. 5

Example 2.—What is the total length, or overall length, of the piece of shafting shown dimensioned in Fig. 5?

Note.—The marks (″) used on the drawing, mean *inches;* thus, $12\frac{3}{4}''$ means $12\frac{3}{4}$ inches.

Solution.—The total length is the sum of the three parts, measuring $12\frac{3}{4}$ inches, $14\frac{5}{8}$ inches, and $7\frac{5}{16}$ inches. Then, applying part I of the rule, the fractions are reduced to a common denominator. The least common denominator of the three fractions is 16. Part II of the rule is now applied, and the whole numbers and the fractions are added separately. The sum of the fractions is $\frac{27}{16} = 1\frac{11}{16}$; the sum of the whole numbers is 33. The sum of both, which is the total, or overall, length of the shaft, is $34\frac{11}{16}$ inches.

$$\begin{aligned} 12\tfrac{3}{4} &= 12\tfrac{12}{16} \\ 14\tfrac{5}{8} &= 14\tfrac{10}{16} \\ 7\tfrac{5}{16} &= 7\tfrac{5}{16} \end{aligned}$$

$$\textit{sum} = 33 + \tfrac{27}{16} = 33 + 1\tfrac{11}{16} = 34\tfrac{11}{16}. \text{ Ans.}$$

Example 3.—A workman has five pieces of belt whose lengths are $9\frac{1}{2}$, $6\frac{1}{4}$, $3\frac{3}{4}$, $3\frac{1}{3}$, and $1\frac{5}{6}$ feet, respectively, and he needs a belt 24 feet long for some special work. Can he make up a belt of the required length by lacing the pieces together, end to end?

Solution.—The least common denominator of the fractions is 12. The fractions reduced to the least common denominator are as follows: $\frac{1}{2} = \frac{6}{12}$; $\frac{1}{4} = \frac{3}{12}$; $\frac{3}{4} = \frac{9}{12}$; $\frac{1}{3} = \frac{4}{12}$; $\frac{5}{6} = \frac{10}{12}$. Therefore, the mixed numbers $9\frac{1}{2} + 6\frac{1}{4} + 3\frac{3}{4} + 3\frac{1}{3} + 1\frac{5}{6}$ are equal to $9\frac{6}{12} + 6\frac{3}{12} + 3\frac{9}{12} + 3\frac{4}{12} + 1\frac{10}{12}$. The sum of the latter numbers is $22\frac{32}{12}$. Reducing the improper fraction $\frac{32}{12}$ to a mixed number, it becomes $2\frac{8}{12}$, in which the fraction may be reduced to $\frac{2}{3}$; thus, $2\frac{8}{12} = 2\frac{2}{3}$. The sum $22 + 2\frac{2}{3} = 24\frac{2}{3}$ is the total length, in feet, of the joined pieces. As the length required is only 24 feet, the pieces can be joined to form the required belt. Ans.

26. Adding Fractions by Means of a Foot Rule.—In practice, it is often required to add dimensions expressed in inches and fractions of an inch. This may be done either by the methods described for the addition of fractions, or by means of a foot rule. To illustrate, let it be supposed that the dimensions $1\frac{1}{2}$, $\frac{5}{8}$, $2\frac{3}{16}$, $\frac{3}{4}$, $\frac{15}{16}$, and $1\frac{3}{8}$ inches are to be added. The several fractions $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{16}$, $\frac{3}{4}$, $\frac{15}{16}$, and $\frac{3}{8}$ are first added, by using a rule graduated in sixteenths of an inch, as shown in Fig. 6. Beginning at the end *a*, count off $\frac{1}{2}$ inch, to the point *b*. From *b*

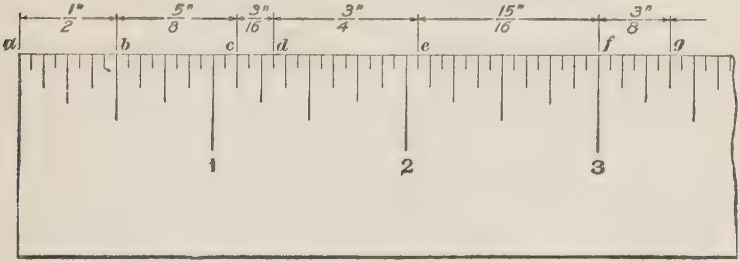

Fig. 6

count off five $\frac{1}{8}$-inch divisions, or $\frac{5}{8}$ inch, to *c*. Next count three $\frac{1}{16}$-inch divisions, or $\frac{3}{16}$ inch, to *d*. From *d* count off three $\frac{1}{4}$-inch spaces, or $\frac{3}{4}$ inch, which is the same as twelve $\frac{1}{16}$-inch divisions, because $\frac{3}{4}=\frac{12}{16}$. This locates the mark *e*. From *e* count off $\frac{15}{16}$ inch, or fifteen $\frac{1}{16}$-inch spaces, to *f*. Finally, from *f* count off three $\frac{1}{8}$-inch divisions, to *g*. The point *g* thus found is at a distance of $3\frac{3}{8}$ inches from *a*; hence, $\frac{1}{2}+\frac{5}{8}+\frac{3}{16}+\frac{3}{4}+\frac{15}{16}+\frac{3}{8}=3\frac{3}{8}$ inches. To this add the whole numbers of inches, 1, 2, and 1, and the total is $3\frac{3}{8}+1+2+1=7\frac{3}{8}$ inches; that is, $1\frac{1}{2}+\frac{5}{8}+2\frac{3}{16}+\frac{3}{4}+\frac{15}{16}+1\frac{3}{8}=7\frac{3}{8}$ inches.

EXAMPLES FOR PRACTICE

1. Find the sum of the following:

(*a*) $\frac{4}{6}, \frac{7}{24}, \frac{5}{8}$.
(*b*) $\frac{2}{3}, \frac{5}{15}, \frac{24}{45}$.

Ans. (*a*) $1\frac{7}{12}$
(*b*) $1\frac{8}{15}$

2. The bolt shown in Fig. 7 has the end threaded for a distance of

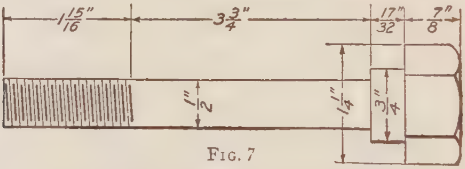

FIG. 7

$1\frac{15}{16}$ inches, is plain for $3\frac{3}{4}$ inches, has a collar $\frac{17}{32}$ inch long, and has a head $\frac{7}{8}$ inch thick. What is the overall length of the bolt? Ans. $7\frac{3}{32}$ inches

3. A helper works $2\frac{3}{4}$ hours on one job, $1\frac{1}{3}$ hours on another, and $\frac{5}{12}$ hour on a third. What is the total time that he worked? Ans. $4\frac{1}{2}$ hours

4. The weights of a number of castings are as follows: $412\frac{3}{4}$ pounds, $270\frac{1}{2}$ pounds, 1,020 pounds, $75\frac{1}{4}$ pounds, and $68\frac{1}{2}$ pounds. What is their total weight? Ans. 1,847 lb.

5. Find the total length of five cuts of cloth, the lengths of the cuts being as follows: $51\frac{1}{4}$ yards, $51\frac{3}{8}$ yards, $55\frac{1}{4}$ yards, $53\frac{5}{8}$ yards, and $48\frac{1}{2}$ yards. Ans. 260 yd.

6. The beam shown in Fig. 8 has rivet holes spaced from left to right at the distances indicated in the drawing. The distances are as follows: $1\frac{15}{16}$ inches, $9\frac{3}{4}$ inches, $6\frac{1}{2}$ inches, $5\frac{3}{8}$ inches, and $2\frac{7}{16}$ inches. What is the length of the beam? Ans. 26 inches

7. A piece of lead pipe work is made up of the following lengths: $39\frac{1}{8}$, $46\frac{1}{4}$, $3\frac{3}{8}$, $9\frac{3}{4}$, and $12\frac{1}{2}$ feet, respectively. What is the total length of the pipe? Ans. 111 ft.

8. During a journey of three days a man travels the following distances: $50\frac{5}{12}$ miles, $56\frac{3}{4}$ miles, and $85\frac{5}{6}$ miles. What is the total length of the journey? Ans. 193 miles

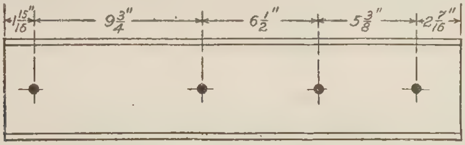

FIG. 8

9. Four castings of the following weights are ordered from a brass foundry: $6\frac{1}{4}$ pounds, $3\frac{1}{2}$ pounds, $5\frac{3}{4}$ pounds, and $8\frac{7}{8}$ pounds. What is the total weight of the castings? Ans. $24\frac{3}{8}$ lb.

10. The stock required for three bolts must be cut from a bar of iron. The finished lengths of the bolts are to be $2\frac{3}{16}$, $1\frac{13}{16}$, and $3\frac{1}{4}$ inches, respectively. Allowing, in all, $\frac{3}{4}$ of an inch of stock for cutting off, for forming heads, and for finishing the ends of the bolts, how long must be the piece of stock? Ans. 8 inches

SUBTRACTION OF FRACTIONS

27. Rule for Subtraction of Fractions.—Reasons were given in Arts. **18** and **25** for the necessity of reducing fractions to a common denominator, before addition can take place. The same reasons hold good in the case of subtraction. For instance, if the fraction $\frac{6}{8}$ were subtracted from $\frac{9}{12}$ by simply finding the differences between the numerators and the denominators, the difference would be $\frac{9}{12}-\frac{6}{8}=\frac{3}{4}$. But $\frac{9}{12}=\frac{3}{4}$ and $\frac{6}{8}=\frac{3}{4}$; therefore, the difference between them cannot be $\frac{3}{4}$, but is instead $\frac{3}{4}-\frac{3}{4}=0$. In other words, *six eighths* cannot be subtracted from *nine twelfths* any more than *six apples* can be subtracted from *nine oranges*. The two fractions must be reduced to a common denominator before one can be subtracted from the other.

The following rule applies to subtraction of fractions:

Rule.—**I.** *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, take 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, take 1 from it, reduce the 1 to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

Example 1.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

Solution.—The least common denominator is 16, and $\frac{3}{8}=\frac{6}{16}$. Then, according to part I of the rule,

$$\frac{13}{16}-\frac{3}{8}=\frac{13}{16}-\frac{6}{16}=\frac{13-6}{16}=\frac{7}{16}. \quad \text{Ans.}$$

Example 2.—From 7 take $\frac{5}{8}$.

Solution.—There is no fraction in the minuend 7, so it is necessary to take 1 from 7 to form a fraction whose denominator is the same as that of the fraction to be subtracted, or 8. As $1=\frac{8}{8}$, it follows that instead of 7, it is possible to write $6+1$, or $6+\frac{8}{8}$, or simply $6\frac{8}{8}$. Then, $6\frac{8}{8}-\frac{5}{8}=6\frac{3}{8}$; that is, $7-\frac{5}{8}=6\frac{3}{8}$.

$$\begin{array}{lr} \textit{minnend} & 7=6\frac{8}{8} \\ \textit{subtrahend} & \frac{5}{8} \\ \hline \textit{remainder} & 6\frac{3}{8} \end{array} \quad \text{Ans.}$$

Example 3.—A block of wood $17\frac{9}{16}$ inches long, as shown in Fig. 9, has a piece *a* $9\frac{15}{32}$ inches long sawed off. What is the length of the remaining piece *b*, neglecting the thickness of the saw cut?

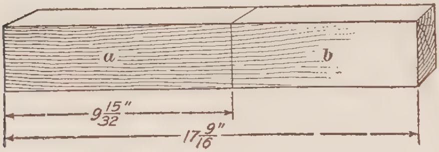

Fig. 9

Solution.—The common denominator of the fractions is 32. $17\frac{9}{16}=17\frac{18}{32}$.

$$\begin{array}{lr} \textit{minuend} & 17\frac{18}{32} \\ \textit{subtrahend} & 9\frac{15}{32} \\ \hline \textit{difference} & 8\frac{3}{32} \end{array} \quad \text{Ans.}$$

That is, the remaining portion *b*, Fig. 9, is $8\frac{3}{32}$ inches long.

Example 4.—A piece $4\frac{7}{16}$ inches long is cut from a bar $9\frac{1}{4}$ inches long. What is the length of the remaining piece?

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$. As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted. Therefore, 1, or $\frac{16}{16}$, is taken from the 9 in the minuend and added to the $\frac{4}{16}$; thus, $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. Subtracting $\frac{7}{16}$ from $\frac{20}{16}$ leaves a remainder of $\frac{13}{16}$. Since 1 was taken from 9, 8 remains. 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$, the rerequired answer.

minuend $9\frac{4}{16}$	or $8\frac{20}{16}$	
subtrahend $4\frac{7}{16}$	$4\frac{7}{16}$	
difference $4\frac{13}{16}$	$4\frac{13}{16}$	Ans.

EXAMPLE 5.—From 9 take $8\frac{3}{16}$.

SOLUTION.—There is no fraction in the minuend from which to subtract the fraction in the subtrahend, so 1, or $\frac{16}{16}$, is taken from 9. Thus, $9 = 8\frac{16}{16}$, and $\frac{16}{16} - \frac{3}{16} = \frac{13}{16}$. Since 1 was taken from 9, only 8 is left, and 8 from 8 $= 0$.

minuend 9	or $8\frac{16}{16}$	
subtrahend $8\frac{3}{16}$	$8\frac{3}{16}$	
difference $\frac{13}{16}$	$\frac{13}{16}$	Ans.

EXAMPLES FOR PRACTICE

1. In the following examples, subtract:

(a) $\frac{4}{30}$ from $\frac{5}{10}$.
(b) $13\frac{1}{4}$ from $30\frac{1}{2}$.
(c) $12\frac{1}{8}$ from 27.

Ans. (a) $\frac{11}{30}$; (b) $17\frac{1}{4}$; (c) $14\frac{7}{8}$

2. A bar of iron $22\frac{1}{8}$ inches long, as shown in Fig. 10, has three pieces cut from it, measuring $6\frac{1}{2}$ inches, $4\frac{7}{8}$ inches, and $2\frac{5}{32}$ inches in length. What length of the original bar remains? Ans. $8\frac{19}{32}$ inches

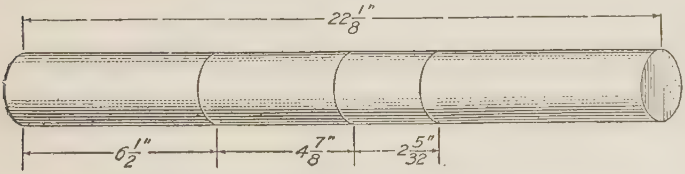

FIG. 10

SUGGESTION.—Add the lengths of the three pieces that are cut off and subtract their sum from the total length.

3. A piece of cast iron $4\frac{1}{8}$ inches thick was planed down to a thickness of $3\frac{31}{32}$ inches. What thickness of metal was removed? Ans. $\frac{5}{32}$ inch

4. At the beginning of a week an engineer had on hand $48\frac{5}{6}$ gallons of cylinder oil. During the week the following quantities were used: $2\frac{1}{4}$ gal-

lons during the first three days, $\frac{3}{4}$ gallon on the fourth day, 1 gallon on the fifth day, $\frac{1}{2}$ gallon on the sixth day, and not any on the seventh day. How much oil remained at the end of the week? Ans. $44\frac{1}{3}$ gal.

5. The main line shaft in a manufacturing plant is driven by an engine capable of developing $250\frac{2}{3}$ horsepower. To operate all the machinery requires $210\frac{3}{4}$ horsepower, and to overcome the friction of the shafting and bearings requires $25\frac{1}{8}$ horsepower. What surplus power is the steam engine capable of developing in case it is required? Ans. $14\frac{19}{24}$ horsepower

6. On a certain day a power plant had on hand $284\frac{1}{3}$ tons of coal. During the day $25\frac{1}{2}$ tons of coal were received, and $50\frac{1}{6}$ tons were consumed. How many tons of coal remained at the end of the day? Ans. $259\frac{2}{3}$ tons

7. A filling box in a textile mill contains $10\frac{3}{4}$ pounds of yarn. When $7\frac{13}{16}$ pounds of the yarn is removed from the box, how many pounds remain? Ans. $2\frac{15}{16}$ lb.

8. At the beginning of a voyage of a large steamship, an engineer's storekeeper had on hand $50\frac{7}{12}$ pounds of various sizes of rod packing. During a period of 6 days, all the hoisting engines, anchor windlasses, steering engines, and other auxiliary machinery were repacked, the quantity used each day being as follows:

Monday..................	$6\frac{1}{8}$ lb.	Thursday.................	$6\frac{5}{8}$ lb.
Tuesday..................	$5\frac{3}{4}$ lb.	Friday....................	$5\frac{7}{8}$ lb.
Wednesday...............	$6\frac{1}{4}$ lb.	Saturday..................	$4\frac{1}{2}$ lb.

How many pounds of packing remained on hand? Ans. $15\frac{11}{24}$ lb.

9. A steamship has to run a distance of $498\frac{3}{4}$ miles. If during the first day she made $374\frac{2}{5}$ miles, how many miles remained to complete the voyage? Ans. $124\frac{7}{20}$ miles

10. A certain factory requires a total motor installation of $27\frac{1}{2}$ horsepower. Three motors have been purchased, one rated at 5 horsepower, a second at $\frac{1}{4}$ horsepower, and a third at $\frac{2}{3}$ horsepower. What must be the combined horsepower of the motors that still remain to be purchased, to give the required total of $27\frac{1}{2}$ horsepower? Ans. $21\frac{7}{12}$ horsepower

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MULTIPLICATION OF FRACTIONS

28. Principle of Multiplication Explained.—Fractions can be multiplied without reducing them to a common denominator. A fraction is multiplied by a whole number when the numerator of the fraction is multiplied by that number. For example, the expression $\frac{3}{4}\times 3$ means add the fraction $\frac{3}{4}$ three times; thus, $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$. As these fractions have a common denominator their sum is found, according to the rule in Art. **25**, by adding the numerators and writing the sum over the common denominator. Hence, $\frac{3}{4}\times3=\frac{9}{4}$, or $2\frac{1}{4}$.

The word "of" when placed between two fractions, or between a fraction and a whole number, means the same as ×, or *times*. For instance, $\frac{1}{2}$ of $\frac{6}{8}$ is equal to $\frac{1}{2}\times\frac{6}{8}$ and means that one-half of $\frac{6}{8}$ shall be taken. As $\frac{6}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$, it follows that one-half of these fractions is $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$. Hence, $\frac{1}{2}\times\frac{6}{8}=\frac{3}{8}$. This example shows the *results* obtained by multiplying $\frac{1}{2}$ by $\frac{6}{8}$, but it does not show *how* the fractions are multiplied to obtain the results. According to the succeeding rule the numerator of one fraction is multiplied by the numerator of the other to obtain the numerator of the product, and the denominator of the one is multiplied by the denominator of the other to obtain the denominator of the product. According to this rule $\frac{1}{2}\times\frac{6}{8}=\frac{6}{16}$, which product reduced to its lowest terms is equal to $\frac{3}{8}$, as before.

29. Rule for Multiplication.—The following rule applies to multiplication of fractions:

Rule.—I. *To multiply either proper or improper fractions, multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

II. *To multiply one mixed number by another, reduce them both to improper fractions and then multiply them as in part I of the rule.*

III. *To multiply a fraction by a whole or by a mixed number, proceed as in part I of the rule, treating the whole number as a numerator or first reducing the mixed number to an improper fraction.*

IV. *To multiply a mixed number by a whole number, first reduce the mixed number to an improper fraction; then multiply the numerator of this fraction by the whole number, and divide the product by the denominator. Or, multiply the whole-number part of the mixed number by the whole-number multiplier, and then the fractional part by the whole-number multiplier, and add the products.*

In the preceding rule reference is made to the multiplication of two numbers only. This was done so as to keep the rule as simple as possible. But it is to be understood that the rule also applies when more than two numbers are to be multiplied together. The application of the rule is illustrated by the following examples:

EXAMPLE 1.—Multiply $\frac{5}{8}$ by $\frac{3}{16}$.

SOLUTION.—Applying part I of the rule, $\frac{5}{8}\times\frac{3}{16}=\frac{15}{128}$. Ans.

EXAMPLE 2.—Multiply $2\frac{3}{4}$ by $4\frac{1}{8}$.

SOLUTION.—Applying part II of the rule, the mixed numbers are reduced to improper fractions. Thus, $2\frac{3}{4}=\frac{11}{4}$, and $4\frac{1}{8}=\frac{33}{8}$; then, $\frac{11}{4}\times\frac{33}{8}=\frac{363}{32}=11\frac{11}{32}$. Ans.

EXAMPLE 3.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.—Part III of the rule is applied. Thus, $\frac{3}{8}\times 2=\frac{3\times 2}{8}=\frac{6}{8}=\frac{3}{4}$. Ans.

EXAMPLE 4.—Multiply $3\frac{4}{5}$ by 5.

SOLUTION.—By the first method in part IV of the rule, $3\frac{4}{5}$ is reduced to an improper fraction and becomes equal to $\frac{19}{5}$; then, $\frac{19}{5}\times 5=\frac{95}{5}=19$. Ans.

By the second method, $5\times 3\frac{4}{5}=5\times 3+5\times\frac{4}{5}=15+\frac{20}{5}=15+4=19$. Ans.

EXAMPLE 5.—Multiply $\frac{3}{8}$ by $\frac{2}{3}$ and by 5.

SOLUTION.—Parts I and III of the rule are applied. Thus, $\frac{3}{8}\times\frac{2}{3}\times 5$ is equal to $\frac{3}{8}\times\frac{2}{3}\times\frac{5}{1}=\frac{30}{24}=1\frac{6}{24}=1\frac{1}{4}$. Ans.

30. Cancelation of Fractions.—It is seen from the preceding rules and examples that when several fractions are multiplied together, a product is found of the numerators and of the denominators, these products representing, respectively, the numerator and the denominator in a new fraction. It follows that the product $\frac{3}{4}\times\frac{8}{9}\times\frac{10}{16}$ may be represented in this form: $\frac{3\times 8\times 10}{4\times 9\times 16}$. That is, if a number of fractions are to be multiplied together, they may be represented as *one* fraction in which all the numerators are written above the line and connected by multiplication signs, and all the denominators are written below the line, likewise connected by multiplication signs.

31. By arranging the fractions in the manner just described, the operation of multiplication may often be shortened. Thus, instead of multiplying the numerators together and then multiplying the denominators together, it may be possible to divide the numerators and the denominators by the *same* number, just as in reducing a fraction to its lowest terms. Doing this will change the *form* of the fraction, but not its *value*, as was proved in Art. **11.** In the example $\frac{3\times8\times10}{4\times9\times16}$ there is a 3 in the numerator and a 9 in the denominator; it is therefore possible to divide by 3. That this division has been done is indicated by drawing a line through the respective numbers and writing the quotient of each division near the dividend. This first step will then be

$$\frac{\overset{1}{\cancel{3}}\times8\times10}{4\times\underset{3}{\cancel{9}}\times16}$$

showing that the divisor 3 is contained once in 3 and 3 times in 9. Again, 8 is above the line and 16 below it, and both of these can be divided by 8. The next step is

$$\frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{8}}\times10}{4\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{16}}}$$

Now, there is a 10 above the line and a 4 below it, and both of these can be divided by 2 without any remainder. The next step then is

$$\frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{8}}\times\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{16}}}$$

This process cannot be carried any farther, because there are no numbers above and below the line that can be divided by the same number without a remainder. Therefore, all the uncrossed numbers in the numerator are multiplied together, or $1\times1\times5=5$; next, all the uncrossed numbers in the denominator are multiplied together, or $2\times3\times2=12$. The first product is then written over the second, giving $\frac{5}{12}$ as the reduced fraction. In other words, $\frac{3}{4}\times\frac{8}{9}\times\frac{10}{16}=\frac{5}{12}$. This can be proved very simply; for example, $\frac{3\times8\times10}{4\times9\times16}=\frac{240}{576}$, which can be reduced to its lowest terms by dividing both terms by 48. Thus, $\frac{240\div48}{576\div48}=\frac{5}{12}$. This shows that the process of dividing numbers above and below the line of a fraction by the *same* number will give the same result as taking the product of all the numerators and dividing it by the product of all the denominators.

The operation of dividing the numbers above and below the line by the same number is called **cancelation,** and when a line is drawn through one of the numbers, as shown, the number is said to be *canceled*. In cancelation, if the quotient is 1, it is not written at all, but it is understood. The preceding example, therefore, would commonly be written

$$\frac{\cancel{3}\times\cancel{8}\times\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{16}}}=\tfrac{5}{12}$$

32. It is important to remember that cancelation can be used only in cases where the numbers in both numerator and denominator are *multiplied* together. Where addition or subtraction is indicated in either term of the fraction, cancelation cannot be used.

The following examples show the advantage of employing cancelation in the multiplication of fractions:

EXAMPLE 1.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16}\times\frac{7}{8}=\dfrac{4\times7}{16\times8}=\frac{28}{128}=\frac{7}{32}$. Ans.

Or, by cancelation, $\dfrac{\cancel{4}\times7}{\underset{4}{\cancel{16}}\times8}=\dfrac{7}{4\times8}=\frac{7}{32}$. Ans.

EXAMPLE 2.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\dfrac{\cancel{4}\times3\times\cancel{16}}{8\times\cancel{4}\times\underset{2}{\cancel{32}}}=\dfrac{3}{8\times2}=\frac{3}{16}$. Ans.

EXAMPLE 3.—What is the product of $32\frac{5}{8}$ and 24?

SOLUTION.—The mixed number $32\frac{5}{8}$ when reduced to an improper fraction becomes $\dfrac{32\times8+5}{8}=\frac{261}{8}$. Applying the first half of part IV of the rule, Art. **29,**

$$\tfrac{261}{8}\times24=\frac{261\times24}{8}=\frac{261\times\overset{3}{\cancel{24}}}{\cancel{8}}=261\times3=783.\quad\text{Ans.}$$

The solution of this example may also be found by means of the method given in the second half of part IV of the rule, Art. **28**, as follows: $32\times24=768$, and $\frac{5}{8}\times24=\frac{5\times\overset{3}{\cancel{24}}}{\cancel{8}}=15$. Then, according to the rule, it is necessary to add the two products, giving $768+15=783$. That is,

$$32\tfrac{5}{8}\times24=783. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the product of:

(*a*) $7\times\frac{3}{19}$.
(*b*) $14\times\frac{5}{16}$.
(*c*) $\frac{21}{32}\times\frac{5}{14}\times\frac{8}{3}$.

Ans. $\begin{cases}(a)\ 1\frac{2}{19}\\(b)\ 4\frac{3}{8}\\(c)\ \frac{5}{8}\end{cases}$

2. How long must a piece of stock be to make 4 bolts, if each bolt requires a piece of stock $3\frac{7}{8}$ inches long, including the allowance for cutting?

Ans. $15\frac{1}{2}$ inches

3. Eleven holes are spaced equally in a straight line, as in Fig. 11.

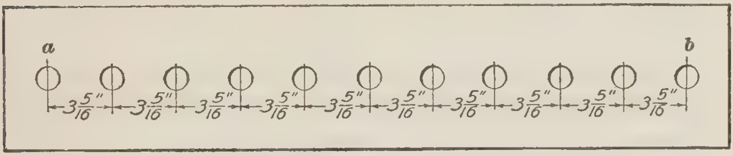

Fig. 11

What is the distance between the centers of the end holes *a* and *b*, if the spacing from center to center of two adjacent holes is $3\frac{5}{16}$ inches?

Ans. $33\frac{1}{8}$ inches

Note.—The number of *spaces* is one less than the number of holes.

4. A single belt can transmit $107\frac{2}{3}$ horsepower, but as it is desired to use more power, a double belt of the same width is substituted for it. Suppose that the double belt is capable of transmitting $1\frac{3}{7}$ times as much power as the single belt; how many horsepower can be used after the change?

Ans. $153\frac{17}{21}$ horsepower

5. At $303\frac{3}{5}$ pounds per mile, what is the weight of $2\frac{5}{8}$ miles of copper wire?

Ans. $796\frac{19}{20}$ lb.

6. The grate of a steam boiler contains $20\frac{1}{2}$ square feet. If the boiler burns $8\frac{3}{10}$ pounds of coal an hour per square foot of grate area and can evaporate $7\frac{1}{2}$ pounds of water an hour per pound of coal burned, how many pounds of water are evaporated by the boiler in 1 hour? Ans. $1{,}276\frac{1}{8}$ lb.

7. If an electrician receiving 70 cents an hour works on a job at different times $8\frac{1}{3}$ hours, $7\frac{1}{4}$ hours, $6\frac{2}{3}$ hours, and $5\frac{1}{2}$ hours, how much does he earn in dollars and cents, 100 cents equaling 1 dollar?
Ans. 19 dollars $42\frac{1}{2}$ cents

8. A mechanic works 56 hours in one week at $50\frac{1}{2}$ cents per hour. How much pay does he receive for that week? Ans. 28 dollars 28 cents

9. Condensed water is discharged by a return pipe at the rate of $73\frac{2}{5}$ gallons per hour. If the discharge takes place during a period of $10\frac{1}{5}$ hours per day, how many gallons will be discharged in $9\frac{3}{4}$ days?
Ans. $7{,}299\frac{63}{100}$ gal.

NOTE.—A return pipe conveys the condensed steam in a heating system back to the steam boiler in which the steam was generated.

10. A foundry turns out 85 cast-iron pulleys weighing $62\frac{5}{8}$ pounds apiece. What is the total amount of metal used? Ans. $5{,}323\frac{1}{8}$ lb.

DIVISION OF FRACTIONS

33. Compound Fractions.—The sign of division used for whole numbers is also used for fractions, as $\frac{1}{2} \div \frac{3}{4}$, or the division may be indicated by writing the dividend above the divisor with a heavy line between them, as $\frac{\frac{1}{2}}{\frac{3}{4}}$.

A fraction whose terms are whole numbers, as $\frac{1}{2}$ or $\frac{3}{4}$, is called a *simple fraction* in distinction from a **compound fraction**, either or both of the terms of which are simple fractions. The fractions $\frac{\frac{1}{2}}{\frac{3}{4}}$, $\frac{1}{\frac{3}{4}}$, and $\frac{\frac{1}{2}}{3}$ are compound fractions.

34. Reciprocal of a Fraction.—Any whole number may be considered as a fraction with 1 as its denominator; thus, 16 may be written $\frac{16}{1}$. The **reciprocal** (pronounced *ree-sip′-ro-kal*) of a fraction is obtained by inverting the fraction; that is, turning it upside down by writing the denominator *above* the line and the numerator *below*. Thus, the reciprocal of 12, which may be written $\frac{12}{1}$, is $\frac{1}{12}$; the reciprocal of 2 is $\frac{1}{2}$, etc. The reciprocal of the fraction $\frac{3}{4}$ is $\frac{4}{3}$, and of $\frac{11}{16}$ is $\frac{16}{11}$.

35. Preliminary Remarks on Division of Fractions. At first it may seem difficult to understand what is meant by dividing a number by a fraction or dividing one fraction by another. An attempt will therefore be made in the following to make the subject clearer. The case of dividing by a whole number will be dealt with first, as, for instance, $12 \div 3$. The question to be answered by this expression is: How many times is the number 3 contained in the number 12? Or, into how many parts may 12 be divided, if each part is to consist of 3 units? Or, by what must 3 be multiplied in order to make the product 12? The answer to all three questions is 4.

As the divisor decreases in value, the quotient will increase, until finally, with a divisor equal to 1, the dividend and the quotient will be equal. For instance, $12 \div 1 = 12$, showing that the number 12 may be divided into 12 parts, each equal to 1. Judging from this fact, one may draw the conclusion that on dividing by a number *smaller* than 1, as, for instance, with a proper fraction, the quotient will become greater than the dividend, and this is found to be the case.

36. Proceeding now to division by fractions, the first example chosen for further consideration is $9 \div \frac{1}{3}$. The problem indicated by this expression is to ascertain how many times the fraction $\frac{1}{3}$ is contained in the number 9. Or, by what number must $\frac{1}{3}$ be multiplied in order to make the product 9? As the number 1 contains 3 parts, each equal to one-third of the number, it follows, that the number of one-thirds contained in 9 must be equal to $9 \times 3 = 27$. In other words, $\frac{1}{3}$ must be multiplied by 27 to make the product 9. This example shows that when dividing by a fraction having 1 as a numerator, the quotient is found by multiplying the dividend by the denominator of the divisor. That is, the divisor is *inverted*, or turned upside down, and used as a multiplier. Thus, $9 \div \frac{1}{3} = 9 \times \frac{3}{1} = 27$. Multiplying by the inverted fraction is equivalent to multiplying by the reciprocal; the latter term was defined in Art. **34.**

The same reasoning holds good in the case of dividing one fraction by another. For instance, in the expression $\frac{4}{5} \div \frac{1}{5}$ it is required to find how many times $\frac{1}{5}$ is contained in $\frac{4}{5}$. As the fraction $\frac{4}{5}$ is 4 times as great as $\frac{1}{5}$, it is evident that $\frac{1}{5}$ is contained 4 times in $\frac{4}{5}$, and that the quotient must be 4. The same result is obtained by inverting the divisor and multiplying; thus, $\frac{4}{5} \div \frac{1}{5} = \frac{4}{5} \times \frac{5}{1} = \frac{20}{5} = 4$.

In this case, also, the quotient is greater than either the divisor or the dividend.

If, in the preceding expression, the divisor $\frac{1}{5}$ is replaced by $\frac{2}{5}$, as $\frac{4}{5} \div \frac{2}{5}$, it is evident that as the divisor has been doubled in size, the number of times it is contained in the dividend is correspondingly decreased. In this case the dividend must be divided by 2; or, $4 \div 2 = 2$, instead of 4, as in the preceding example. This result is also obtained by multiplying by the inverted divisor, or $\frac{4}{5} \div \frac{2}{5} = \frac{4}{5} \times \frac{5}{2} = \frac{20}{10} = 2$.

After these preliminary explanations the subject of dividing fractions will be considered more in detail.

37. Rule for Dividing Fractions.—One fraction can be divided by another fraction without reducing them to a common denominator.

In dividing fractions the following rule applies:

Rule.—*To divide by a fraction, multiply by its reciprocal. Reduce mixed numbers to improper fractions before dividing.*

Example 1.—Divide $\frac{3}{4}$ by $\frac{5}{16}$.

Solution.—In this case, the divisor is $\frac{5}{16}$, and its reciprocal is $\frac{16}{5}$. Then, according to the rule,

$$\frac{3}{4} \div \frac{5}{16} = \frac{3}{4} \times \frac{16}{5} = \frac{3 \times \overset{4}{\cancel{16}}}{\cancel{4} \times 5} = \frac{12}{5} = 2\frac{2}{5}. \quad \text{Ans.}$$

Example 2.—Divide $\frac{3}{4}$ by 3.

Solution.—The divisor is 3, and its reciprocal is $\frac{1}{3}$. According to the rule, the fraction is to be multiplied by this reciprocal; then,

$$\frac{3}{4} \div 3 = \frac{3}{4} \times \frac{1}{3} = \frac{\cancel{3} \times 1}{4 \times \cancel{3}} = \frac{1}{4}. \quad \text{Ans.}$$

Example 3.—Divide 48 by $\frac{3}{16}$.

Solution.—The divisor is $\frac{3}{16}$ and its reciprocal is $\frac{16}{3}$. Then,

$$48 \div \frac{3}{16} = 48 \times \frac{16}{3} = \frac{\overset{16}{\cancel{48}} \times 16}{\cancel{3}} = 256. \quad \text{Ans.}$$

Example 4.—Divide $13\frac{3}{4}$ by $2\frac{1}{2}$.

Solution.—First reduce both mixed numbers to improper fractions; thus, $13\frac{3}{4} = \frac{55}{4}$ and $2\frac{1}{2} = \frac{5}{2}$. Then

$$13\frac{3}{4} \div 2\frac{1}{2} = \frac{55}{4} \div \frac{5}{2} = \frac{55}{4} \times \frac{2}{5} = \frac{\overset{11}{\cancel{55}} \times \cancel{2}}{\underset{2}{\cancel{4}} \times \cancel{5}} = \frac{11}{2} = 5\frac{1}{2}. \quad \text{Ans.}$$

EXAMPLE 5.—A bar of steel $23\frac{5}{8}$ inches long is divided into 7 equal parts. What is the length of each part?

SOLUTION.—The length of each part must be $23\frac{5}{8}\div 7$. Now, $23\frac{5}{8}=\frac{189}{8}$. The reciprocal of 7 is $\frac{1}{7}$. Then, applying the rule,

$$\frac{189}{8}\div 7=\frac{189}{8}\times\frac{1}{7}=\frac{\overset{27}{\cancel{189}}\times 1}{8\times\cancel{7}}=\frac{27}{8}=3\frac{3}{8}$$

Therefore, the length of each part is $3\frac{3}{8}$ inches. Ans.

EXAMPLES FOR PRACTICE

1. In the following examples, divide:

(a) 15 by $6\frac{3}{7}$.
(b) 172 by $\frac{4}{5}$.
(c) $\frac{103}{6}$ by $14\frac{2}{3}$.

Ans. $\begin{cases}(a)\ 2\frac{1}{3}\\ (b)\ 215\\ (c)\ 1\frac{15}{88}\end{cases}$

2. How many gears, each $1\frac{1}{8}$ inches thick, can be set side by side in a space 45 inches long? Ans. 40

3. A distance measuring $34\frac{7}{32}$ inches is divided into 15 equal parts. What is the length of each part? Ans. $2\frac{9}{32}$ inches

4. A boiler plate containing 24 square feet of surface weighs $362\frac{4}{10}$ pounds. What is its weight per square foot? Ans. $15\frac{1}{10}$ lb.

5. A certain boiler has $927\frac{1}{2}$ square feet of heating surface, which is equal to 35 times the area of the grate. What is the area of the grate in square feet? Ans. $26\frac{1}{2}$

6. If the distance around the rim of a locomotive driving wheel is $13\frac{1}{12}$ feet, how many revolutions will the wheel make in traveling 682 feet? Ans. $52\frac{20}{157}$ rev.

7. A tank containing 1,830 gallons of water supplies a family whose daily consumption, according to a meter, averages $521\frac{1}{2}$ gallons. How many days' supply does the tank contain? Ans. $3\frac{531}{1043}$ days

8. If one horsepower will drive $3\frac{3}{4}$ cotton looms of a certain kind, how much power will be required for a weave room containing 600 looms? Ans. 160 horsepower

9. How many bolts can be obtained from a steel bar $15\frac{3}{4}$ inches in length, if each bolt requires a piece $3\frac{15}{16}$ inches long? Ans. 4 bolts

10. A piece of land having an area of $12\frac{3}{4}$ acres is to be divided into plots, each plot containing $\frac{3}{8}$ of an acre. How many plots will there be? Ans. 34 plots

COMBINED PROCESSES

MULTIPLICATION AND DIVISION COMBINED WITH ADDITION AND SUBTRACTION

38. Order of Operations.—When the numerator or the denominator of a fraction contains several numbers or fractions connected by signs that indicate different processes, such processes must be performed in the order used in the case of whole numbers; that is, unless otherwise indicated, multiplication and division must precede addition and subtraction. The method to be used is shown in the following examples:

EXAMPLE 1.—Find the value of $\dfrac{\frac{1}{4}+\frac{2}{3}\times 9}{10\frac{15}{16}-\frac{3}{4}\times 12}$.

SOLUTION.—As multiplication must precede addition, first find the product of $\frac{2}{3}\times 9$. Add this product to $\frac{1}{4}$ and divide the sum by the difference between $10\frac{15}{16}$ and the product of $\frac{3}{4}\times 12$. The numerator, or dividend, is $\frac{2}{\cancel{3}}\times\overset{3}{\cancel{9}}=6$; $\frac{1}{4}+6=6\frac{1}{4}=\frac{25}{4}$. The denominator, or divisor, is

$$\frac{3}{\cancel{4}}\times\overset{3}{\cancel{12}}=9;\ 10\tfrac{15}{16}-9=1\tfrac{15}{16}=\tfrac{31}{16}$$

Therefore,

$$\frac{\frac{1}{4}+\frac{2}{3}\times 9}{10\frac{15}{16}-\frac{3}{4}\times 12}=\tfrac{25}{4}\div\tfrac{31}{16}=\frac{25}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{31}=\frac{25\times 4}{31}=\tfrac{100}{31}-3\tfrac{7}{31}.\quad \text{Ans.}$$

EXAMPLE 2.—Find the value of $\dfrac{1}{\frac{1}{4}+\frac{1}{5}+\frac{1}{7}+\frac{1}{10}}$.

SOLUTION.—The denominator $\frac{1}{4}+\frac{1}{5}+\frac{1}{7}+\frac{1}{10}=\frac{35}{140}+\frac{28}{140}+\frac{20}{140}+\frac{14}{140}=\frac{97}{140}$. Therefore, the whole fraction

$$\frac{1}{\frac{1}{4}+\frac{1}{5}+\frac{1}{7}+\frac{1}{10}}=\frac{1}{\frac{97}{140}}=1\div\tfrac{97}{140}=1\times\tfrac{140}{97}=\tfrac{140}{97}=1\tfrac{43}{97}.\quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Find the value of the following expressions:

(*a*) $\dfrac{2}{\frac{1}{3}+\frac{1}{8}+\frac{1}{12}}$.

(*b*) $4+\dfrac{1}{\frac{1}{9}+\frac{1}{18}+\frac{1}{36}+\frac{1}{72}}$.

(*c*) $\dfrac{1}{\frac{1}{4}+\frac{1}{6}+\frac{1}{12}+\frac{1}{18}}+\frac{2}{3}\times 3\frac{3}{8}$.

Ans. $\begin{cases} (a)\ 3\frac{9}{13} \\ (b)\ 8\frac{4}{5} \\ (c)\ 4\frac{1}{20} \end{cases}$

FACTORS AND AVERAGES

39. Factors.—In some calculations it is necessary to *factor* a fraction or a whole number, that is, to separate the fraction or the number into its factors. The **factors** of a number are simply those numbers which, when multiplied together, will equal the number. Thus, 2 and 3 are factors of 6, because $2\times3=6$; 2 and 5 are factors of 10, because $2\times5=10$; 3 and 4 are factors of 12, because $3\times4=12$. But 6 and 2 are also factors of 12, because $6\times2=12$; also, 3, 2, and 2 are factors of 12, because $3\times2\times2=12$.

A fraction is factored by finding the factors of its numerator and denominator. Thus, the factors of $\frac{10}{16}$ are $\frac{5}{4}$ and $\frac{2}{4}$, because

$$\frac{10}{16}=\frac{5\times2}{4\times4}=\frac{5}{4}\times\frac{2}{4}.$$

Some numbers have no factors except themselves and 1; thus, the factors of 5 are 1 and 5, because $1\times5=5$. The factors of 7 are 1 and 7; of 11 1 and 11; of 13, 1 and 13; and so on. Such numbers are called prime numbers, as explained in Art. **6**.

The same number may often be factored in several different ways; thus, $24=6\times4$, or $3\times2\times4$, or 3×8, or $6\times2\times2$, or $3\times2\times2\times2$, because the product of each of these groups of numbers is 24. Similarly, the fraction $\frac{24}{30}$ has the factors $\frac{4}{6}\times\frac{6}{5}$, $\frac{3}{10}\times\frac{8}{3}$, $\frac{2}{3}\times\frac{12}{10}$, and many others.

40. Averages.—It often becomes necessary to find the average of several numbers; for instance, a number of men will require different lengths of time in which to do a piece of work, and it may be required to find the average time.

The **average** of several terms is equal to their sum divided by the number of terms.

EXAMPLE 1.—Find the average of the numbers 6, 12, 8, 10, 11, and 7.

SOLUTION.—There are six numbers, and their sum is $6+12+8+10+11+7=54$. Then, the average is $54 \div 6 = 9$. Ans.

EXAMPLE 2.—If five gear-wheels weigh 28, 12, 36, 25, and 14 pounds, respectively, what is the average weight per wheel?

SOLUTION.—There are five wheels, and the sum of their weights is $28+12+36+25+14=115$ pounds. The average is therefore $115 \div 5 = 23$ pounds. Ans.

It will be noted that if the average is multiplied by the number of things it will give the sum. Thus, in example 1 the average of six numbers is 9; hence, the sum of the six numbers is $9 \times 6 = 54$. In example 2, the average weight of the gears is 23 pounds; hence, the weight of the five gears is $23 \times 5 = 115$ pounds.

EXAMPLES FOR PRACTICE

1. A workman finishes 23 pieces of work on Monday, 26 on Tuesday, 19 on Wednesday, 27 on Thursday, and 20 on Friday. What is the average per day? Ans. 23

2. Six workmen engaged on the same class of work require 48, 42, 56, 50, 45, and 51 minutes, respectively. What is the average time for the work? Ans. $48\frac{2}{3}$ minutes

3. If the average weight of eight castings is $334\frac{1}{2}$ pounds each, what is the total weight of the castings? Ans. 2,676 lb.

4. Six rivets measure, respectively, $2\frac{1}{4}$, $2\frac{15}{16}$, $2\frac{7}{8}$, $2\frac{3}{8}$, $2\frac{7}{16}$, and $2\frac{1}{2}$ inches in length. What is the average length? Ans. $2\frac{9}{16}$ inches

5. What prime numbers are factors of: (*a*) 210? (*b*) 1,001?

$$\text{Ans.}\begin{cases}(a)\ 2, 3, 5, \text{ and } 7\\(b)\ 7, 11, \text{ and } 13\end{cases}$$

DECIMALS

FUNDAMENTAL OPERATIONS

DEFINITIONS, NUMERATION, AND NOTATION

DEFINITIONS OF TERMS AND NUMBERS

1. Decimal Fractions.—In many branches of industry and in nearly all books of a scientific nature, it is found advantageous to use a form of fraction known as a *decimal,* or *decimal fraction,* as by its use the main fundamental processes of arithmetic are greatly simplified.

Decimals are *tenth* fractions; that is, they indicate *tenth* parts of a unit or their subdivisions into *hundredths, thousandths,* etc. The denominator of a decimal consists, therefore, always of the figure 1 followed by one or more ciphers, as 10, 100, 1,000, etc.; for this reason it is possible to use an abbreviated method in writing decimal fractions and to dispense entirely with the method adopted in writing common fractions. Thus, instead of drawing a line below the numerator and writing the denominator below it, the line and the denominator are omitted in a decimal fraction and replaced by a point.

2. Decimal Point.—The **decimal point** (.) used in decimals is written at the left of the number expressing the numerator. For example, .3 is a decimal, and it is read *three tenths.* Its value is the same as the fraction $\frac{3}{10}$; that is, $.3 = \frac{3}{10}$. Every decimal consists of a decimal point followed by one or more figures. For instance, .25, .625, .73056, etc.

NOTE.—In English technical literature the decimal point is above the base line and before the middle of the figure, as ·32. On the European continent the decimal point is, in general, replaced by a comma placed on the base line in the usual position.

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3. Decimal Places.—Each figure to the right of a decimal point occupies a **decimal place.** These decimal places are of great importance in decimals, as the place occupied by a figure serves as a means for indicating the value of its denominator. When referring to the number of figures after a decimal point, it is customary to state the number of *places* occupied by them. Thus, in the decimals .47 and .372, there are two and three places, respectively.

4. Numerator and Denominator.—The **numerator** of a decimal fraction is the number that follows the decimal point, and the **denominator** is the figure 1 followed by as many ciphers as there are places in the decimal. To illustrate, take the decimal .42. The number following the decimal point is 42, which is the numerator; the figures after the decimal point occupy two places, that is, the decimal has *two places*. Hence, the denominator is 1 followed by two ciphers, or 100. Then, the value of .42 is $\frac{42}{100}$, and it is read *forty-two hundredths*.

5. Changing Value of Denominator.—The value of the denominator of a decimal fraction depends on the number of decimal places. Thus:

One decimal place expresses *tenths*
Two decimal places express *hundredths*
Three decimal places express *thousandths*
Four decimal places express *ten-thousandths*
Five decimal places express *hundred-thousandths*
Six decimal places express *millionths*

The following series of decimals shows how the number of places in a decimal fraction affects the value of its denominator, the same numerator, 3, being used in each decimal:

.3 $= \frac{3}{10}$ = 3 tenths
.03 $= \frac{3}{100}$ = 3 hundredths
.003 $= \frac{3}{1000}$ = 3 thousandths
.0003 $= \frac{3}{10000}$ = 3 ten-thousandths
.00003 $= \frac{3}{100000}$ = 3 hundred-thousandths
.000003 $= \frac{3}{1000000}$ = 3 millionths

From these examples it is seen that to change a decimal to a common fraction, the number in the decimal is written as the *numerator*, and the numeral 1 followed by as many ciphers as there are decimal places in the decimal is written as the *denominator*. It is important to remember these facts. For example, let it be required to write the decimal .5625 as a common fraction. The number 5625 is the numerator of the fraction; there are four decimal places, hence the denominator is 1 and four ciphers, or 10,000, and the complete fraction becomes $\frac{5625}{10000}$.

6. Mixed Numbers.—Very often there are figures written to the left of the decimal point as well as to the right of it, as, for example, 12.5, 7.25, and 18.125. These are called **mixed numbers,** because each consists of a whole number and a fraction combined. The decimal point simply separates the whole number at its left from the decimal at its right. The number 12.5 is read *twelve and five tenths;* 7.25 is read *seven and twenty-five hundredths;* 18.125 is read *eighteen and one hundred twenty-five thousandths.* Another method of reading these numbers is as follows: The number 12.5 is read *twelve point five;* 7.25 is read *seven point twenty-five;* 18.125 is read *eighteen point one hundred twenty-five.*

Sometimes decimals are written with a cipher to the *left* of the decimal point, as 0.6. The cipher in this case merely shows that there is no whole number. A decimal thus written has the same value, and is read in the same way, as though the cipher were omitted.

NUMERATION AND NOTATION

7. Numeration of Decimals.—The relation of decimals and whole numbers is clearly shown by the following table:

hundreds of millions	tens of millions	millions	hundreds of thousands	tens of thousands	thousands	hundreds	tens	units	decimal point	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths	ten-millionths	hundred-millionths
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

As stated in Art. **1**, decimals are *tenth* fractions, that is, they are based on the scale of ten, each division and subdivision being divided in tenths. In the preceding table the starting point is the *units* place in both decimals and whole numbers. The decimals *decrease* on the scale of ten to the *right*, and the whole numbers *increase* on the scale of ten to the *left*. The first figure to the left of units is *tens*, and the first figure to the right of units is *tenths*. The second figure to the left of units is *hundreds*, and the second figure to the right is *hundredths*. The third figure to the left is *thousands*, and the third to the right is *thousandths*, and so on. The figures equally distant from units place correspond in name, the decimals having the ending *ths*, to distinguish them from the whole numbers.

8. In **reading** a decimal, the number indicating the numerator is read first as if it were a whole number, and then the denominator is stated. Thus, .725 is read *seven hundred twenty-five thousandths;* .0175 is read *one hundred seventy-five ten-thousandths.*

In reading mixed numbers, the whole number is read first, followed by the word *and* and then the decimal. For example, the mixed number in the table of Art. **7** is read *nine hundred eighty-seven million six hundred fifty-four thousand three hundred twenty-one and twenty-three million four hundred fifty-six thousand seven hundred eighty-nine hundred-millionths.*

9. Notation of Decimals.—In **writing** a decimal, the number indicating the numerator is written first and the position of the decimal point is then located, so as to give as many decimal places as there are ciphers in the denominator. The places are counted from the right-hand figure of the numerator toward the left, and ciphers are prefixed to the numerator if this is necessary to obtain the required number of places before placing the decimal point. Locating the decimal point is called **pointing off the decimal.**

EXAMPLE.—Express seventeen thousandths as a decimal.

Solution.—The numerator is 17; the denominator is 1,000, in which there are 3 ciphers. From Art. **4** and the preceding rule there should be as many places in the decimal as there are ciphers in the denominator; that is, there must be three places. As the number 17 requires only two places, it must be preceded by a cipher to obtain the required three decimal places, making it .017. Ans.

10. In writing decimals the following points should be remembered:

1. *Annexing ciphers to the right of a decimal or removing ciphers from the right of a decimal does not change its value.*

For example, $.5 = .50$, because $\frac{5}{10} = \frac{50}{100}$, also $.1200 = .12$, because $\frac{1200}{10000} = \frac{12}{100}$.

2. *Inserting a cipher between the decimal point and the first figure of the decimal divides the decimal by 10.*

For example, $.5 \div 10 = .05$, because $.5 = \frac{5}{10}$ and $\frac{5}{10} \div 10 = \frac{5}{10} \times \frac{1}{10} = \frac{5}{100}$, or .05. Likewise, $.05 \div 10 = .005$; $.005 \div 10 = .0005$; etc. The last statement may, therefore, also be made in the following form:

Moving the decimal point to the left in a number divides the number by 10 for each place that the point is moved.

3. *Taking away a cipher from the left of a decimal multiplies the decimal by 10.* For example, $.0005 \times 10 = .005$, because $.0005 = \frac{5}{10000}$ and $\frac{5}{10000} \times 10 = \frac{50}{10000} = \frac{5}{1000} = .005$. Likewise, $.005 \times 10 = .05$; $.05 \times 10 = .5$; etc.

The last statement may also be given the following wording:

Moving the decimal point to the right in a number multiplies the number by 10 for each place the point is moved.

11. In some cases, a mixed number containing a decimal may be more conveniently expressed in the form of a common improper fraction. To do so, it is only necessary to write the entire number, omitting the decimal point, as the numerator of the fraction, and the denominator of the decimal part as the denominator of the fraction. Thus, $127.483 = \frac{127483}{1000}$; for, $127.483 = 127\frac{483}{1000} = \frac{127000}{1000} + \frac{483}{1000} = \frac{127000+483}{1000} = \frac{127483}{1000}$.

EXAMPLES FOR PRACTICE

Express decimally:

(*a*) Fourteen ten-thousandths.	Ans.	(*a*) .0014
(*b*) Forty-seven millionths.		(*b*) .000047
(*c*) Four and two tenths.		(*c*) 4.2
(*d*) Seven hundred twenty-five hundred-thousandths.		(*d*) .00725

ADDITION OF DECIMALS

12. Arranging the Numbers.—In the addition of decimals, tenths are placed under tenths, hundredths under hundredths, etc.; this, of course, brings the decimal points in a *vertical* line, that is, one directly under another. Then addition is performed exactly as in the case of whole numbers. Hence, in placing the numbers to be added, it is only necessary to take care that the *decimal points are in the same vertical line*. In adding whole numbers, the right-hand figures are always in line; but in adding decimals, the right-hand figures will not be in line unless the decimals contain the same number of figures. To insure that the right-hand figures of whole and mixed numbers are always placed in a vertical line, a whole number is sometimes represented as a mixed number by placing a decimal point at its right end, it being understood that the decimal places are occupied by ciphers. In example 1, Art. **13,** the numbers 242 and 6 are treated in this manner.

The similarity in the arrangement of whole numbers and decimals is seen from the three succeeding examples, of which the first shows the addition of whole numbers, the second that of decimals, and the third the addition of mixed numbers. In the first example the right-hand figures are all in line, but this is not the case with the examples containing decimals.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

13. Rule for Adding Decimals.—To add decimals, proceed according to the following rule:

Rule.—*Place the numbers to be added so that the decimal points will be directly under one another. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

The application of the rule is shown in the succeeding examples.

Example 1.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

Solution.—

$$\begin{array}{r@{.}ll} 242 & & \\ & 36 & \\ 118 & 725 & \\ 1 & 005 & \\ 6 & & \\ 100 & 1 & \\ \hline 468 & 190 & \text{Ans.} \end{array}$$

Example 2.—A bar is marked off into 8 parts measuring 1.25, 4.3125, 2.305, 7.6, 10.4375, 5.5625, .875, and 3.0625 inches in length. What is the total length of the bar?

Solution.—The length of the bar is the sum of the lengths of the parts. The numbers are arranged with the decimal points in a vertical line, and added. Thus:

$$\begin{array}{r@{.}ll} 1 & 25 & \\ 4 & 3125 & \\ 2 & 305 & \\ 7 & 6 & \\ 10 & 4375 & \\ 5 & 5625 & \\ & 875 & \\ 3 & 0625 & \\ \hline 35 & 4050 & \text{Ans.} \end{array}$$

The total length, therefore, is 35.405 inches.

EXAMPLES FOR PRACTICE

1. In the following examples, find the sum of:

(*a*) .2143, .105, 2.3042, and 1.1417. Ans. (*a*) 3.7652

(*b*) 783.5, 21.473, .2101, and .7816. Ans. (*b*) 805.9647

2. Four round pieces of bar steel, when measured accurately, are found to have lengths of 11.25, 7.625, 1.3125, and 5.4375 inches (abbreviated in.). What is their total length when placed end to end?
Ans. 25.625 in.

3. The estimated weights of the parts of a return-tubular boiler were as follows: Shell, 3,626 pounds; tubes, 3,564.5 pounds; manhole cover, ring, and yoke, 270.34 pounds; stays, etc., 1,089.4 pounds; steam nozzles, 236.07 pounds; handhole covers and yokes, 120.25 pounds; feedpipe, 34.75 pounds; boiler supports, 350.6 pounds. What was the total estimated weight of the boiler? Ans. 9,291.91 lb.

4. The five sides of a field measure, respectively, 8.13 rods, 4.63 rods, 7.88 rods, 4.76 rods, and 9.29 rods. What is the total length of the sides? Ans. 34.69 rods

5. There are four electric lamps connected to a circuit. The lamps require current as follows: 1 ampere, .5 ampere, .3 ampere, and .25 ampere, respectively. What is the total current in amperes required by the four lamps? Ans. 2.05 amperes

6. A sample of coke was burned to test its quality. The coke was found to contain 5.79 pounds of ash, .597 pound of sulphur, and 93.613 pounds of carbon. What was the weight of the sample? Ans. 100 lb.

7. The inside width of a square steel box is 6.065 inches, and the thickness of the walls is .280 inch. What is the outside width of the box? Ans. 6.625 in.

NOTE.—*Two* opposite walls of the box must be considered in this example.

8. During a five-day voyage a steamer passes over the following distances: 384.75 miles, 372.825 miles, 356.5 miles, 392.625 miles, and 345.25 miles. What was the total distance covered? Ans. 1,851.95 miles

9. A farmer has 10.5 acres in one field, 8.75 acres in another, and 30.25 acres in a third field. How many acres does he possess in all?
Ans. 49.5 acres

10. A merchant sold to four customers the following quantities of cloth: 5.125 yards, 15.5 yards, 9.3125 yards, and 6.625 yards. What is the total length of the cloth that was sold? Ans. 36.5625 yd.

11. By carefully measuring the six sides of a tract of land it was found that the first side measured 537.68 feet, the second 87.36 feet, the third 836.39 feet, the fourth 732.12 feet, the fifth 237.26 feet, and the sixth 523.69 feet; what is the exact distance around the property?
Ans. 2,954.50 ft.

SUBTRACTION OF DECIMALS

14. Rule for Subtracting Decimals.—In the subtraction of decimals, tenths are placed under tenths, hundredths under hundredths, etc., bringing the decimal points in a vertical line, as in addition of decimals.

Rule.—*Place the subtrahend under the minuend, so that the decimal point of the subtrahend will be directly under that of the minuend. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them, and subtract as in the case of whole numbers.

Example 1.—Subtract .132 from .3063.

Solution.—

$$\begin{array}{rl} \textit{minuend} & .3063 \\ \textit{subtrahend} & \underline{.132} \\ \textit{difference} & .1743 \quad \text{Ans.} \end{array}$$

It is understood that the fourth decimal place of the subtrahend is occupied by a cipher.

Example 2.—A bar 7.895 inches long has a piece .725 inch long cut away from one end. What length remains?

Solution.—The length remaining must be the difference between 7.895 inches and .725 inch.

$$\begin{array}{rl} \textit{minuend} & 7.895 \\ \textit{subtrahend} & \underline{.725} \\ \textit{difference} & 7.170 \text{ or } 7.17 \quad \text{Ans.} \end{array}$$

That is, the length remaining is 7.17 inches.

Example 3.—A block 11 inches thick has .625 inch planed from it. What is its thickness then?

Solution.—The resulting thickness is the difference between 11 and .625 inches.

$$\begin{array}{rl} \textit{minuend} & 11.000 \\ \textit{subtrahend} & \underline{.625} \\ \textit{difference} & 10.375 \quad \text{Ans.} \end{array}$$

That is, the thickness is then 10.375 inches.

EXAMPLES FOR PRACTICE

1. In the following examples, from:

(*a*) 407.385 take 235.0004.
(*b*) 22.718 take 1.7042.

Ans. (*a*) 172.3846
(*b*) 21.0138

2. A block of steel 1.0625 inches thick is cut down to a thickness of .9375 inch. What thickness of metal is removed? Ans. .125 in.

3. If a bar 3.25 inches long has a piece 1.625 inches long cut away, what is the length of the remainder? Ans. 1.625 in.

4. If the temperature of steam at 5 pounds pressure is 227.964 degrees and at 100 pounds pressure is 337.874 degrees, how many degrees hotter is the steam at the higher pressure? Ans. 109.91

5. In a cistern that will hold 326.5 barrels of water there are 178.625 barrels. How much does it lack of being full? Ans. 147.875 barrels

6. A man left to his son and to his brother portions of his estate amounting to .33 of the whole. If the portion received by the son was .25 of the estate, what portion did the brother receive? Ans. .08

7. One bar is 12.0013 inches and another bar 5.9938 inches long. What is the difference in length? Ans. 6.0075 in.

8. A liquid-measure quart has 57.75 cubic inches, and a dry-measure quart has 67.200625 cubic inches. Find the difference in cubic inches between the two kinds of quart measures. Ans. 9.450625 cubic in.

9. One cubic foot of water weighs 62.37 pounds, and one cubic foot of ice weighs 57.5 pounds. How much heavier is the water than the ice? Ans. 4.87 lb.

10. In testing a sample of coke weighing 100 pounds it was found to contain 5.78 pounds of ash and .586 pound of sulphur; the remainder was carbon. How much carbon did the sample contain?

Ans. 93.634 lb.

MULTIPLICATION OF DECIMALS

15. Rule for Multiplication.—Multiplication of decimals is similar to multiplication of whole numbers. The *right-hand* figure of the multiplier is placed under the *right-hand* figure of the multiplicand, irrespective of the relative positions of the two decimal points; in other words, the points need not be in a vertical line. The number of decimal places in the product depends on the total number of decimal places found in the multiplier and the multiplicand. The preceding statement may be formulated as a rule, as follows:

Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as with whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, beginning at the right, and prefixing ciphers if necessary.*

16. Application of Rule.—The application of the preceding rule is shown in the solutions of the following examples:

Example 1.—Multiply .825 by 13.

Solution.—In this example, there are three decimal places in the multiplicand and none in the multiplier; therefore, three decimal places are pointed off in the product, beginning to count at the *right*.

$$\begin{array}{rrl} \textit{multiplicand} & .825 & \\ \textit{multiplier} & 13 & \\ \cline{2-2} & 2475 & \\ & 825 & \\ \cline{2-2} \textit{product} & 10.725 & \text{Ans.} \end{array}$$

Example 2.—What is the product of 426 and .005?

Solution.—In this example there are three decimal places in the multiplier and none in the multiplicand; therefore, three decimal places are pointed off in the product, counting from the right.

$$\begin{array}{rrl} \textit{multiplicand} & 426 & \\ \textit{multiplier} & .005 & \\ \cline{2-2} \textit{product} & 2.130 \text{ or } 2.13 & \text{Ans.} \end{array}$$

17. When a decimal, serving as a multiplicand or as a multiplier, contains one or more ciphers next to the decimal point, as .0049, such ciphers have no effect on the figures in the product, but they do affect the position of its decimal point.

When multiplying decimals such as .3700, the ciphers at the right-hand end may be dropped, as they only make more figures to deal with and do not change the value of the product in any way. The following examples show the application of these statements:

EXAMPLE 1.—Multiply 1.205 by 1.15.

SOLUTION.—In the multiplicand there are three decimal places and in the multiplier two places, making in all five places to point off in the product.

$$\begin{array}{rrl} \textit{multiplicand} & 1.205 & \\ \textit{multiplier} & 1.15 & \\ \hline & 6025 & \\ & 1205 & \\ & 1205 & \\ \hline \textit{product} & 1.38575 & \text{Ans.} \end{array}$$

EXAMPLE 2.—Multiply .232 by .001.

SOLUTION.—In this example the multiplicand is multiplied by the figure 1 of the multiplier, which gives 232 as the product. The sum of the numbers of decimal places in the multiplicand and multiplier is six, and six places must be pointed off in the product. But, as there are only three figures in the product, it is necessary to prefix three ciphers, as shown, to obtain the required number of decimal places.

$$\begin{array}{rrl} \textit{multiplicand} & .232 & \\ \textit{multiplier} & .001 & \\ \hline \textit{product} & .000232 & \text{Ans.} \end{array}$$

EXAMPLE 3.—Multiply .92500 by .313.

SOLUTION.—According to the preceding statement the last ciphers in the decimal .92500 could be omitted without affecting the product. The correctness of this statement will be proved by first multiplying the decimals in their present forms and then multiplying without the ciphers in the multiplicand, the products being the same in each case.

In the solution (a) there must be eight decimal places in the product, eight being the sum of the decimal places in the multiplicand and multiplier. If the two ciphers are omitted from the multiplicand, as in (b), only six decimal places are required in the product. The two products are equal, because the two ciphers to the right of the decimal in the solution (a) have no value.

$$\begin{array}{rrl} & (a) & \\ \textit{multiplicand} & .92500 & \\ \textit{multiplier} & .313 & \\ \hline & 277500 & \\ & 92500 & \\ & 277500 & \\ \hline \textit{product} & .28952500 & \text{Ans.} \end{array} \qquad \begin{array}{rrl} & (b) & \\ \textit{multiplicand} & .925 & \\ \textit{multiplier} & .313 & \\ \hline & 2775 & \\ & 925 & \\ & 2775 & \\ \hline \textit{product} & .289525 & \text{Ans.} \end{array}$$

EXAMPLES FOR PRACTICE

1. In the following examples, find the product of:

(*a*) .000492×4.1418.
(*b*) 4,003.2×1.2

Ans. { (*a*) .0020377656
(*b*) 4,803.84

2. Four equal distances of 2.375 inches are marked off, end to end, on a piece of work. What is the total distance marked off? Ans. 9.5 in.

3. If 1 cubic inch of cast brass weighs .295 pound, what is the weight of a brass casting containing 768 cubic inches? Ans. 226.56 lb.

4. If a steam pump delivers 2.38 gallons of water per stroke and runs at 51 strokes a minute, how many gallons of water will it pump in 58½ minutes? Ans. 7,100.73 gallons

5. Wishing to obtain the weight of a connecting-rod from a drawing it was calculated that the rod contained 294.8 cubic inches of wrought iron, 63.5 cubic inches of brass, and 10.4 cubic inches of Babbitt. Assuming the weight of wrought iron to be .278 pound per cubic inch, of brass .303 pound, and of Babbitt .264 pound, what was the weight of the rod? Ans. 103.94 lb.

NOTE.—Two decimal places in the answer is sufficiently accurate in this case.

6. Three men *A, B,* and *C* bought a tract of land for 42,685 dollars. *A* paid .35 of this price, *B* .44, and *C* .21. How much money did each invest?

Ans. { *A,* 14,939.75 dollars
B, 18,781.40 dollars
C, 8,963.85 dollars

7. A tank contains 86 gallons of a solution of nitric acid. If in making each gallon of solution .012 gallon of nitric acid was used, how much acid was required for the 86 gallons? Ans. 1.032 gallons

8. The cost of manufacturing a certain article is 132 dollars. Of this amount .6 was spent in the foundry, .3 on drilling, and the remainder on filing. What is the cost of each operation?

Ans. { Foundry, 79.20 dollars
Drilling, 39.60 dollars
Filing, 13.20 dollars

9. If a man can shovel 1.7 cubic yards of clay into a cart in 1 hour, how many cubic yards of clay will he shovel in 4.75 days of 8 hours each? Ans. 64.6 cubic yards

10. How many bricks will be laid by a bricklayer in 13.75 days, if he can lay 1,240 bricks per day? Ans. 17,050 bricks

DIVISION OF DECIMALS

18. Rule for Division.—Division of decimals is very similar to division of whole numbers. The numbers are written and divided in the same way, the decimal point being disregarded while dividing. Before dividing, however, it may be necessary to annex ciphers to the decimal part of the dividend so as to make the number of decimal places in the dividend equal to or greater than the number in the divisor. Sometimes it is necessary to annex ciphers to the decimal part of the dividend to make it large enough to contain the divisor. The rule to be used is as follows:

Rule.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there is a remainder, the remainder can be placed as the numerator in a fraction with the divisor as its denominator and considered as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still is a remainder, terminate the quotient by the plus sign (+), which shows that it can be carried farther.*

19. Division Without a Remainder.—The application of the preceding rule is shown in the solutions of the following examples in which the divisor is contained in the dividend without a remainder. The quotient obtained by such division is known as an *exact* decimal.

Example 1.—Divide .625 by 25.

Solution.—The dividend has 3 decimal places and the divisor none; therefore there are 3−0=3 decimal places in the quotient. One cipher has to be prefixed to the number in the quotient, to make the 3 decimal places required.

```
divisor dividend quotient
    25).625(.025  Ans.
         50
         ---
         125
         125
         ---
remainder  0
```

Example 2.—Divide 6.035 by .05.

Solution.—In this example the dividend is divided by 5 in the divisor, as if the cipher were not before it. The dividend has 3 decimal places and the divisor 2; the difference being 1, one decimal place is pointed off in the quotient.

```
        divisor dividend quotient
          .05 ) 6.035 ( 120.7   Ans.
                5
                -
                10
                10
                --
                  35
                  35
remainder          0
```

Example 3.—Divide .125 by .005.

Solution.—In this example there are the same number of decimal places in the dividend as in the divisor; therefore, no decimal places are pointed off in the quotient.

```
        divisor dividend quotient
          .005 ) .125 ( 25   Ans.
                  10
                  --
                   25
                   25
                   --
remainder           0
```

Example 4.—Divide 326 by .25.

Solution.—As 326 is a whole number, it must have annexed to it a decimal portion consisting of two ciphers in order to make the number of decimal places in the dividend and divisor equal. As a consequence, there are no decimal places in the quotient.

```
.25 ) 326.00 ( 1304   Ans.
      25
      --
       76
       75
       --
        100
        100
        ---
          0
```

Example 5.—Divide .0025 by 1.25.

Solution.—According to Art. **18**, the divisor and dividend may be considered as whole numbers in dividing. When considered as such, it is seen that the dividend 25 will not contain the divisor 125; a cipher must therefore be annexed to the number 25, thus making the dividend 250. As 125 is contained twice in 250, the quotient is 2. In its present form the dividend contains 5 decimal places; there are 2 decimal places in the divisor. The difference, 5−2=3, is the number of the required decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient.

```
1.25 ) .00250 ( .002   Ans.
          250
          ---
            0
```

20. Short division should be used not only for whole numbers, but also for decimals, whenever it is advantageous to apply it. For instance, examples 2 and 3 in the preceding series can easily be solved by short division. Example 3 would be solved as follows:

Example.—Divide .125 by .005.

Solution.—The explanation accompanying the solution of example 3, Art. **19**, applies also here.

$$\begin{array}{rl} \textit{divisor} & \textit{dividend}\\ .005) & \underline{.125} \\ \textit{quotient} & 25 \quad \text{Ans.}\end{array}$$

21. Division With a Remainder.—In the foregoing examples, the divisor was contained in the dividend without any remainder. The method to follow in case there is a remainder is shown in the following examples:

Example.—What is the quotient of 199 divided by 15?

Solution.—In solution (a) the dividend and the divisor are treated as whole numbers, and the remainder, 4, is used as the numerator in a fraction where the divisor, 15, is the denominator. The quotient is a mixed number consisting of the whole number 13 and the fraction $\frac{4}{15}$, or $13\frac{4}{15}$

$$\begin{array}{rl}
 & (a)\\
 & 15)\,199\,(13+\tfrac{4}{15},\text{ or }13\tfrac{4}{15}\quad\text{Ans.}\\
 & \underline{15}\\
 & 49\\
 & \underline{45}\\
\textit{remainder} & 4
\end{array}
\qquad
\begin{array}{rl}
 & (b)\\
 & 15)\,199.000\,(13.266+\quad\text{Ans.}\\
 & \underline{15}\\
 & 49\\
 & \underline{45}\\
 & 40\\
 & \underline{30}\\
 & 100\\
 & \underline{90}\\
 & 100\\
 & \underline{90}\\
\textit{remainder} & 10
\end{array}$$

In solution (b) the dividend is treated as a decimal, a decimal point being placed to the right of the dividend and a number of ciphers added corresponding to the number of decimal places required in the quotient. In this case 3 ciphers are added, but it is noted that after including these in the division there is still a remainder of 10; in fact, this example belongs to one of those cases in which the division does not end. If the fraction $\frac{4}{15}$ in solution (a) is reduced to a decimal (by the method to be explained later), it becomes .266. This shows that the quotients obtained in solutions (a) and (b) are equal, or, $13\frac{4}{15}=13.266$.

22. In many cases of decimal division there is a remainder, so that the quotient can be continued without a limit. Decimals of this kind are known as **interminate.** In the case of an interminate decimal it is necessary to decide how many decimal places are required in the quotient. Three decimal places are sufficient for most practical purposes, and more than four or five are rarely needed. After the number of places required has been decided on, the work of division is carried one place farther. This extra figure, which is subsequently omitted, serves as a means for a closer adjustment of the last figure in the required decimal. Thus, if the extra figure is 5 or a greater number, the preceding figure is increased by 1, a minus (−) sign being annexed to it to show that the quotient is not quite as large as indicated.

For instance, if in the last example in Art. **21** it had been required to obtain the answer correct to 4 decimal places, the work should have been carried to 5 places, obtaining 13.26666. Here, the fifth, or extra, figure is larger than 5, hence in omitting it, the fourth figure is increased by 1, and the required answer is 13.2667−.

If the extra figure in the decimal is less than 5, the preceding figure is left unaltered, when omitting the extra figure, but a plus sign (+) is written in its place to indicate that the true value of the quotient is slightly greater than that given. For example, if it is desired to retain 4 decimal places in the number 73.41823 it would be expressed as 73.4182+, the plus sign indicating that the true result is slightly larger than that shown.

These remarks apply to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain 3 decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain 5 decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

23. In works of a scientific nature and in calculations dealing with minute quantities, where it is desirable to make a distinction between exact and interminate decimals, it is customary to annex a cipher to the last place of the exact decimal. Thus, by adding a cipher to the decimal .375 and writing .3750 it is intended to convey the information that the decimal is without a remainder.

In tables consisting of exact and interminate decimals it is sometimes necessary, for the sake of uniformity, to add several ciphers to an exact decimal. Thus, in the following column there are added two ciphers to the exact decimal 5, so as to obtain the same number of places as in the adjoining decimals.

$$\begin{array}{r}14.392\\12.500\\27.183\end{array}$$

24. Repeating Decimals.—When in dividing decimals the divisor is not contained in the dividend without a remainder, the division may be continued indefinitely; that is, the quotient is interminate. In such cases one figure of the quotient may continue to repeat itself. Thus, in dividing 1 by 6 the quotient is .1666..., the figure 6 continuously repeating itself. The decimal is known as a **repeating decimal,** or **repeater.** The fact that a figure is a repeater is indicated by placing a dot over it; thus, the preceding quotient is written $.1\dot{6}$. An instance of a repeating decimal was found in the example of Art. **21.**

Instead of only one figure repeating itself, it is possible for two or more to do so. For instance, on finding the quotient of $1 \div 7$ by short division, as $\begin{array}{l}7\,)\,\underline{1.000000000000}\\ \quad .14285\dot{7}142857\end{array}$, it is found that a whole group of figures keeps on repeating, the group consisting of the figures 1,4,2,8,5,7. This repeating decimal, which is sometimes called a **circulating decimal,** is written $.\dot{1}4285\dot{7}$, the two dots indicating the first and the last figure of the repeating group of figures.

EXAMPLES FOR PRACTICE

1. In the following examples, divide:

(*a*) 101.6688 by 2.36.
(*b*) 187.12264 by 123.107.

Ans. $\begin{cases}(a)\ 43.08\\(b)\ 1.52\end{cases}$

2. A bar 24.375 inches long is divided into equal parts measuring 1.625 inches in length. How many parts are there? Ans. 15

3. In the manufacture of a number of machines 612 pounds of bronze was required. If each machine used 12.75 pounds, how many machines were there? Ans. 48

4. The cost of 18.75 tons of coal was 60.75 dollars. What was the cost per ton? Ans. 3.24 dollars

5. A keg of boiler rivets weighs 100 pounds and contains 595 rivets. What is the weight of one of the rivets? Ans. .168+ lb.

6. A bar, shown on a drawing, measures 18.75 inches in length. If it has to be divided into 6 equal parts, what will be the length of each part? Ans. 3.125 in.

7. A spool contains 75.6 pounds of copper wire. How many coils, weighing 18.9 pounds each, can be made from the wire on the spool? Ans. 4 coils

8. Five gallons of ready-mixed paint are required to cover a surface measuring 1,377.5 square feet. What will be the size of the surface, measured in square feet, that may be covered by 1 gallon of paint? Ans. 275.5 square feet

9. If a carpenter can lay 5,750 shingles in 5.75 days, how many shingles can he lay in 1 day? Ans. 1,000 shingles

REDUCTION OF FRACTIONS AND DECIMALS

REDUCING FRACTIONS TO DECIMALS

25. Rule for Reduction.—Common fractions must frequently be changed to decimals; for example, fractions of an inch must be changed to equivalent decimals. The following rule is used:

Rule.—*To reduce a common fraction to a decimal, place a decimal point after the numerator, annex ciphers to the right of the point, and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed to the numerator.*

EXAMPLE 1.—Find the decimal that is equivalent to the fraction $\frac{3}{4}$.

Solution.—Using short division the solution is as shown. As two ciphers were annexed to the numerator, it follows from the rule that two decimal places must be pointed off in the quotient.

$$\begin{array}{r} 4\,)\underline{3.00} \\ .75 \end{array}$$

or $\frac{3}{4}=.75$ Ans.

Example 2.—What decimal is equivalent to $\frac{7}{8}$?

Solution.—

$$\begin{array}{l} 8\,)\,7.000\,(.875 \\ \underline{6\,4} \\ \,6\,0 \\ \,\underline{5\,6} \\ \,4\,0 \\ \,\underline{4\,0} \\ \,0 \end{array}$$

or $\frac{7}{8}=.875$ Ans.

Example 3.—What decimal is equivalent to $\frac{4}{11}$?

Solution.—Performing the division as shown, it is found that there is a remainder and that the division may be carried on indefinitely. It is also seen that the quotient is a repeating decimal. Complying with the rules given in Art. **22**, the answer may be written in either of the following forms: .3636+, or .364.

$$\begin{array}{l} 11\,)\,4.0000\,(.3636 \\ \underline{3\,3} \\ \,7\,0 \\ \,\underline{6\,6} \\ \,4\,0 \\ \,\underline{3\,3} \\ \,7\,0 \\ \,\underline{6\,6} \\ \,4 \end{array}$$

or $\frac{4}{11}=.3636$ Ans.

26. Table of Decimal Equivalents.—The decimal equivalents given in Table I should be calculated by applying the rule in Art. **25.** In each case the numerator should contain a number of ciphers sufficient to allow the division to be completed without a remainder, since none of the decimals are repeating decimals. The table is convenient for reference, but it is well to memorize the first portion marked "8ths."

TABLE I

DECIMAL EQUIVALENTS

8ths

$\frac{1}{8}=.125$
$\frac{1}{4}=.25$
$\frac{3}{8}=.375$
$\frac{1}{2}=.5$
$\frac{5}{8}=.625$
$\frac{3}{4}=.75$
$\frac{7}{8}=.875$

16ths

$\frac{1}{16}=.0625$
$\frac{3}{16}=.1875$
$\frac{5}{16}=.3125$
$\frac{7}{16}=.4375$
$\frac{9}{16}=.5625$
$\frac{11}{16}=.6875$
$\frac{13}{16}=.8125$
$\frac{15}{16}=.9375$

32ds

$\frac{1}{32}=.03125$
$\frac{3}{32}=.09375$
$\frac{5}{32}=.15625$
$\frac{7}{32}=.21875$
$\frac{9}{32}=.28125$
$\frac{11}{32}=.34375$
$\frac{13}{32}=.40625$
$\frac{15}{32}=.46875$
$\frac{17}{32}=.53125$
$\frac{19}{32}=.59375$
$\frac{21}{32}=.65625$
$\frac{23}{32}=.71875$
$\frac{25}{32}=.78125$
$\frac{27}{32}=.84375$
$\frac{29}{32}=.90625$
$\frac{31}{32}=.96875$

EXAMPLES FOR PRACTICE

Reduce the following common fractions to decimals:

(*a*) $\frac{4}{5}$		(*a*) .8
(*b*) $\frac{7}{9}$		(*b*) .778−
(*c*) $\frac{8}{15}$	Ans.	(*c*) .533+
(*d*) $\frac{23}{40}$		(*d*) .575
(*e*) $\frac{19}{26}$		(*e*) .731−

REDUCING DECIMALS TO FRACTIONS

27. Rules for Reduction.—To reduce a decimal to a common fraction of the same value, the following rule is used:

Rule.—*To reduce a decimal to a common fraction, place under the figures of the decimal the figure 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

Example 1.—Reduce .125 to a fraction.

Solution.— $.125=\frac{125}{1000}=\frac{25}{200}=\frac{5}{40}=\frac{1}{8}$. Ans.

Example 2.—Reduce .875 to a fraction.

Solution.— $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$. Ans.

28. In practice it often happens that a decimal must be reduced to a specified fractional part of a unit, as, for instance, to sixteenths or thirty-seconds of an inch. When, as in this case, the denominator of the fraction is given, the following rule is applied:

Rule.—*To reduce a decimal to a fraction with a given denominator, multiply the decimal by the given denominator and beneath the product place that denominator.*

It will be noted that the method outlined in the preceding rule is equivalent to that of dividing a unit into any number of parts desired, and of these parts taking a number of parts equivalent to the decimal fraction. For instance, if it is required to find the equivalent of .75 inch in sixteenths of an inch, the unit is divided into sixteenths, as $\frac{16}{16}$; $\frac{16}{16} \times .75 = \frac{16 \times .75}{16} = \frac{12}{16}$. Thus, .75 inch is equal to $\frac{12}{16}$ inch.

Example 1.—If a steel plate plate is .5827 inch thick, what is its equivalent thickness in 64ths of an inch?

Solution.—According to the example, the decimal .5827 is to be reduced to a fraction having 64 as its denominator. Applying the rule,

$$.5827 \times \frac{64}{64} = \frac{.5827 \times 64}{64} = \frac{37.2928}{64}$$

As the numerator in the latter fraction is a mixed number, it must be replaced by the nearest equivalent whole number. 37.2928 is nearer to 37 than to 38 because the difference between 37.2928 and 37, or .2928, is smaller than the difference between 38 and 37.2928, which is .7072. So 37 is chosen and the decimal is omitted. It follows that .5827 inch $= \frac{37}{64}$ inch, nearly. Ans.

Example 2.—Change .3917 to 12ths.

Solution.—According to the requirements the decimal .3917 is to be changed to an equivalent proper fraction having 12 as a denominator. Applying the rule,

$$.3917 \times \frac{12}{12} = \frac{.3917 \times 12}{12} = \frac{4.7004}{12}$$

The number 4.7004 being nearer 5 than 4, the number 5 is chosen as the nearest equivalent. That is, $.3917 = \frac{5}{12}$, nearly. Ans.

EXAMPLES FOR PRACTICE

1. Reduce the following to common fractions:

(*a*) .25.
(*b*) .625
(*c*) .3125.
(*d*) .04.
(*e*) .06.

Ans.
(*a*) $\frac{1}{4}$
(*b*) $\frac{5}{8}$
(*c*) $\frac{5}{16}$
(*d*) $\frac{1}{25}$
(*e*) $\frac{3}{50}$

2. Express:

(*a*) .625 in 8ths.
(*b*) .3125 in 16ths.
(*c*) .15625 in 32ds.
(*d*) .77 in 64ths.
(*e*) .81 in 48ths.
(*f*) .923 in 96ths.

Ans.
(*a*) $\frac{5}{8}$
(*b*) $\frac{5}{16}$
(*c*) $\frac{5}{32}$
(*d*) $\frac{49}{64}$
(*e*) $\frac{39}{48}$
(*f*) $\frac{89}{96}$

DECIMAL CURRENCY

29. Examples of Calculations.—In the United States and some other countries the **dollar** is the unit in which money values are expressed. The dollar is 100 **cents**; that is, 1 cent is one-hundredth of a dollar. Represented as a decimal 1 cent =.01 dollar, and 25 cents=.25 dollar. The sign $ is the *dollar mark,* and is usually placed in front of a number representing dollars or cents, when the latter number is given as a decimal part of a dollar. For example, $12 is read *twelve dollars,* and $.25, or $0.25, is read *twenty-five cents.* The expression $12.25 is read *twelve dollars and twenty-five cents.* The decimal point is used to separate the dollars from the cents. All to the left of the decimal point represents dollars, and the two figures to the right of the decimal point represent cents, the cents being hundredths of a dollar.

Calculations in which dollars and cents are used are made in the same way, and according to the same rules, as calculations using decimals, because an expression like $1.15 is a decimal, and represents $1\frac{15}{100}$ dollars. The following examples, with their solutions, will serve to illustrate the methods of using dollars and cents in calculations:

Example 1.—A workman's wages are \$2.85 a day. How much does he earn in 26 days?

Solution.—In 26 days he will earn 26 times as much as in 1 day, or 26×\$2.85=\$74.10. Ans.

Example 2.—The total cost of making 95 pieces of work was \$235.60. What was the cost per piece?

Solution.—The cost per piece is equal to the total cost divided by the number of pieces, or \$235.60÷95=\$2.48. Ans.

Example 3.—If a man receives \$36.94 on pay day and immediately pays bills amounting to \$19.67, how much has he left?

Solution.—He will have left the difference between what he received and what he paid out, or \$36.94−\$19.67=\$17.27. Ans.

Example 4.—A certain piece of work requires four men to complete it. If these men are paid \$12.80, \$21.16, \$13.54, and \$6.15, what is the cost of the labor?

Solution.—The labor cost must be the sum of the four amounts, or \$12.80+\$21.16+\$13.54+\$6.15=\$53.65. Ans.

EXAMPLES FOR PRACTICE

1. A workman is paid \$27.60 for 6 days' work. How much does he earn per day? Ans. \$4.60

2. If a certain article costs \$.45, what will 48 of these cost? Ans. \$21.60

3. Three different grades of pig iron were purchased at \$14.25, \$19.65, and \$17.40 a ton. What was the average cost per ton? Ans. \$17.10

4. A man works 112 hours at \$.46 an hour, and draws \$28.75 on his pay. How much is still due him? Ans. \$22.77

5. How much remains when 67 cents is taken from \$10.36? Ans. \$9.69

6. If a man receives \$3.20 per day of 8 hours, how much does he receive after working 16½ hours? Ans. \$6.60

7. If a certain product costs 8¾ cents per pound, what will be the cost of 350 pounds? Ans. \$30.62½

8. An ice-making plant has an output of 50 tons daily. If the ice costs \$3.60 per ton, what is the value of the output for 320 days? Ans. \$57,600

9. If lath costs $.55 per bundle, what must be paid for 45 bundles? Ans. $24.75

10. If a carpenter earns $350 in 12.5 weeks, what are his average wages per week? Ans. $28

PER CENT. AND PERCENTAGE

DEFINITIONS

30. Explanation of Term Per Cent.—The term **per cent.** is frequently used in connection with calculations of various kinds, as, for example, in expressing the composition of metals and other substances, profits on investments, etc. The term is an abbreviation of the Latin words *per centum,* and it means *by the hundred,* or *in the hundred.* The meaning of the term may be seen more clearly from the following explanations and examples.

If a man has $100 and spends $50 of it, then he has spent $\frac{50}{100}$ or 50 per cent. of his money. If he spends $20 more, then he has spent $\frac{50+20}{100}=\frac{70}{100}$ or 70 per cent. of the money. Supposing that he spends $10 more, he has spent 80 per cent. and $20 is left, which amounts to $\frac{20}{100}$, or 20 per cent., of the money he had.

Again, if it is stated that the population of Scranton, Pa., increased 6 per cent. in the period from 1910 to 1920, it is equivalent to saying that the increase was 6 in every hundred; that is, for every 100 in 1910, there were 6 more, or 106, in 1920.

31. As another example let it be supposed that the composition of a certain specimen of solder is such that a bar of solder, weighing 10 pounds, contains 6 pounds of lead. Then the weight of the lead is $\frac{6}{10}$ of the weight of the bar. But, $\frac{6}{10}=\frac{60}{100}$ and $\frac{60}{100}$ is 60 per cent. of the bar. That is, the bar contains 60 per cent. of lead.

Example 1.—Find the weight of tin in 150 pounds of solder, if there is 40 per cent. of tin, by weight, in the solder.

Solution.—The weight of tin is 40 per cent., or 40 hundredths, of the whole weight of solder. Therefore, the weight of the tin is $\frac{40}{100}$ of 150 pounds, or $\frac{40}{100}\times150$ pounds$=\frac{40\times150}{100}=60$ pounds. Ans.

EXAMPLE 2.—A tank contains 75 gallons of a sulphuric-acid solution. If there is 47 per cent. of sulphuric acid in the solution, how many gallons of acid were used?

SOLUTION.— 47 per cent. of 75 gallons is $75\times\frac{47}{100}=75\times.47=35.25$ gallons of sulphuric acid. Ans.

32. Sign of Per Cent.—The **sign** of per cent. is %; thus, 2% means 2 per cent. and is read *two per cent.* The figure before the sign % indicates how many per cent. is meant, or the number of hundredths. Thus, 6% is read *6 per cent.,* and means 6 hundredths; $12\frac{1}{2}$% is read *twelve and a half per cent.,* and means $12\frac{1}{2}$ hundredths.

1% of a number means $\frac{1}{100}$ of that number; 5% of a number is $\frac{5}{100}$ of the number; and so on. Thus, 1% of 200 is 2, because $\frac{1}{100}$ of 200 is $\frac{1}{100}\times200=2$. If a man is receiving 60 cents an hour for his work and he gets an increase of 10% in his wages, his increase amounts to 10% of 60, or $\frac{10}{100}\times60=6$ cents; then, his new rate of pay is $60+6=66$ cents an hour.

33. Omission of Per Cent. Sign.—The per cent. sign, %, may be dropped and the number of per cent. expressed in several other ways. For example, 12% may be written 12 per cent. Or, the per cent. may be changed to a common fraction, thus, $6\%=\frac{6}{100}$; $25\%=\frac{25}{100}$; $12\frac{1}{2}\%=\frac{12\frac{1}{2}}{100}=\frac{12.5}{100}=\frac{125}{1000}$; $51.6\%=\frac{51.6}{100}=\frac{516}{1000}$. In all of these examples the per cent. sign is omitted.

34. Per Cent. Expressed as a Decimal.—If the per cent. is to be changed to a decimal instead of to a common fraction, the % sign is dropped and the number of decimal places is increased by two by moving the decimal point two places to the left. For example, suppose that 25% is to be changed to a decimal having the same value. Mentally, this number may be supposed to be written 25.0%. The sign % is dropped and the number of decimal places in the number is increased by two by moving the decimal point two places to the left, giving .250, or .25; that is, 25% = .25. That the method just given is correct may be proved by means of Art. **33.**

According to the latter, $25\% = \frac{25}{100}$; but, $\frac{25}{100} = .25$. Therefore, $25\% = \frac{25}{100} = .25$.

Suppose that 2.5% is to be changed to a decimal. There is already one decimal place in the number, and the two more that must be prefixed when the sign % is dropped make a total of three decimal places; then, $2.5\% = .025$, a cipher being used to give the necessary number of places. Again, $.25\% = .0025$. The number of decimal places to be pointed off in this case is $2+2=4$, and so two ciphers must be prefixed.

35. When a per cent. is used in a calculation, it is always changed to a fraction or to a decimal. It is necessary, therefore, that the preceding rules, explaining the method of changing per cent. to a fraction and to a decimal, should be thoroughly understood. Table II shows various per cents. expressed as decimals and as common fractions.

TABLE II

PER CENTS. EXPRESSED AS DECIMALS AND FRACTIONS

Per Cent.	Decimal	Fraction	Per Cent.	Decimal	Fraction
1%	.01	$\frac{1}{100}$	150%	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500%	5.00	$\frac{500}{100}$ or 5
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}\%$	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}\%$	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}\%$	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}\%$	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}\%$	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}\%$	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}\%$	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

36. Percentage.—By the term **percentage** is meant the product obtained by multiplying a number by a per cent., expressed as a decimal. For instance, if it is desired to find a number that is 5% of 45, the per cent. is expressed as a decimal, becoming .05. The product $45 \times .05 = 2.25$ is the percentage. Note that *per cent.* and *percentage* do not mean the same thing.

CALCULATIONS INVOLVING PER CENT. AND PERCENTAGE

37. Finding Percentage, When Number and Per Cent. Are Known.—Calculations of percentage are often required in cases where the number is known and also the per cent. These calculations are performed according to the following rule:

Rule.—*To find the percentage of a number, multiply the number by the per cent. expressed as a decimal.*

Example 1.—What percentage is obtained by taking 36% of 125?

Solution.—According to the rule, 36% is first changed to a decimal, thus becoming .36. Then, 125×.36=45. Therefore, 36% of 125 gives a percentage of 45. Ans.

Example 2.—From a stock of 300 machines, 76% was sold. What was the percentage sold?

Solution.—The number sold was 76% of 300. Expressed as a decimal, 76%=.76. Then, 300×.76=228. The percentage sold was 228 machines. Ans.

Example 3.—A brass casting weighs 348 pounds. The brass is made up of 65% of copper and 35% of zinc. (*a*) What is the percentage of copper in the casting? (*b*) What is the percentage of zinc in the casting, both percentages being expressed in pounds?

Solution.—(*a*) The weight of copper is 65% of the whole, or 65% of 348. Expressed as a decimal, 65%=.65. The percentage of copper is, then,

348×.65=226.2 lb. Ans.

(*b*) The percentage of zinc is

348×.35=121.8 lb. Ans.

Another way of finding the percentage of zinc is to subtract the percentage of copper from the weight of the casting, or 348−226.2 =121.8 pounds, as before.

38. Finding Per Cent., When Number and Percentage Are Known.—When several numbers are combined into one whole, it is frequently required to find what per cent. each number is of the whole number. These numbers represent, in reality, percentages and the question is to ascertain what per cent. of the whole is represented by each percentage. For example, if a certain weight of copper and a certain weight of tin are melted together to form bronze, it may be necessary to know the per cent. of copper and the per cent. of tin. The constituent parts of copper and tin represent percentages, and the problem amounts, in reality, to this: Given the weight of the casting and the percentages of copper and of tin to find what per cent. of copper and of tin there are in the casting. The following rule applies to problems of this class:

Rule.—*To find what per cent. of the whole is represented by a given percentage, divide the percentage by the whole, and multiply the result by 100.*

EXAMPLE 1.—What per cent. is employed to obtain 16 as a percentage of 64?

SOLUTION.—In this example, 64 is the whole and 16 the percentage. Then, applying the rule, 16÷64=.25. Finally, .25×100=25. Therefore, the percentage 16 is 25% of 64. Ans.

EXAMPLE 2.—In a bronze casting weighing 98 pounds there is a percentage of copper equal to 83.3 pounds. What per cent. of copper is there in the casting?

SOLUTION.—Applying the rule, 83.3÷98=.85, and .85×100=85%. Ans.

EXAMPLE 3.—From a stock of 300 motors a percentage of 228 was sold. What per cent. was sold?

SOLUTION.—Following the rule, 228÷300=.76, and .76×100=76; that is, 76% was sold. Ans.

EXAMPLE 4.—A certain alloy contains percentages of copper, tin, and zinc, equal to 144 pounds, 27 pounds, and 9 pounds, respectively. Find (*a*) the per cent. of copper, (*b*) the per cent. of tin, and (*c*) the per cent. of zinc in the alloy.

NOTE.—An *alloy* is a solidified combination of two or more metals, mixed while in a molten condition.

SOLUTION.—The total weight of the alloy is equal to the sum of the percentages, or 144+27+9=180 pounds.

(*a*) The percentage of copper being 144 pounds, by the rule, 144÷180 =.8, and .8×100=80%; that is, there is 80% of copper in the alloy. Ans.

(*b*) The percentage of tin being 27 pounds, 27÷180=.15, and .15× 100=15%; that is, there is 15% of tin in the alloy. Ans.

(*c*) The percentage of zinc is 9 pounds. Hence, by the rule, 9÷180=.05, and .05×100=5%; that is, there is 5% of zinc in the alloy. Ans.

39. Finding Number, When Percentage and Per Cent. Are Known.—Sometimes a number is given which is known to represent a certain percentage of another number, not known. It is desired to find the latter, the per cent. employed being known. The number is found according to the following rule:

Rule.—*To find a number of which the percentage and the per cent. are known, divide the percentage by the per cent. expressed as a decimal.*

Example 1.—At 68 per cent. the percentage of a certain number is 101. What is the number?

Solution.—According to the rule, 68% is first changed to a decimal, thus becoming .68. Then, $101 \div .68 = 148\frac{9}{17}$. Therefore, the number, of which the percentage is 101 at 68%, is $148\frac{9}{17}$. Ans.

Example 2.—If the percentage of a certain number is 784 at $83\frac{1}{3}\%$, what is the number?

Solution.—The per cent. is first expressed as a decimal. For this purpose it is convenient to reduce $83\frac{1}{3}\%$ to an improper fraction; thus, $83\frac{1}{3}\% = \frac{83\frac{1}{3}}{100} = \frac{\frac{250}{3}}{100} = \frac{250}{3} \times \frac{1}{100} = \frac{250}{300}$. According to the rule, 784 is to be divided by $\frac{250}{300}$, or $784 \div \frac{250}{300} = 784 \times \frac{300}{250} = 940.8$. The required number is, therefore, 940.8. Ans.

40. Sum of Percentages Equal to Whole Number. It should be noted that when the composition of a body is given in per cents. of its parts, the sum of the per cents. is equal to 100. For instance, in example 3, Art. **37**, there is 65% of copper and 35% of zinc, or a total of $65 + 35 = 100\%$. Also, it is important to note that the *sum* of the numbers that represent the percentages of a thing or quantity must be equal to the number that represents the whole thing or quantity under consideration. For instance, in example 3, Art. **37**, there are percentages of copper and zinc weighing 226.2 and 121.8 pounds, respectively. The sum of these percentages, or $226.2 + 121.8 = 348$ pounds, is equal to the weight of the brass casting. Also, if a certain percentage of a thing or quantity is taken away, the remaining percentage or percentages must be equal to the difference between the whole quantity and the percentage removed. Thus, supposing a water tank holding 8,000 gallons has lost 12% by leakage; the amount remaining is $100 - 12 = 88\%$. This can easily be proved. If 12% leaks away, the percentage lost is, according to Art. **37,** 8,000×.12 =960 gallons, and the percentage left in the tank is 8,000−960 =7,040 gallons. Applying the rule in Art. **38,** 7,040÷8,000 =.88, or 88%. In other words, the water remaining is 88% of the original amount, which is exactly the same result as was obtained by subtracting 12% from 100%.

EXAMPLES FOR PRACTICE

1. What per cent. of
(*a*) 360 is 90?
(*b*) 125 is 25?
(*c*) 47 is 94?

Ans. (*a*) 25%
(*b*) 20%
(*c*) 200%

2. In a manufacturing plant an average of 3,640 pounds of coal per day was consumed. Alterations were made that resulted in a saving of 250 pounds per day. What was the per cent. of coal saved?

Ans. 6.9%, nearly

3. If the speed of an engine running at 128 revolutions per minute should be increased 6¼%, how many revolutions would it then make?

Ans. 136 revolutions

NOTE.—Calculate the number of revolutions corresponding to 6¼% and add these to the present number.

4. Mr. A borrows $1,100 from his neighbor, Mr. B. This sum is 18½% of the total amount that Mr. B has on interest in the bank. How much money has Mr. B? Ans. $5,945.95

NOTE.—The $1,100 borrowed is a percentage and the total amount is found by the rule, Art. **39.**

5. If an electric generator is rated at 250 kilowatts and is guaranteed to be capable of developing 25% overload, (*a*) what is this overload? (*b*) What is the total output of the generator while developing this overload?

Ans. (*a*) 62.5 kilowatts
(*b*) 312.5 kilowatts

NOTE.—A kilowatt, abbreviated K. W., is a unit of electric power.

6. A railway system has 540 cars in operation, out of which 45 must be replaced by new ones. (*a*) What per cent. of cars must be replaced? (*b*) What per cent. of them are still serviceable?

Ans. (*a*) 8⅓%
(*b*) 91⅔%

7. A tank containing 10,000 gallons of water loses in a given time 160 gallons by leakage. (*a*) What per cent. is lost by leakage? (*b*) What per cent. remains in the tank?

Ans. (*a*) 1.6%
(*b*) 98.4%

8. One cubic yard of a certain concrete mixture requires 4 bags of cement, each bag containing 100 pounds. A certain quantity of lime is

to be added to this mixture, amounting to 3 per cent. of the weight of cement used. How many pounds of lime will be required for each cubic yard of concrete made? Ans. 12 lb.

9. The gas bill for last month was 25% higher than the bill for this month, which amounts to $6.40. What was the amount of last month's bill? Ans. $8.00

NOTE.—25% of $6.40 is to be added to $6.40.

ACCURACY OF CALCULATIONS

REQUIRED NUMBER OF SIGNIFICANT FIGURES

41. Accuracy of Measurements.—Results may be obtained by arithmetical operations that are absolutely correct. But, absolute accuracy in the practical application of the results is in many cases an impossibility. Measurements are liable to error, however carefully made, owing to imperfections of the measuring instruments, erroneous methods of applying them, and unavoidable errors in reading their indications. For example, the measurement of a distance between two given points would appear to be a simple operation, but on attempting to set off a distance of, say, 35.12 inches, thus measuring the length to the hundredth part of an inch, one will appreciate the attending difficulties. The distance, as first set off, may appear as absolutely correct, but let the operation be repeated, taking every precaution to avoid errors, and the results may differ by several hundredths of an inch.

Results based on the average of a series of measurements cannot be expected to be of a greater degree of accuracy than that of the instruments with which such measurements were made. Conversely, it is a waste of time and labor to use very accurate and sensitive instruments to obtain results that are required to be only approximately correct.

42. Significant Figures.—The preceding remarks lead to the consideration of the number of *significant figures* that should be retained in the readings, calculations, and results in order to obtain results within the accuracy of the measurements used. By **significant figures** are meant the figures of a number between the first digit at the left and the last digit at the right, it being understood that a cipher is considered as a figure but not as a digit.

For example, the numbers 20,467, 28.321, 45.673, and .00010569 each have five significant figures. If it were necessary to use only four significant figures, these numbers would be written 20,470, 28.32, 45.67, and .0001057, respectively; that is, if the figure dropped is 5 or greater, the next figure to the left is increased by 1. If the dropped figure is less than 5, the next figure to the left is not changed.

A cipher may be a significant figure. For example, the numbers 405.63, 300.64, and .12005 each have five significant figures. When one or more ciphers are located between two digits, as in the preceding examples, the ciphers become significant figures.

43. The part of a number containing significant figures is called the **significant part** of the number. Thus, in the number .00812, the significant part is 812, and in the number 170.3, the significant part is 1703.

In speaking of the significant figures or of the significant part of a number, the figures are considered in their proper order, from the first digit at the left to the last digit at the right, but no attention is paid to the decimal point. Hence, all numbers that differ only in the position of the decimal point have the same significant part. For example, .002103, 21.03, 2,103 have the same significant figures 2, 1, 0, and 3, and the same significant part 2103.

44. Number of Significant Figures Required.—The degree of accuracy with which calculations should be determined, either in whole numbers or in decimals, depends on the accuracy of the means employed for applying the results to practice or for obtaining the data employed in the calculations. It is evidently useless to employ great accuracy in calculating a practical problem, when the measuring instrument, by which the result is to be applied to practice, is incapable of a sufficiently accurate reading. For instance, it would be absurd to give a dimension in thousandths of an inch, when the rule employed in that particular case did not measure even thirty-seconds; or, to calculate air pressure to hundredths of a pound, when the pressure gauge cannot be read with certainty to tenths of a pound.

45. Generally speaking, in most practical problems, it is not necessary to use more than five significant figures, and in most cases only three to four figures are required. For instance, in a length of 812.75 feet, it would not be expected that the measurement would be accurate to within an inch. Hence, in the fraction .75, which means 9 inches, the last figure could be dropped and the figure 7 be replaced by 8. The length would now be indicated by the number 812.8 feet, which would be sufficiently accurate. As another example, let it be supposed that some chemical is weighed on an ordinary scale and is found to weigh 8 pounds 9 ounces = 8.5625 pounds. Under the circumstances one would not expect the reading to be accurate within an ounce; hence, the figure 5 could be dropped, and the weight be given as 8.563 pounds, which is sufficiently accurate. Again, in calculating the thickness of a steel plate as .81875 inch, this number would contain more decimals than could well be applied in practice, as the measuring instrument usually employed for measuring such dimensions cannot be read to more than 4 decimal places; thus, .81875 could be reduced to .8188 and be accurate enough for all practical purposes.

Again, to take an example referring to greater magnitudes, the average distance of the moon from the earth is 238,000 miles, a number with three significant figures. Considering the great distance and the difficulties involved in obtaining accurate results, the number is considered to be sufficiently accurate.

WEIGHTS AND MEASURES

DENOMINATE NUMBERS

ENGLISH MEASURES

DEFINITIONS

1. Varieties of Measures.—A **measure** is a standard unit, established by law or custom, by means of which a quantity of any kind may be measured. For example, the inch and the mile are *measures of length;* the pint and the gallon are *measures of capacity,* as used for liquids; the ounce and the ton are *measures of weight;* the second and the month are *measures of time,* and so on.

2. Denominate Numbers.—When a number is used in connection with measures, it becomes a **denominate number,** that is, a *named* number. Numbers are said to be of the *same denomination* when they are combined with similarly named units of measure, as 2 pounds and 7 pounds. A pound is a *unit of measure;* a foot, an hour are also units of measure, but of different kinds.

If a denominate number consists of units of measure of but one denomination, it is called a **simple denominate number** or a **simple number.** For example, 14 inches is a simple number, as it contains units of measure of but one denomination, namely, inches. The denominate numbers 16 cents, 10 hours, 6 gallons are all *simple* numbers, but they are not of the same *kind.*

If simple numbers of different denominations but of the same kind of measure are combined, the combination is called

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a compound denominate number, or a compound number. For example, 3 yards 2 feet 7 inches is a compound number, as it is a combination of the different denominations, yards, feet, and inches; these denominations are of the *same* kind, being all unit measures of length. Other examples of compound numbers are: 2 pounds 3 ounces 10 grains; 5 gallons 3 quarts 1 pint; 10 hours 14 minutes 32 seconds.

A unit measure that is larger than another unit measure is said to be of a **higher denomination;** if smaller than another unit measure, it is of a **lower denomination.** Thus, a *foot* is of a higher denomination than an *inch* and of a lower denomination than a *yard.*

3. Systems of Measures.—In modern practice two systems of measures are employed: the *English system* and the *metric system.* The **English system** is in general use in the United States, Great Britain, and Canada. The **metric system** is used on the European continent and to some extent in the United States, as, for instance, in chemistry and pharmacy. This system is a decimal system; that is, the values of the different units of the same kind increase or decrease by tens, 10 units of each denomination making 1 unit of the next higher denomination.

ABBREVIATIONS

4. Abbreviations of Units.—In writing denominate numbers, it is convenient to use *abbreviations* instead of writing out the name of the unit in full. For example, the names *inch* and *inches* may be abbreviated to *in.;* thus, write 5 *in.* instead of 5 *inches.* Similarly, 8 *ft.* means 8 *feet.* Sometimes, the marks (′) and (″) are used for feet and inches, respectively, in particular on drawings; thus 6′ 2″ means 6 feet 2 inches. The abbreviations commonly used for the various units are given in the following tables, each table showing the relation between various units of measure of the same kind. These tables should be memorized, including the abbreviations. The practice of writing the tables from memory will be found helpful.

MEASURES OF EXTENSION

LINEAR MEASURE

5. Definitions.—Measures of extension are used to measure lengths of lines, areas of surfaces, and contents of solids. When speaking of lines it is to be understood that a *mathematical line* has no breadth or thickness and merely indicates extension in one direction; that is, *length.* A *straight line* is frequently defined as the shortest distance between two points. The lengths of lines are expressed in **linear measures.**

TABLE OF LINEAR MEASURE

		Abbreviations
12 inches (in.)	=1 foot	ft.
3 feet	=1 yard	yd.
$5\frac{1}{2}$ yards $16\frac{1}{2}$ feet	=1 rod	rd.
320 rods 5,280 feet	=1 statute mile	mi.
6,080 feet	=1 nautical mile	naut. mi.
3 nautical miles	=1 nautical league	naut. l.

Note.—The statute mile is used for measuring distances on land, lakes, and rivers. The nautical mile is used for measuring distances on the ocean.

Example 1.—A piece of shafting is 7 feet long. What is its length in inches?

Solution.—According to the table, 1 ft.=12 in.; therefore, 7 ft. must be equal to

$$7\times12=84 \text{ in.} \quad \text{Ans.}$$

Example 2.—The width of a door is 48 inches. What is the width in feet?

Solution.—According to the table, 12 in.=1 ft.; therefore, the number of feet in 48 in. is

$$48\div12=4 \text{ ft.} \quad \text{Ans.}$$

Example 3.—How many feet are there in 3 statute miles?

Solution.—According to the table, 5,280 ft.=1 mi.; therefore, 3 mi. must be equal to

$$3\times5{,}280=15{,}840 \text{ ft.} \quad \text{Ans.}$$

LAND MEASURE

6. Surveyor's Linear Measure.—**Surveyor's measure,** also known as **land measure,** is used for measuring land, as in locating and laying out railways, roads, and tracts or building plots.

TABLE OF SURVEYOR'S LINEAR MEASURE

		Abbreviations
7.92 inches	=1 link	li.
100 links 66 feet	=1 chain	ch.
80 chains	=1 statute mile	mi.

The surveyor's chain (also called Gunter's chain) of 100 links, each 7.92 inches long, has been used very extensively in the past and is still common, but its use is decreasing. Chains or steel tapes 50 or 100 feet in length, graduated in feet and decimals of a foot and sometimes in feet and inches, are being employed to a large extent at the present time.

Note.—In Mexico, and in those parts of the United States that belonged to Mexico previous to 1845, the old Mexican measures of length are sometimes used, and referred to in early surveys. The principal units are:

1 vara=2.75 feet=33 inches, English measure
5,000 vara=1 league=2.604 miles, English measure

Example 1.—How many inches are there in 50 links?

Solution.—According to the table 1 li.=7.92 in.; therefore, 50 links must be equal to

$$50\times7.92=396 \text{ in. Ans.}$$

Example 2.—A line set out by a surveyor is found to measure 40 chains in length. What is the equivalent length in feet?

Solution.—According to the table, 1 ch.=66 ft.; therefore, 40 chains must be equal to

$$40\times66=2{,}640 \text{ ft. Ans.}$$

Example 3.—A boundary line between two estates is found to measure 440 chains in length. What is the equivalent length in statute miles?

Solution.—According to the table, 1 mi.=80 ch. The problem consists of finding how many times 80 ch. is contained in 440 ch.; or,

$$440\div80=5\tfrac{1}{2} \text{ mi. Ans.}$$

SQUARE MEASURE

7. Definitions.—The term **area** means the extent of a surface within its boundary lines For example, the area of a floor is the extent of the visible surface limited by the four surrounding walls. As area refers to extent of surface, the unit of measurement must also be a surface. The **unit of area** is a *square*, which is a surface of the form shown in Fig. 1.

By the term **square** is meant a four-sided figure in which the sides are of equal length and at *right angles* to one another; that is, any two adjoining sides meet each other squarely. A **square inch** is a square that is 1 inch long on each side. A **square foot** is a square that measures 1 foot on each side, as in Fig. 1. This may be made to represent square inches by dividing each side into twelve equal parts, 1 foot being equal to 12 inches, and drawing lines across the square from the points of division on one side to those on the opposite side. The square will then appear as in Fig. 2, divided into a number of small squares. As each side of the square was divided into twelve equal parts, each part is $\frac{1}{12}$ foot long, or $\frac{1}{12}\times 12 = 1$ inch long, and each little square therefore measures 1 inch on each side; that is, each little square is 1 square inch. Now, if the total number of little squares is counted, it will be found to be 144. In other words, there are 144 square inches in 1 square foot.

FIG. 1

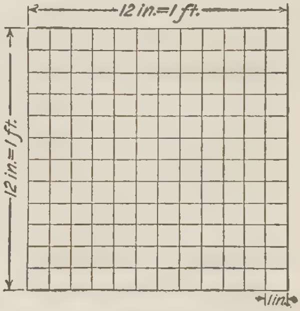


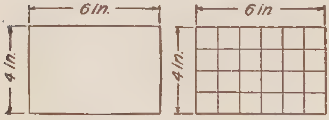
FIG. 2

8. Application of Square Measure.—A square inch, a square foot, etc. belong to the units of **square measure,** which is employed for measuring the extent of areas, such as floors, building lots, estates, etc. Square measure is also used to determine the surface areas of objects, such as engine cylinders, condensers, pipes, etc. Small areas are measured in square inches or square feet, and larger areas in square yards, square rods, acres, and square miles. The accompanying table gives the principal units of square measure.

TABLE OF SQUARE MEASURE

		Abbreviations
144 square inches (sq. in.)	=1 square foot	sq. ft.
9 square feet	=1 square yard	sq. yd.
30¼ square yards	=1 square rod	sq. rd.
160 square rods	=1 acre	A.
640 acres	=1 square mile	sq. mi.

9. Finding Areas of Squares and Rectangles. Instead of finding the area of a square by counting the number of small squares contained by it, as in Art. **7,** it is possible to find the area more easily by simply multiplying the length of the square by its width. Thus, referring to Fig. 2, there are 12 rows of the little squares and 12 squares in each row or 12×12=144 square inches, which is the same result as obtained by counting. When this method of multiplying the length by the width is employed it is important to note that the length and the width must be in the *same* units. In the case just mentioned, both the length and the width were in inches, and the area as a result is in square inches. With the length and the width in feet, the area will be 1 foot×1 foot=1 square foot.



(a) (b)

FIG. 3

Sometimes the area to be measured is a **rectangle;** that is, a figure similar to that shown in Fig. 3 (*a*). It differs from a square in so far that not *all* the sides are of equal length, but only those sides that are opposite each other. The area of a rectangle is found in the same way as that of a square; hence, the following rule applies to both figures:

Rule.—*To find the area of a square or a rectangle, multiply the length by the width, both being expressed in the same units.*

Thus, if the rectangle in Fig. 3 (*a*) is 6 inches long and 4 inches wide, its area is 6×4=24 square inches. This can be proved as shown in (*b*), by dividing each side into inches, drawing lines across the figure from the points of division, and counting the number of squares thus formed. Each of these squares is 1 inch on each side, or 1 square inch in area, and there are 24 in all, or 24 square inches.

10. Difference Between the Terms Square Feet and Feet Square.—Distinction must be made between expressions, such as "4 square feet" and "4 feet square," as they do not mean the same thing. When a square is referred to as being equal to *4 square feet,* reference is had to its *area.* It follows from the preceding rule that each of its sides must be of a length equal to 2 feet, in order that the area may be 2×2=4 square feet.

The expression "4 feet square" does not refer to the *area* of the square, but to *length of the sides,* and means that each side of the square is equal to 4 feet in length. The area of the latter square is 4×4=16 square feet.

11. Application of Rule for Finding Areas of Squares and Rectangles.—The application of the rule in Art. **9** is illustrated by means of the following examples:

Example 1.—How many square inches are there in a rectangle 28 inches long and 13 inches wide?

Solution.—According to the rule, the area is

28×13=364 sq. in. Ans.

Example 2.—A sheet of copper is 4 feet long and 3 inches wide. What is the area of its surface?

Solution.—Both dimensions must be expressed in the same units; reducing 4 ft. to inches, the result is 12×4=48 in. Applying the rule, the area is

48×3=144 sq. in., or 1 sq. ft. Ans.

Example 3.—The base of an office cabinet is 6¼ feet long and 2½ feet wide. What area of floor space does it cover?

Solution.—According to the rule, the area is

$$6\tfrac{1}{4}\times 2\tfrac{1}{2}=\frac{25}{4}\times\frac{5}{2}=\frac{125}{8}=15\tfrac{5}{8}\text{ sq. ft.}\quad\text{Ans.}$$

12. Surveyor's Square Measure.—The square measure used by surveyors differs somewhat from that given in Art. **8.** The various units employed are included in the following table:

TABLE OF SURVEYOR'S SQUARE MEASURE

		Abbreviations
10,000 square links (sq. li.)	=1 square chain	sq. ch.
10 square chains	=1 acre	A.
640 acres	=1 square mile	sq. mi.
36 square miles	=1 township	Tp.

Note.—The acre contains 4,840 square yards or 43,560 square feet, and it is equal to the area of a square measuring 208.71 feet on each one of the sides.

Example 1.—How many square links are there in 11 square chains?

Solution.—According to the table, 1 sq. ch. is equal to 10,000 sq. li.; therefore, 11 sq. ch. must be equal to

11×10,000=110,000 sq. li. Ans.

Example 2.—A piece of land contains 85 square chains. What is the equivalent value in acres?

Solution.—There are 10 sq. ch. in an acre; therefore, the number of acres contained in 85 sq. ch. must be

85÷10=8.5 A. Ans.

Example 3.—How many acres are there in a township?

Solution.—As one township contains 36 sq. mi. and each square mile contains 640 A., it follows that the number of acres contained in one township must be

36×640=23,040 A. Ans.

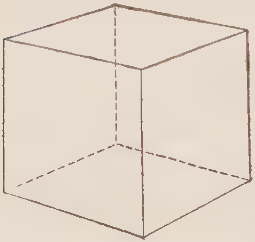


Fig. 4

CUBIC MEASURE

13. Definitions.—A **solid** is a figure that has length, breadth, and thickness. The boundaries of a solid are surfaces. A solid bounded by six squares, as Fig. 4, is a **cube.** If the solid is bounded by six rectangular surfaces it is a **rectangular solid.**

The space included between the bounding surfaces of a solid is called the **cubical contents, capacity,** or **volume** of the solid.

The **unit of volume** is a cube, the edges of which are of a length equal to that of the corresponding unit of length. Thus, 1 **cubic inch** measures 1 inch on each edge; 1 **cubic foot** measures 1 foot on each edge, etc.

TABLE OF CUBIC MEASURE

		Abbreviations
1,728 cubic inches (cu. in.)	=1 cubic foot	cu. ft.
27 cubic feet	=1 cubic yard	cu. yd.
128 cubic feet	=1 cord	cd.
16½ to 25 cubic feet	=1 perch	P.

14. The **standard cord** is a measure of wood; it is a pile 8 feet long, 4 feet wide, and 4 feet high. In some localities it is customary to consider the cord as a pile of wood 8 feet long and 4 feet high, the length of the wood, that is, the width of the pile, being left out of consideration.

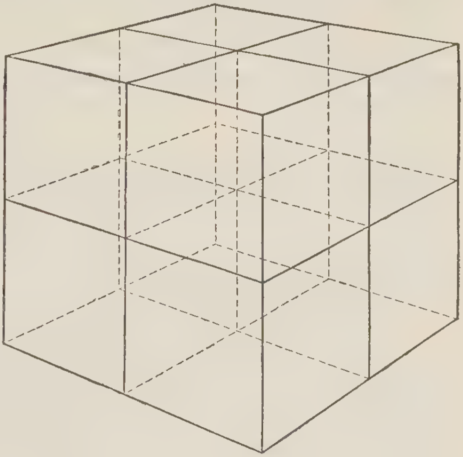


Fig. 5

The **perch,** a measure of stone and masonry, is but rarely used at present. Its cubical contents varies, according to locality, from 16½ to 25 cubic feet. The dimensions will vary accordingly; if the contents is 24¾ cubic feet, the perch would be 16½ feet, or 1 rod, long, 1½ feet wide, and 1 foot high.

15. Rule for Calculating Volume.—When writing the three dimensions of a solid, as 3 feet wide by 2 feet deep by 6 feet long, the word "by" is frequently represented by the symbol ×, and the preceding expression is written: 3 ft. ×2 ft.×6 ft.

The volume of any rectangular solid is the product of its three dimensions. For example, a box with inside dimensions 2 ft.×4 ft.×8 ft. contains 2×4×8=64 cubic feet; an iron bar 10 feet (or 10×12 inches) long, and with end surfaces that are each 1 inch square contains 10×12×1×1=120 cubic inches.

A cube 2 feet on each edge is a 2-foot cube, and contains 2×2×2=8 cubic feet, as shown in Fig. 5. A cube 12 inches, or 1 foot, on each edge is 1 cubic foot, or a 12-inch cube, and contains 12×12×12=1,728 cubic inches. A cube 3 feet, or 1 yard, on each edge is 1 cubic yard, or a 3-foot cube, and contains 3×3×3=27 cubic feet. Tables of cubic measure can be calculated from tables of linear measure in this manner.

The preceding method of calculating the volume of a rectangular solid may be stated as a rule, as follows:

Rule.—*To find the volume of a cube or a rectangular solid, multiply together the length, breadth, and depth, all expressed in the same units.*

If the dimensions are stated in inches, the volume is given in cubic inches; if in feet, the volume is given in cubic feet, and so on.

Example 1.—A sand bin is 14 feet long, 8 feet wide, and 6 feet high. How many cubic feet does it contain?

Solution.—According to the rule, it contains

14×8×6=672 cu. ft. Ans.

Example 2.—How many cubic inches of metal are there in a block 12½ inches long, 8½ inches wide, and 4 inches thick?

Solution.—Applying the rule, the volume is

$$12\tfrac{1}{2}\times 8\tfrac{1}{2}\times 4=\frac{25}{2}\times\frac{17}{2}\times\frac{4}{1}=425 \text{ cu. in.} \quad \text{Ans.}$$

Example 3.—A box contains 86,400 cubic inches. What is the contents expressed in cubic feet?

Solution.—According to the table, 1 cu. ft. contains 1,728 cu. in. The problem is, therefore, to ascertain how many times 1,728 is contained in 86,400, or,

86,400÷1,728=50 cu. ft. Ans.

16. Board Measure.—A special branch of cubic measure is known as **board measure.** In measuring sawed lumber, the unit of measure is the **board foot,** which is equal to the contents of a board 1 foot square and 1 inch thick. A board foot is, therefore, equal to one-twelfth of a cubic foot. Boards less than 1 inch thick are usually reckoned as though the thickness were 1 inch; but in buying and selling, the actual thickness is considered in fixing the price.

The following rule may be used for finding the number of feet, board measure (B. M.), in a piece of lumber:

Rule.—*To find the number of feet, board measure, in a piece of lumber, multiply together the length, in feet, the width, in inches, and the thickness, in inches, and divide the product by 12, thicknesses less than 1 inch being considered as 1 inch.*

Example 1.—How many feet, B. M., are there in a board 14 feet long, 8 inches wide, and 1 inch thick?

Solution.—According to the rule the number of feet, B. M., is equal to $\frac{14\times8\times1}{12}=9\frac{1}{3}$ ft. B. M. Ans.

Example 2.—How many feet, B. M., are there in a plank 10 feet 6 inches long, 15 inches wide, and 3 inches thick?

Solution.—Length = 10 ft. 6 in. $=10\frac{1}{2}$ ft. = 10.5 ft. The number of feet, B. M., is

$$\frac{10.5\times15\times3}{12}=39.375=39\tfrac{3}{8}\text{ ft. B. M.}$$

Example 3.—Find the number of feet, B. M., in 20 pieces of siding $\frac{1}{2}$ inch thick, $5\frac{1}{2}$ inches wide, and 10 feet 9 inches long.

Solution.—Since the thickness is less than 1 in., the boards are considered to be 1 in. thick when finding the amount of material in them. Length $=10\frac{9}{12}$ ft. = 10.75 ft., width $=5\frac{1}{2}$ in. = 5.5 in. The required length is

$$\frac{20\times10.75\times5.5\times1}{12}=\frac{1{,}182.5}{12}=98.5\text{ ft. B. M. Ans.}$$

Example 4.—How many feet, B. M., of 2-inch planking are needed for a barn floor 18 ft.×25 ft.?

Solution.—The planks are considered as 25 ft. long. Reducing the width to inches and considering the whole floor as one plank, its width is equal to 18×12=216 in. Proceeding as in the rule, the number of feet required is

$$\frac{25\times216\times2}{12}=900 \text{ ft. B. M. Ans.}$$

Example 5.—A board is 9½ ft.×4 in.×1¼ in. How many feet, B. M., does it contain?

Solution.—Applying the rule,

$$\frac{9\frac{1}{2}\times4\times1\frac{1}{4}}{12}=3.96 \text{ ft. B. M., nearly. Ans.}$$

MEASURES OF CAPACITY

LIQUID MEASURES AND DRY MEASURES

17. Liquid Measure.—The term **capacity** means cubical contents and is used here in connection with measures to indicate the relative amount of space they possess for measuring purposes.

Liquid measure is used for measuring liquids. Liquids in small quantities are measured in pints and quarts; in larger quantities, the gallon is the more common unit, though quantities are sometimes stated in barrels, but rarely in hogsheads. The standard barrel, as used in measuring capacity, contains 31½ gallons, but the barrels ordinarily used vary greatly in size.

One United States standard liquid gallon, known as the *Winchester* or wine gallon, contains 231 cubic inches. One gallon of pure water weighs 8.355, or approximately 8⅓ pounds. One cubic foot contains 7.481, or approximately 7.5, gallons, and 1 cubic foot of water, at 62° Fahrenheit, weighs 62½ pounds, nearly.

In some English-speaking countries the beer or ale gallon of 282 cubic inches' capacity is used for measuring the liquids mentioned.

The British imperial gallon, used in Canada, contains 277.463 cubic inches, and the weight of such a gallon of pure water is 10 pounds. One imperial gallon equals approximately 1.2 United States liquid gallons.

TABLE OF LIQUID MEASURE

English System

		Abbreviations
4 gills (gi.)	=1 pint	pt.
2 pints	=1 quart	qt.
4 quarts	=1 gallon	gal.
31½ gallons	=1 barrel	bbl.
2 barrels	=1 hogshead	hhd.

18. Dry Measure.—Dry articles, such as fruit, grain, vegetables, etc. are measured by **dry measure.** The standard unit is the United States bushel, otherwise known as the *Winchester bushel,* which contains 2,150.42 cubic inches; and ½ peck (dry gallon) contains 268.8 cubic inches.

One British imperial bushel contains 2,219.704 cubic inches, or 1.0322 United States bushels.

The quart used in dry measure is not the same as the quart used in liquid measure. The dry quart contains 67.2 cubic inches, and the liquid quart only 57¾ cubic inches.

TABLE OF DRY MEASURE

		Abbreviations
2 pints (pt.)	=1 quart	qt.
8 quarts	=1 peck	pk.
4 pecks	=1 bushel	bu.

19. The application of the preceding tables is shown in the following examples:

Example 1.—How many quarts are there in 2 barrels?

Solution.—According to the table of liquid measure there are 4 qt. in 1 gal., and 31½ gal. in 1 bbl. Therefore, in 2 bbl. there are

$$2\times 31.5=63.0 \text{ gal.}$$

and

$$63\times 4=252 \text{ qt. Ans.}$$

Example 2.—How many hogsheads will be required for 315 gallons of molasses?

Solution.—According to the table of liquid measure, 1 hhd.=2 bbl., and 1 bbl.=31½ gal. In 2 bbl. there are 2×31½=63 gal. Therefore, 315 gal. require

$$315\div 63=5 \text{ hhd. Ans.}$$

Example 3.—A man bought 10 bushels of potatoes and intends to sell them by the quart. How many quarts of potatoes has he on hand?

Solution.—According to the table of dry measure, 1 bu.=4 pk., and 1 pk.=8 qt., so that in 1 bu. there are 4×8=32 qt.; in 10 bu. there are

10×32=320 qt. Ans.

MEASURES OF WEIGHT

AVOIRDUPOIS AND TROY MEASURES

20. Standard Units of Weight.—There are two English measures of weight in general use, the **avoirdupois** (av.) **weight,** for coarse, heavy substances, such as coal, iron, copper, hay, and grain, and the **Troy weight,** for finer and more valuable substances, such as gold, silver, and jewels. Besides there is the **apothecaries' weight,** which is used by physicians in prescribing and by druggists in compounding medicines. Medicines are bought and sold by avoirdupois weight.

TABLE OF AVOIRDUPOIS WEIGHT

		Abbreviations
27.3438 grains (gr.)	=1 dram	dr.
16 drams	=1 ounce	oz.
16 ounces (oz.)	=1 pound	lb.
100 pounds	=1 hundredweight	cwt.
20 hundredweight 2,000 pounds	=1 ton	T.

TABLE OF TROY WEIGHT

		Abbreviations
24 grains (gr.)	=1 pennyweight	dwt.
20 pennyweights	=1 ounce	oz.
12 ounces	=1 pound	lb.

TABLE OF APOTHECARIES' WEIGHT

		Abbreviations
20 grains (gr.)	=1 scruple	sc.
3 scruples	=1 dram	dr.
8 drams	=1 ounce	oz.
12 ounces	=1 pound	lb.

21. One **avoirdupois pound** equals approximately 1.2153 **Troy pounds,** the former containing 7,000 Troy grains and the latter 5,760 grains. One avoirdupois ounce contains 7,000÷16=437.5 grains, and 1 Troy ounce contains 24×20=480 grains. It is to be noted that 1 avoirdupois grain is equal to 1 Troy grain and to 1 apothecaries' grain.

22. The **ton** containing 2,000 pounds is sometimes called the *short ton* to distinguish it from the *long ton,* containing 2,240 pounds, or 20 hundredweights of 112 pounds each. The short ton is in general use except in the following cases: The *long* ton is used in the United States custom houses, and at the mines for weighing anthracite. It is also used by wholesale dealers in weighing steel rails and a few other iron products, when shipped in bulk. The denomination *hundredweight* is rarely used in practical work, especially with short-ton measures.

23. Gross Weight and Net Weight.—The **gross weight** of an article means the total weight, including the package, crate, or containing vessel; the **net weight** means the weight of the article alone. For example, the gross weight of a reel of wire is the weight of the wire and reel together, and the net weight of the wire is the gross weight less the weight of the reel.

24. Application of Tables of Weight.—The application of the preceding tables of weights is illustrated by the following examples:

Example 1.—If a short ton of coal costs $6.00, how much is the cost per pound?

Solution.—According to the table there are 2,000 lb. in 1 T. As 600 cents is the price of 2,000 lb., it follows that the price of 1 lb. is $\frac{1}{2000}$ of 600, or 600÷2,000=.3, or $\frac{3}{10}$ cent.

If the price is expressed in terms of dollars, the operation and answer will be: 6.00÷2,000=.003 dollar. Ans.

Example 2.—If a bag of sugar weighs 800 ounces, what is its weight in pounds?

Solution.—According to the table, 1 lb.=16 oz.; therefore, dividing 800 by 16 will give the equivalent number of pounds; thus,

800÷16=50 lb. Ans.

Example 3.—How many grains are there in 1 pound of gold?

Solution.—According to the table 1 lb. Troy=12 oz., and 1 oz. is equal to 20 dwt. of 24 gr.; hence, 1 lb. is equal to

$$12 \times 20 \times 24 = 5{,}760 \text{ gr.} \quad \text{Ans.}$$

MEASURES OF TIME

UNITS AND STANDARD TIME

25. Units of Time.—The principal unit of time is the **second.** A minute contains 60 seconds, and a day contains 24 hours, or 86,400 seconds. The time required for the earth to make one complete revolution around the sun is called 1 **solar year,** and is exactly 365 days 5 hours 48 minutes 49.7 seconds.

TABLE OF MEASURES OF TIME

		Abbreviations
60 seconds (sec.)	=1 minute	min.
60 minutes	=1 hour	hr.
24 hours	=1 day	da.
7 days	=1 week	wk.
365 days 12 months	=1 common year	yr.
366 days	=1 leap year	
100 years	=1 century	C.

26. As the solar year contains more than 365 days, the fractional part is omitted in the **common year,** the latter containing 365 days. To compensate for this omission the fractional parts of several years are added periodically to one year, which is then known as a **leap year.** It contains 366 days, the extra day being added to the month of February.

When the number expressing a year is divisible by 4 and is not divisible by 100, that year is a leap year. A centennial year is a leap year, if the number expressing the year is divisible by 400. Thus, 1912, 1916, and 1920 are leap years, because each is exactly divisible by 4 and is not divisible by 100. The centennial year 1900 was not a leap year, because the number 1900 is not divisible by 400. The year 2000 will be a leap year, because the number is divisible by 400.

27. Standard Time.—Five **standard time divisions** are in use in the United States, Canada, and Mexico, and each division is bounded by lines running north and south. These divisions named from east to west are *Atlantic, Colonial,* or *Intercolonial; Eastern; Central; Mountain;* and *Pacific.* The United States lies almost wholly in the four divisions last named.

Time of day is usually stated as A. M., meaning *ante meridiem,* or *before noon,* and P. M., meaning *post meridiem,* or *after noon,* followed by the name of the division when this might otherwise be uncertain. Thus, the time of day may be stated as 10:45 A. M., Central time, or 11:45 A. M., Eastern time. The difference in time in adjacent divisions is exactly 1 hour, and is earlier in each westward division. Thus, when it is noon in Washington, D. C., the time is 11 A. M. in Chicago, 10 A. M. in Denver, and 9 A. M. in San Francisco.

28. Months and Number of Days.—The numbers and names of the months and the number of days in each are as follows:

No.	Name	Number of Days	No.	Name	Number of Days
1.	January (Jan.)	31	7.	July	31
2.	February (Feb.)	28 or 29	8.	August (Aug.)	31
3.	March (Mar.)	31	9.	September (Sept.)	30
4.	April (Apr.)	30	10.	October (Oct.)	31
5.	May	31	11.	November (Nov.)	30
6.	June	30	12.	December (Dec.)	31

For many business purposes, 30 days is considered to be 1 month.

MEASURES OF CURRENCY

AMERICAN AND ENGLISH

29. Units of Money.—The term **currency** refers to money, such as coins, bank notes, etc. The United States measure of money being based on the decimal system, it is possible to write fractions of dollars as decimals. The number of dollars is written as a whole number followed by a decimal fraction representing the number of cents. Thus, the expression 7 dollars and 25 cents is written $7.25. Sometimes, the decimal contains a third place representing *mills;* but the mill is not a coin. It is simply used as a convenient means for representing a tenth part of a cent. For instance, the expression $.065 is read *6 cents and 5 mills.*

Numbers representing pounds, English money, are preceded by the sign £ and followed by numbers representing other denominations each with its abbreviation; thus, £4 12s. 6d. is read *4 pounds 12 shillings 6 pence,* the word *pence* being plural of *penny.*

£1=$4.866½ A crown=5s.
A guinea=21s. A florin =2s.

The following tables give measures of currency as used in the United States and Great Britain. In Canada the units of money are the same as in the United States.

TABLE OF UNITED STATES MONEY

		ABBREVIATIONS
10 mills (m.)	=1 cent	c. or ct.
10 cents	=1 dime	d.
10 dimes	=1 dollar	$
10 dollars	=1 eagle	E.

TABLE OF ENGLISH MONEY

		ABBREVIATIONS
4 farthings (far.)	=1 penny	d.
12 pence	=1 shilling	s.
20 shillings	=1 pound, or sovereign	£

MEASURES OF ARCS AND ANGLES

CIRCULAR AND ANGULAR MEASURES

30. Definitions.—The measure of arcs and angles is called **circular,** or **angular, measure,** and is applied to the measurement of arcs, angles, and circles. A **circle** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the **center.** The line that bounds a circle is called its **circumference.** Any straight line drawn from the center to the circumference, as *c a, c b,* or *c d,* Fig. 6, is called a **radius;** if a radius is extended to meet the circumference also on the opposite side of the center, as *b c e,* it is called a **diameter.** Any portion of the circumference of a circle, as *b d,* is called a **circular arc,** or simply an **arc.**

The whole circumference of any circle is supposed to be divided into 360 equal parts, and each of these equal parts is called a **degree.** Since 1 degree is $\frac{1}{360}$ of any circumference, it follows that the length of an arc of 1 degree will be different in circles of different diameters, but the length of the circumference of any circle will always be 360 times as large as the length of the arc of 1 degree. Or, the arc of 1 degree is always $\frac{1}{360}$ of the circumference, whatever the size of the circle.

31. An **angle** is the opening between two straight lines that meet at a point. Thus, in Fig. 6, the lines *b c* and *d c* form an angle at the point *c.* These lines are the **sides** of the angle and the point *c* at which they meet is called the **vertex** of the angle.

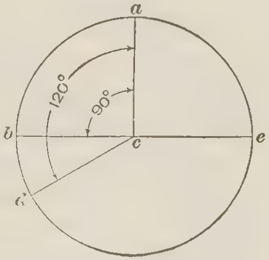


FIG. 6

If any circle is drawn with its center at the vertex of an angle, that portion of the circumference included between the sides of the angle is known as the intercepted arc. Such an angle is measured by its intercepted arc. For example, if the arc *a d* contains 120 degrees, the angle *a c d* is an angle of 120 degrees. For the same reason the angle *a c b* is equal to 90 degrees if the arc *a b* contains 90 degrees.

If a circumference is divided into four equal parts, each of the parts is called a **quadrant.** The arc *a b* is a quadrant and it contains 90 degrees.

A degree and its subdivisions, minutes and seconds, are indicated by the signs (°), (′), (″), both for angles and arcs; thus, 32° 15′ 10″ is read *thirty-two degrees fifteen minutes ten seconds.*

TABLE OF ANGULAR MEASURE

		Abbreviations
60 seconds (″)	=1 minute	′
60 minutes	=1 degree	°
90 degrees	=1 right angle or quadrant,	∟ or rt. ∠
360 degrees	=1 circle	cir.

MEASURES OF TEMPERATURE

FAHRENHEIT AND CENTIGRADE SCALES

32. Thermometer.—The **temperature** of a body is the measure of its degree of sensible heat; hot bodies are said to have *high* temperatures and cold bodies *low* temperatures. Temperature is usually measured by means of an instrument called a **thermometer.** It consists of a glass tube closed at the upper end and having at the lower end a bulb filled with mercury. When the thermometer is placed in contact with a hot body, the heat causes the mercury to expand, or take up more space, and so it rises in the hollow tube. If the body is cold, the mercury contracts, and occupies less space than before, and the column then descends or grows shorter. The greater the temperature, the higher the mercury rises in the tube; and the lower the temperature, the lower the mercury descends in the tube.

33. Thermometer Scales.—Two thermometric scales are in common use; one is the *Fahrenheit scale,* and the other the *centigrade scale.* A thermometer with these two scales, one on each side of the tube, is shown in Fig. 7. The one on the left, with *F* at the top, is the Fahrenheit scale, and the one at the right, with *C* above it, is the centigrade scale. The Fahrenheit thermometer, named after its inventor, is the one most commonly used in the United States. The centigrade thermometer is used largely in scientific work. The difference between the two is that the boiling point of water is marked 212 on the Fahrenheit scale and 100 on the centigrade; and the freezing point is 32 on the Fahrenheit scale and 0 on the centigrade. Each small division on each scale is a *degree* of temperature. The abbreviation for Fahrenheit is Fahr. or F., and the abbreviation for centigrade is Cent. or C.; thus, 180° F. means 180 degrees on the Fahrenheit scale, and 65° C. means 65 degrees on the centigrade scale.

34. In graduating thermometers, that is, making the scales of values on them, two fixed points of temperature are almost universally employed. These are the temperatures of melting ice, and of the steam of water boiling in an uncovered vessel. In graduating the centigrade scale the distance between these two points is divided into 100 equal graduations or degrees. The freezing point is called zero, or 0°, and the boiling point 100°. Degrees of the same value are carried above and below the boiling and freezing points, respectively. Distinction is made between the degrees above zero and those below by placing a minus sign before the latter; thus, 10° below zero is marked −10° and is read *minus ten degrees.*

35. In graduating the Fahrenheit scale, the distance between the freezing and the boiling point is divided into 180 equal parts, and degrees of the same size are carried above and below the boiling and freezing points. Fahrenheit, who proposed this scale, assumed that the greatest cold obtainable was 32° below the freezing point, and accordingly took that point as the zero and reckoned from it upwards. The freezing point thus became 32° F. and the boiling point 32° + 180° = 212°. Degrees below the Fahrenheit zero are provided with a minus sign, as are similarly placed degrees on the centigrade scale. On many Fahrenheit thermometers, the space between the freezing point and boiling point is divided into 90 equal parts, so that the division marks will not be so close together. In such a case, each division corresponds to 2 degrees, but each main division is numbered as in Fig. 7.

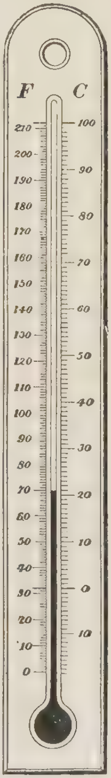


FIG. 7

36. Conversion of Thermometer Readings.—From the fact that 180° F. = 100° C., it follows that 9° F. = 5° C. and 1° F. = $\frac{5}{9}$° C., or 1° C. = $\frac{9}{5}$° F. These relative values form the basis of the following rules by which Fahrenheit temperatures may be changed into their corresponding centigrade values, or vice versa:

Rule I.—*To find the Fahrenheit temperature, multiply the centigrade temperature by $\frac{9}{5}$ and add 32 to the product.*

Rule II.—*To find the centigrade temperature, subtract 32 from the Fahrenheit temperature and multiply the remainder by $\frac{5}{9}$.*

Example 1.—What is the equivalent of 85° C. on the Fahrenheit scale?

Solution.—Applying rule I, the temperature is

$$(85\times\tfrac{9}{5})+32=185° \text{ F. Ans.}$$

Example 2.—If a temperature is 68° F., what is the equivalent on the centigrade scale?

Solution.—According to rule II, 32 is subtracted from 68 giving the remainder 36; then,

$$36\times\tfrac{5}{9}=20° \text{ C. Ans.}$$

37. Converting Variations of Temperature.—Distinction must be made between a *stationary* temperature and a *variation* of temperature when converting from one scale into the other. For example, suppose that the temperature of an electric generator rises 40° C.; since, according to Art. **36**, 1° C. = $\frac{9}{5}$° F., it follows that 40° C. = $\frac{9}{5}\times 40 = 72$° F., the equivalent change in Fahrenheit degrees. Again, suppose that the temperature of a room decreases from 180° F. to 99° F., or 81° F.; since 1° F. = $\frac{5}{9}$° C., 81° F. = $\frac{5}{9}\times 81 = 45$° C., the equivalent variation in centigrade degrees. In other words, when converting a variation of temperature, the height of the freezing point above 0° F., or 32°, is not considered.

MISCELLANEOUS MEASURES

38. Collective Terms.—The terms *pair, couple, gross, great gross,* and *score* are used in connection with some articles or things; for example, a pair of shoes, a dozen screws, a gross of shears, and a score of years. Paper is usually bought and sold by the *quire, ream,* or *bundle.*

TABLE OF MISCELLANEOUS MEASURES

		Abbreviations
2 things	=1 couple, or 1 pair	pr.
12 things	=1 dozen	doz.
12 dozen	=1 gross	gr.
12 gross	=1 great gross	g. gr.
20 things	=1 score	
24 sheets	=1 quire	qr.
20 quires	=1 ream	rm.
2 reams	=1 bundle	bdl.
5 bundles	=1 bale	B.

METRIC MEASURES

LINEAR MEASURE

39. Principal Units.—In the **metric system** a uniform scale of 10 is used throughout, as in the ordinary scale of numbers and in United States money. The name of this system is derived from the *meter,* the unit from which all the other units are derived. The metric system of measures is in general use on the continent of Europe and is used to some extent in the United States.

The unit of length is the **meter,** which has a length equal to 39.37 inches, nearly. One hundredth of a meter is called a **centimeter;** and one thousandth of a meter, a **millimeter.**

In the United States, engineers and mechanics are likely to meet with metric units in constructing machinery that has been designed abroad and is being built to order in this country. The units used on the drawings are usually the centimeter and the millimeter.

40. The relative lengths of centimeters and inches may be seen from Fig. 8 in which the upper scale is divided into inches and subdivisions, and the lower scale into centimeters and millimeters. The portion of the scale shown contains about 9 centimeters; each centimeter divided into 10 equal parts to represent millimeters. It requires 100 centimeters, or more than 10 times the length shown in the illustration, to make 1 meter. A millimeter is equal to .03937 inch and a centimeter is equal to .3937 inch; that is, 1 inch is equal to $\frac{1}{.03937}$ $=25.4$ millimeters, or 2.54 centimeters. The **kilometer** is 1,000 meters and is used in measuring great distances. It is equal to about $\frac{5}{8}$ mile.

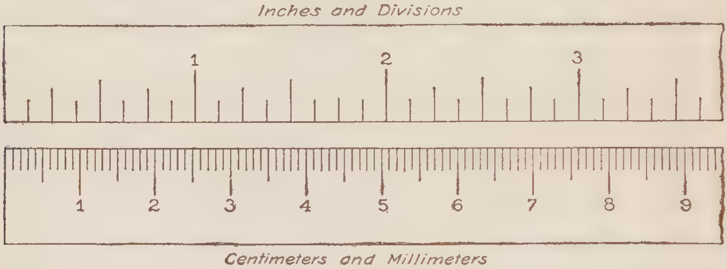


FIG. 8

The following equivalents, or conversion factors, are in most cases approximate: 1 inch = 25.4 millimeters = 2.54 centimeters; 1 foot = 30.5 centimeters = .305 meter; 1 meter = 39.37 inches = 3.28 feet; 1 mile = 1.61 kilometers; 1 kilometer = .6214 mile = 3,281 feet.

TABLE OF LINEAR MEASURE

		ABBREVIATIONS
10 millimeters (mm.)	= 1 centimeter	cm.
10 centimeters	= 1 decimeter	dm.
10 decimeters	= 1 meter	m.
1,000 meters	= 1 kilometer	km.

SQUARE MEASURE AND CUBIC MEASURE

41. Units of Square Measure.—Areas such as floors, ceilings, and ordinary surfaces are measured in square meters; countries and states, in square kilometers; and small areas, in square centimeters. Land is measured in hectares, 1 *hectare* being equal to 10,000 square meters, which is equivalent to 2.471 acres.

The principal metric units of square measure are the **square centimeter** (sq. cm.), which equals .155 square inch, and the **square meter** (sq. m.), which equals 1.195 square yards. One square inch = 6.45 square centimeters, and 1 square yard = .836 square meter.

TABLE OF SQUARE MEASURE

		Abbreviations
100 square millimeters (sq. mm.)	= 1 square centimeter	sq. cm.
100 square centimeters	= 1 square decimeter	sq. dm.
100 square decimeters	= 1 square meter	sq. m.

42. Units of Cubic Measure.—The metric units used in practical work are chiefly the **cubic centimeter** (cu. cm.), which equals .06102 cubic inch, and the **cubic meter** (cu. m.), or **stere,** which equals 1.31 cubic yards. One cubic inch = 16.388 cubic centimeters, and 1 cubic yard = .765 cubic meter.

TABLE OF CUBIC MEASURE

		Abbreviations
1,000 cubic millimeters (cu. mm.)	= 1 cubic centimeter	cu. cm.
1,000 cubic centimeters	= 1 cubic decimeter	cu. dm.
1,000 cubic decimeters	= 1 cubic meter	cu. m.

LIQUID AND DRY MEASURE

43. Units of Capacity.—Metric units of capacity are the same for both liquids and dry substances. The **liter** (l.) is the principal unit of capacity; it is equal in volume to a cube whose edge is 10 centimeters in length. One **hectoliter** equals 100 liters. One liter = 1,000 cubic centimeters = 2.1 pints (liquid) = .91 quart (dry). One hectoliter = 100 liters = 210 pints (liquid) = 91 quarts (dry). One quart (dry) = 1.1 liters; one gallon = 3.785 liters. The volume of 1 liter is equal to 1 cubic decimeter.

TABLE OF LIQUID AND DRY MEASURE

		Abbreviations
10 milliliters (ml.)	=1 centiliter	cl.
10 centiliters	=1 deciliter	dl.
10 deciliters	=1 liter	l.
10 liters	=1 decaliter	Dl.
10 decaliters	=1 hectoliter	Hl.
10 hectoliters	=1 kiloliter	Kl.

MEASURE OF WEIGHT

44. Units of Weight.—The **gram** is the principal metric unit of weight; it is the weight of 1 cubic centimeter of pure distilled water at its temperature of maximum density, or 39.2° F., and is equal to 15.432 grains, Troy. The gram is used in weighing small quantities of gold, silver, etc., also letters (for postage). For greater weights the **kilogram** is used; it is equal to the weight of 1 liter of pure water at a temperature of 39.2° F. One kilogram = 1,000 grams = 2.2046 pounds avoirdupois, and 1 pound (av.) = .4536 kilogram. One metric **ton** = 1,000 kilograms; its weight is equal to that of 1 cubic meter of water at a temperature of 39.2° F.

TABLE OF MEASURE OF WEIGHT

		Abbreviations
10 milligrams (mg.)	=1 centigram	cg.
10 centigrams	=1 decigram	dg.
10 decigrams	=1 gram	g.
10 grams	=1 decagram	Dg.
10 decagrams	=1 hectogram	Hg.
10 hectograms	=1 kilogram	kg.
1,000 kilograms	=1 metric ton	T.

Note.—It is to be noted that abbreviations of any units greater than the principal units meter, gram, and liter are capitalized. Thus, decaliter and decagrams are abbreviated *Dl.* and *Dg.*, respectively; and kilometer and kilogram are abbreviated *Km.* and *Kg.*, respectively. However, usage has established *km.* and *kg.* as the most common forms of abbreviation for the latter two units named.

OPERATIONS WITH DENOMINATE NUMBERS

REDUCTION

REDUCTION OF ENGLISH UNITS

45. Reduction to Higher or Lower Denominations.—The **reduction** of denominate numbers is the process of changing their denomination without changing their value. Reduction is divided into two cases. In one case, the number is reduced to units lower than the highest named in the number; in the other case, units of a low denomination are reduced to a higher one. The first is called *reduction descending;* the second, *reduction ascending.*

To change 2 miles 120 rods to feet would require an application of reduction descending, and to change 243,079 pounds to tons would involve reduction ascending.

46. Reduction Descending.—Reduction to a lower denomination, or reduction descending, is made by multiplying the number of the higher denomination by the number of units of the lower denomination contained in one unit of the higher denomination. For example, to reduce 2 hours to minutes, 2 is multiplied by 60, as 1 hour contains 60 minutes. Thus, $2\times60=120$ minutes. In reductions to lower denominations the following rule is applied:

Rule.—*To reduce a compound denominate number to a lower denomination, multiply the number representing the higher denomination by the number of units in the next lower denomination required to make one unit of the higher denomination, and to this product add the number of units of the lower denomination.*

EXAMPLE 1.—Reduce 14 degrees 18 minutes 17 seconds to seconds.

Solution.—In order that the rule may be conveniently applied the numbers are arranged in the manner shown.

	14° 18′ 17″
Multiply degrees by number of minutes in 1 degree	60
	840′
Add minutes given in example	18
	858′
Multiply by number of seconds in 1 minute	60
	51480″
Add seconds given in example	17
	51497″ Ans.

Example 2.—How many seconds in 1 day?

Solution.—

Number of hours in 1 day	24
Multiply by number of minutes in 1 hr.	60
	1440 min.
Multiply by number of seconds in 1 min.	60
	86400 sec. Ans.

47. The preceding examples involve *more* than two different denominations. In practice, as a rule, only two denominations are considered, in which case the method of reduction can be much simplified. The following examples will show the method of procedure:

Example 1.—Reduce 64 quarts 1 pint to pints.

Solution.—Referring to the table of liquid measure, it is found that 1 qt.=2 pt. 64 qt. is equal to 64×2, or 128 pt.; therefore, 64 qt. 1 pt. =128 pt.+1 pt.=129 pt. Ans.

Example 2.—Reduce 6 tons 268 pounds to pounds.

Solution.—One ton contains 2,000 lb. Then, by the rule, 6×2,000 =12,000 lb., and 12,000+268=12,268 lb. Ans.

Example 3.—Change 5 hours 24 minutes to minutes.

Solution.—There are 60 min. in 1 hr. Therefore, according to the rule, 5×60=300 min., and 300+24=324 min. Ans.

Example 4.—How many inches in 3 feet 6½ inches?

Solution.—The number 3 is the higher denomination. According to the rule, this must be multiplied by 12, because it takes 12 of the lower units, or inches, to make one unit of the higher denomination, or 1 ft. Multiplying, 3×12=36 in. To this product is now added the 6½ in. in the given number, making a total of 36+6½=42½ in. Therefore, 3 ft. 6½ in.=42½ in. Ans.

Example 5.—How many square inches in a surface whose area is 3 square feet 28 square inches?

Solution.—In 1 sq. ft. there are 144 sq. in. Then, by the rule, 3×144=432 sq. in., and 432+28=460 sq. in. Ans.

EXAMPLES FOR PRACTICE

1. Reduce:

(*a*) 4 rd. 2 yd. 2 ft. to ft.
(*b*) 4 bu. 3 pk. 2 qt. to qt.

$$\text{Ans.}\begin{cases}(a)\ 74\text{ ft.}\\(b)\ 154\text{ qt.}\end{cases}$$

2. The length of an electric transmission line is 1 mi. 45 rd. 11 ft.; find the length, in feet, of the wire for the complete (two-wire) circuit. Ans. 12,067 ft.

3. A student purchased 8 lb. 5 oz. (av.) of chemicals for experimental work. What was its value at 8 cents an ounce? Ans. $10.64

4. If the bearings of a machine hold 3 pints of oil, how many times can they be filled from 2 barrels? Ans. 168 times

5. The pressure of the atmosphere at sea level is at a given time 14.7 pounds per square inch. What is the pressure per square foot? Ans. 2,116.8 lb.

---

48. Reduction Ascending.—Reduction to a higher denomination, or reduction ascending, is made by dividing the number of the lower denomination by the number of units of the lower denomination required to make one unit of the higher denomination. For example, to reduce 32 ounces (av.) to pounds, 32 is divided by 16, because 1 pound contains 16 ounces; thus, 32÷16=2 pounds. In reductions to higher denominations the following rule is applied:

Rule.—*To reduce a denominate number to a higher denomination, divide the number representing the denomination given by the number of units of this denomination required to make one unit of the higher denomination. The remainder will be of the same denomination, but the quotient will be of the higher. The quotient and the remainder together are the required result.*

Example 1.—Reduce 211 inches to yards, feet, and inches.

Solution.—There are 12 in. in 1 ft.; therefore, 211 in. divided by 12=17 ft. and 7 in. remainder. There are 3 ft. in 1 yd.; therefore, 17 ft. divided by 3=5 yd. and 2 ft. remainder. The last quotient and the two remainders constitute the answer, 5 yd. 2 ft. 7 in.

```
12)211(17 ft.
   12
   ──
    91
    84
    ──
     7 in. rem.
 3)17(5 yd.
   15
   ──
    2 ft. rem.
```

Example 2.—Reduce 3,557 pints to higher denominations.

Solution.—There are 2 pt. in 1 qt.; therefore, 3,557 pt. divided by 2=1,778 qt. and 1 pt. remainder. There are 4 qt. in 1 gal.; therefore, 1,778 qt. divided by 4=444 gal. and 2 qt. remainder. There are 31.5 gal. in 1 bbl.; therefore, 444.0 gal. divided by 31.5=14 bbl. and 3.0 gal. remainder.

The answer is made up of the last quotient and all the remainders; thus, 14 bbl. 3 gal. 2 qt. 1 pt. Note that the remainder after the last division is not 30 gal. but 3 gal., the decimal point being placed vertically under the point in the dividend.

2) 3557 (1778 qt.
2
15
14
15
14
17
16
1 pt. rem.

4) 1778 (444 gal.
16
17
16
18
16
2 qt. rem.

31.5) 444.0 (14 bbl.
315
1290
1260
3.0 gal. rem.

Example 3.—Reduce 18,000 square inches to square feet.

Solution.—

Divide by the number of square inches in 1 sq. ft. } 144) 18000 (125 sq. ft. Ans.
144
360
288
720
720

Example 4.—How many weeks in 18 days?

Solution.—It takes 7 days to make 1 week, that is, 7 units of the lower denomination to make 1 unit of the higher. By the rule, 18 is to be divided by 7 to get the result; thus,

7) 18 (2 weeks
14
4 days

The result is therefore 2 wk. 4 da. Ans.

Example 5.—Reduce 14,728 pounds to tons.

Solution.—One ton is equal to 2,000 lb. Then, according to the rule, 14,728 is to be divided by 2,000; thus,

2000) 14728 (7 T.
14000
728 lb.

Therefore, 14,728 lb.=7 T. 728 lb. Ans.

Example 6.—Reduce 945 pints to higher denominations.

Solution.—The solution is found in the manner shown in example 2. The final quotient and the various remainders are as follows: 3 bbl.

23.5 gal. 1 pt. The decimal .5 gal. must be reduced to the next lower denomination by multiplying it by the number of quarts in 1 gal., or $.5\times4=2$ qt. Hence, the final answer is: 3 bbl. 23 gal. 2 qt. 1 pt.

EXAMPLES FOR PRACTICE

1. Reduce the following to units of higher denominations. If fractions appear in the results in any but the lowest denomination, reduce them to lower denominations.

(*a*) 7,460 sq. in. Ans. (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
(*b*) 2,395 pt. (liq.) Ans. (*b*) 9 bbl. 15 gal. 3 qt. 1 pt.

2. How many tons in 459,875 lb.? Ans. 229.9375 T.

3. If 36 cu. ft. of coal weighs 1 T., how much space is filled by 7 T. 1,590 lb.? Ans. 280.62 cu. ft.

4. How many rails, each 30 feet long, will be required to lay a railroad track 26 miles long? Ans. 9,152

Note.—Two lines of rails are to be considered.

REDUCTION OF METRIC UNITS

49. Reduction to Higher or Lower Denominations. In the metric system it is easy to reduce the values of different denominations to the next lower or to the higher denomination, as all that is necessary is to multiply or divide by 10 for linear measures. If one or more of the intermediate denominations are omitted, the factor 10 is increased to 100 or to 1,000, as the case may be. For instance, on reducing meters to centimeters, the number of meters is multiplied by 100, as 1 meter = 100 centimeters. Reducing kilograms to grams, the number of kilograms is multiplied by 1,000, as 1 kilogram = 1,000 grams. A few examples will show the method of procedure.

Example 1.—Reduce 8 kilograms to grams.

Solution.—From Art. **44,** one kg. = 1,000 g.; hence,

$$8\text{ kg.}=8\times1{,}000\text{ g.}=8{,}000\text{ g.}\quad\text{Ans.}$$

Example 2.—Express 7.65 cubic meters in cubic centimeters.

Solution.—From the table of cubic measure, 1 cu. m. = 1,000 cu. dm., and 1 cu. dm. = 1,000 cu. cm. It follows that 7.65 cu. m. $=7.65\times1{,}000\times1{,}000$ cu. cm. = 7,650,000 cu. cm. Ans.

Example 3.—Reduce 97.56 grams to kilograms.

Solution.—From Art. **44,** one kg.=1,000 g. It follows that

$$97.56 \text{ g.}=\frac{97.56}{1,000}=.09756 \text{ kg.} \quad \text{Ans.}$$

Example 4.—What is the weight of 37 cubic millimeters of water?

Solution.—From the table of cubic measure, 1 cu. cm.=1,000 cu. mm. It follows that

$$37 \text{ cu. mm.}=\tfrac{1}{1000}\times 37 \text{ cu. cm.}=.037 \text{ cu. cm.}$$

From Art. **44,** 1 cu. cm. of water weighs 1 g.; therefore, the weight of 37 cu. mm. of water is

$$.037\times 1 \text{ g.}=.037 \text{ g.} \quad \text{Ans.}$$

REDUCTION FROM ONE SYSTEM TO ANOTHER

50. Conversion of Metric Units to English Units. In changing metric units to English units, the general rule to follow is to multiply the number of metric units by the equivalent of that unit in the desired English units. The method can most easily be illustrated by examples, as follows:

Example 1.—A machine part has a diameter of 115 millimeters. What is its diameter in inches?

Solution.—From Art. **40,** the equivalent of 1 mm. is .03937 in. Therefore, the equivalent of 115 mm. is

$$115\times .03937=4.52755 \text{ in., or } 4\tfrac{17}{32} \text{ in., nearly.} \quad \text{Ans.}$$

Example 2.—If a vessel has a volume of 248 cubic centimeters, what is its volume in cubic inches?

Solution.—From Art. **42,** the equivalent of 1 cu. cm. is .06102 cu. in. The volume of the vessel in cubic inches, therefore, is

$$248\times .06102=15.13 \text{ cu. in.} \quad \text{Ans.}$$

51. Conversion of English Units to Metric Units. In order to change English units to metric units divide the number of English units by the equivalent of the desired metric unit in English units; or, as multiplication is simpler than division, multiply the number of English units by the number of metric units equivalent to the English unit.

Example 1.—If a bar is $6\tfrac{3}{8}$ inches long, what is its length in millimeters?

Solution.—According to Art. **40**, 1 in. contains 25.4 mm. Therefore,

$$6\tfrac{3}{8} \times 25.4 = 162 \text{ mm., very nearly. Ans.}$$

Example 2.—If a tank of oil weighs 280 pounds, what is its weight in kilograms?

Solution.—By Art. **44**, 1 lb.=.4536 kg.; hence,

$$280 \times .4536 = 127 \text{ kg. Ans.}$$

EXAMPLES FOR PRACTICE

1. A disk has a diameter of 3¼ inches. What is its diameter in millimeters? Ans. 82.55 mm.

2. Find the length in inches of a piece of wire that is 280 millimeters long. Ans. 11 in.

3. A box containing 12½ cubic inches contains how many cubic centimeters? Ans. 205 cu. cm., nearly

4. A bag of flour weighs 12 kilograms. What is its weight in pounds? Ans. 26½ lb., nearly

52. Conversion Factors.—As a convenient means for reducing the units of one system to those of another a list of the most frequently used *conversion factors* is given herewith. Many of these are factors shown in connection with the preceding tables. The term **conversion factor** means here a multiplier which if used with a given number of one system converts it into an equivalent number of another system. It is stated in Art. **40** that 1 inch=25.4 millimeters. The number 25.4 is a conversion factor by which any number of inches may be converted to millimeters. For example, 7.5 inches are converted to millimeters by using 25.4 as a multiplier; thus, 7.5 inches=7.5×25.4 millimeters=190.5 millimeters.

List of Conversion Factors

Inches×25.4=millimeters; millimeters÷25.4=inches.
Inches×2.54=centimeters; centimeters÷2.54=inches.
Feet×.305=meters; meters×3.28=feet.
Miles×1.61=kilometers; kilometers×.6214=miles.
Square inches×6.45=square centimeters; square centimeters×.155=square inches.

Square yards×.836=square meters; square meters×1.195=square yards.

Cubic inches×16.4=cubic centimeters; cubic centimeters×.061=cubic inches.

Cubic yards×.765=cubic meters, or steres; cubic meters×1.31 =cubic yards.

Quarts (dry)×1.1=liters; liters÷1.1=quarts (dry).

Gallons×3.785=liters; liters×.2642=gallons.

Pounds (av.)×.4536=kilograms; kilograms×2.2=pounds.

53. Application of Conversion Factors.—The application of the preceding list to the conversion of numbers from units of one system to units of another system may be seen from the following examples:

Example 1.—How many meters in 4,375 feet?

Solution.—As feet×.305=meters, 4,375 ft.=4,375×.305=1,334.375 m. Ans.

Example 2.—How many pounds in 723 kilograms?

Solution.—As kilograms×2.2=pounds, 723 kg.=723×2.2=1,590.6 lb. Ans.

Example 3.—Reduce 231 kilometers to miles.

Solution.—As kilometers×.6214=miles, 231 kilometers=231×.6214 =143.5 mi. Ans.

REDUCTION OF DENOMINATE NUMBERS TO DECIMALS

54. Rule for Reduction.—In calculations employing denominate numbers, it is often convenient, and in some cases necessary, to reduce a denominate number to a decimal part of the next higher denomination, as inches to a decimal of a foot, minutes to a decimal of an hour, and so forth. The method employed for the reduction of decimal denominate numbers corresponds in its main features with that explained in Arts. **45** to **48.**

Considering first the particular case of reducing 6 inches to a decimal part of a foot, it is to be noted that 1 inch is $\frac{1}{12}$ part of a foot. Therefore, 6 inches is $\frac{6}{12}$ of a foot. Reducing $\frac{6}{12}$ to a decimal, its equivalent value is the quotient of 6÷12=.5. Hence, 6 inches is equivalent to .5 foot. Similarly, as 7 inches is equal to $\frac{7}{12}$ of a foot, and $\frac{7}{12}$ reduced to a decimal is equal to .5833, it follows that $\frac{7}{12}$ foot is equal to .5833 foot.

In a similar manner 15 minutes may be reduced to a decimal of an hour. As 1 hour = 60 minutes, 15 minutes = $\frac{15}{60}$ hour. On reducing $\frac{15}{60}$ to a decimal its equivalent value is found to be 15 ÷ 60 = .25; therefore, 15 minutes = .25 hour.

The method followed in the preceding examples is embodied in the following rule:

Rule.—*To reduce a denominate number to a decimal of a higher denomination divide the given number by the number of units required to make one unit of the higher denomination. The quotient, expressed as a decimal, will be the equivalent decimal value of the given number in terms of the higher denomination.*

55. Application of Rule.—The application of the preceding rule is shown in the following examples:

Example 1.—In the expression 26 feet 9¾ inches reduce the inches to a decimal of a foot and add the decimal to the whole number.

Solution.—According to the rule, the number 9¾ must be divided by 12. This division will be made easier by reducing the number to an improper fraction or to a whole number and a decimal. Following the latter method, it is found that 9¾ = 9.75. On dividing 9.75 by 12 the quotient is .8125. It follows that 9¾ in. = .8125 ft., and that the complete expression is 26 + .8125 = 26.8125 ft. Ans.

Example 2.—Reduce 36 degrees 27 minutes 48 seconds to degrees and decimal of a degree.

Solution.—Applying the rule, 48″ must be reduced to a decimal of a minute by dividing 48 by 60, the number of seconds in a minute, and expressing the quotient as a decimal. Thus, 48 ÷ 60 = .8; hence, 48″ = .8′. This decimal must be added to the given minutes, the result being 27 + .8 = 27.8′. This number must now be reduced to a decimal of a degree. Applying the rule again, the number 27.8 must be divided by the number of minutes in one degree, or 60. Thus, 27.8 ÷ 60 = .463; therefore, 27.8′ is equal to .463°. It follows that the expression 36° 27′ 48″ is equivalent to 36.463°. Ans.

Example 3.—Reduce 21 gallons 3¾ quarts 1½ pints to gallons and a decimal part of a gallon.

Solution.—There are 2 pt. in a quart, therefore 1½ pt. reduced to a decimal of a quart is equal to $\frac{1.5}{2} = .75$ qt. Adding this decimal to 3¾ qt., the sum is 4.5 qt. These quarts are reduced to a decimal of a gallon by dividing by the number of quarts in 1 gal., or 4; 4.5 ÷ 4

=1.125. Therefore, 4.5 qt. is equal to 1.125 gal. On adding this number to the given number of gallons, the sum is 22.125 gal. It follows that 21 gal. 3¾ qt. 1½ pt. is equal to 22.125 gal. Ans.

EXAMPLES FOR PRACTICE

Reduce each of the following denominate numbers to a decimal part of the next higher denomination.

1. 7½ inches Ans. .625 ft.
2. 13 ounces (av.) Ans. .8125 lb.
3. 46 minutes (time) Ans. .767 hr.
4. 2¾ quarts (liquid) Ans. .6875 gal.
5. 120 square inches Ans. .833 sq. ft.

REDUCTION OF A DECIMAL DENOMINATE NUMBER TO A NUMBER OF A LOWER DENOMINATION

56. Rule for Reduction.—Sometimes it is required to reduce the decimal part of a denominate number to one or more numbers of a lower denomination. For instance, the decimal of a foot is to be reduced to inches, decimal of an hour to minutes and seconds, and so forth.

Since the reduction of a number to a decimal of a higher denomination is made by *division*, the reverse process is effected by multiplication. For example, to reduce .75 foot to inches, .75 must be multiplied by 12, since there are 12 inches in 1 foot; thus, .75×12=9. Therefore, .75 foot is equivalent to 9 inches. The following rule applies to this kind of reduction:

Rule.—*To reduce a decimal denominate number to a number of a lower denomination, multiply the decimal by the number of units required to make one unit of the higher denomination. The product will be the equivalent value in terms of the lower denomination.*

57. Application of Rule.—The following examples show the application of the preceding rule:

Example 1.—Reduce 36.464 hours to hours, minutes, and seconds.

Solution.—According to the rule, the decimal is to be multiplied by 60, as there are 60 min. in 1 hr.; .464×60=27.84; therefore, .464 hr.

=27.84 min. The decimal .84 must now be multiplied by 60, since there are 60 sec. in 1 min. Thus, .84×60=50.4; that is, .84 min.=50.4 sec. It follows that 36.464 hr. is equal to 36 hr. 27 min. 50.4 sec. Ans.

EXAMPLE 2.—Reduce 6.75 feet to feet and inches.

SOLUTION.—The decimal .75 is multiplied by 12, since there are 12 in. in 1 ft., or .75×12=9; that is, .75 ft.=9 in. and 6.75 ft.=6 ft. 9 in. Ans.

EXAMPLE 3.—Reduce 5.453 feet to feet, inches, and sixteenths of an inch.

SOLUTION.—The decimal .453 is multiplied by 12; thus, .453×12 =5.436, showing that .453 ft.=5.436 in. The decimal .436 is reduced to sixteenths by multiplying by $\frac{16}{16}$, since there are 16 sixteenths in 1 inch. Thus, $.436\times\frac{16}{16}=\frac{6.976}{16}$; therefore, .436 in.=6.976 sixteenths of an inch. As it is desirable that the latter number shall be a whole number, the decimal part is omitted, and as the decimal is greater than .5, the whole number is increased by 1; thus $\frac{6.976}{16}=\frac{7}{16}=.436$ in., approximately. It follows that 5.453 ft.=5 ft. $5\frac{7}{16}$ in. Ans.

EXAMPLE 4.—Reduce 4.29 inches to inches and thirty-seconds of an inch.

SOLUTION.—The decimal .29 is reduced to thirty-seconds by multiplying by $\frac{32}{32}$, there being 32 thirty-seconds in 1 inch. Then, $.29\times\frac{32}{32}=\frac{9.28}{32}=\frac{9}{32}$ in., approximately; the decimal .28 being less than .5, it is simply dropped. Therefore, 4.29 in.=$4\frac{9}{32}$ in. Ans.

EXAMPLES FOR PRACTICE

Reduce the following decimal values to units of lower denominations:

1. .625 feet to inches Ans. $7\frac{1}{2}$ in.
2. .8125 pound (av.) to ounces Ans. 13 oz.
3. .767 hour to minutes Ans. 46 min.
4. .6875 gallon to quarts Ans. $2\frac{3}{4}$ qt.
5. .833 square foot to square inches Ans. 120 sq. in.

ADDITION OF COMPOUND NUMBERS

58. Rule for Adding Compound Numbers.—Compound numbers formed of denominations belonging to the same kind may be added by applying the following rule:

Rule.—*Place the numbers so that like denominations are in the same column. Begin at the right-hand column, and add. Divide the sum by the number of units of this denomination required to make one unit of the higher denomination. Place the remainder under the column added, and carry the quotient to the next column.*

ft.	in.	
1	3½	
2	7¾	
1	11⅛	
3	8⅞	
9	7¼	Ans.

Example 1.—Four bars measure 1 foot 3½ inches, 2 feet 7¾ inches, 1 foot 11⅛ inches, and 3 feet 8⅞ inches, respectively; what is the combined length?

Solution.—The numbers are arranged as shown. On adding the numbers in the first column the sum is found to be 31¼ in., or 2 ft. 7¼ in. The 7¼ in. is put down and the 2 ft. is added to the other column, giving 9 ft. The combined length of the bars is 9 ft. 7¼ in. Ans.

Example 2.—A workman spends 1 hour 40 minutes on one piece of work, 2 hours 35 minutes on another, and 3 hours 10 minutes on a third. What is his time on all three?

Solution.—The sum of the numbers in the right-hand column is 85 min., which is equal to 1 hr. 25 min. The 25 min. is set down and the 1 hr. is carried over and added to the number in the other column. Therefore, the total time is 7 hr. 25 min. Ans.

hr.	min.	
1	40	
2	35	
3	10	
7	25	Ans.

EXAMPLES FOR PRACTICE

1. Find the sum of 7 feet 6 inches, 3 feet 7½ inches, 1 foot 9¾ inches, and 2 feet 2 inches. Ans. 15 ft. 1¼ in.

2. Find the total weight of three lots of scrap iron that weigh 2 tons 765 pounds, 4 tons 1,240 pounds, and 1 ton 800 pounds. Ans. 8 T. 805 lb.

3. A carpenter works 2 hours 45 minutes on one job, 1 hour 5 minutes on another, and 3 hours 25 minutes on another. What is the total time worked? Ans. 7 hr. 15 min.

4. Along the end of a factory, as shown by the diagram, Fig. 9, four machines *a*, *b*, *c*, and *d* are set in line. The distances between the machines and between the end machines and the wall, and the lengths of the machines are as marked. What is the distance from one side wall to the other? Ans. 37 ft.

5. Of four oil tanks situated in a power plant, the first tank contains 167 gallons 3 quarts; the second contains 186 gallons 1 quart; the third contains 108 gallons 2 quarts, and the fourth contains 123 gallons 3 quarts. What is the total quantity of oil in the tanks?

Ans. 586 gal. 1 qt.

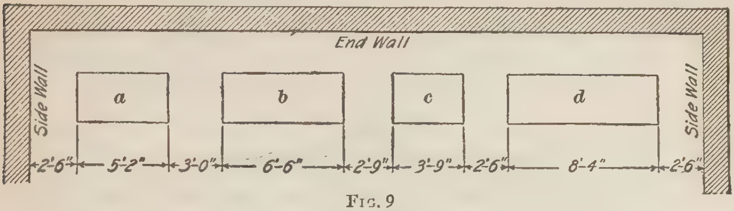


FIG. 9

SUBTRACTION OF COMPOUND NUMBERS

59. Rule for Subtracting Compound Numbers. Compound numbers may be subtracted by applying the following rule:

Rule.—*Place the smaller quantity under the larger quantity, with like denominations in the same column. Beginning at the right, subtract the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be taken from the minuend of the next higher denomination, reduced, and added to it.*

EXAMPLE 1.—A pile of coal containing 3 tons 728 pounds has 1 ton 566 pounds taken away. How much remains?

SOLUTION.—Setting down the numbers according to the rule, and subtracting, the remainder is found, as shown.

That is, 2 T. 162 lb. remain. Ans.

	T.	lb.
minuend	3	728
subtrahend	1	566
remainder	2	162

EXAMPLE 2.—A length of pipe 4 feet 6 inches long has a piece 1 foot 8 inches long cut off. What length remains?

ft.	in.
4	6
1	8

SOLUTION.—Arranging the numbers as shown, the remainder is found in the following manner: As it is impossible to subtract 8 from 6, 1 ft. is borrowed from the 4 ft. and added to the 6 in. after converting the foot to inches. Thus, 12+6=18 in. The 1 ft. taken away from the 4 ft. leaves 3 ft.

The numbers will then appear as shown in the second arrangement.

ft.	in.
3	18
1	8
2	10

Arranged in this form, 8 may be taken from 18, leaving 10, and 1 from 3, leaving 2. The remainder is therefore 2 ft. 10 in. Ans.

NOTE.—The operation of taking 1 ft. away from the 4 ft., reducing it to inches, and adding it to the 6 in., would generally be done mentally, and would not be written down as shown.

EXAMPLES FOR PRACTICE

1. From a piece of timber 10 feet 8 inches long two pieces are sawed off, one measuring 2 feet 6 inches and the other 3 feet 8 inches. Neglecting the width of the saw cuts, what length of timber remains? Ans. 4 ft. 6 in.

2. A glass tube 3 feet 5 inches long, as shown in Fig. 10, is marked

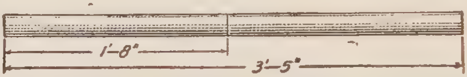


FIG. 10

to be cut off into two parts, the shorter of which is 1 foot 8 inches. What is the length of the longer part? Ans. 1 ft. 9 in.

3. If 2 gallons 3 quarts 1 pint of oil is drawn from a tank containing 8 gallons 1 quart 1½ pint, how much oil remains in the tank? Ans. 5 gal. 2 qt. ½ pt.

4. A certain lever may move through an angle of 45° 26′ 43″. If it has moved through an angle of 30° 14′ 55″, through what angle must it yet move to reach its final position? Ans. 15° 11′ 48″

MULTIPLICATION OF COMPOUND NUMBERS

60. Rule for Multiplying Compound Numbers. Compound numbers may be multiplied by other numbers by applying the following rule:

Rule.—*Multiply the number representing the lower denomination by the given multiplier and reduce the product to the higher denomination. Write the remainder under the lower denomination, and add the quotient to the product obtained by multiplying the higher denomination by the multiplier.*

EXAMPLE 1.—What length of stock is needed to make 16 bolts, if each bolt requires a length of 1 foot 3⅛ inches?

Solution 1.—The stock required must be 16 times the length required for one bolt.

Arranging the numbers as shown, $3\frac{1}{8}$ is multiplied by 16; thus, $3\frac{1}{8}\times16=\frac{25}{8}\times16=\frac{400}{8}=50$. But, 50 in. is equal to $\frac{50}{12}=4$ ft. 2 in. The remainder 2 is placed under 16 and the 4 ft. is carried over and added to the next product. Multiplying 1 ft. by 16, the product is 16 ft., to which is added 4 ft., or 16+4=20 ft. The length of stock must therefore be 20 ft. 2 in. Ans.

ft.	in.	
1	$3\frac{1}{8}$	
	16	
20 ft.	2 in.	Ans.

ft.	in.	
1	$3\frac{1}{8}$	
	16	
16 ft.	50 in.	
=20 ft.	2 in.	Ans.

Solution 2.—The problem may also be solved in the following manner: Multiplying 1 ft. $3\frac{1}{8}$ in. by 16 gives 16 ft. and $16\times3\frac{1}{8}$ in.; but $16\times3\frac{1}{8}$ in. $=16\times\frac{25}{8}=\frac{400}{8}=50$ in. But 50 in. =4 ft. 2 in., and the whole product is therefore 16 ft.+4 ft. 2 in.=20 ft. 2 in. Ans.

Example 2.—Multiply 6 lb. 14 oz. (av.) by 12.

Solution.—The numbers 6 pounds and 14 ounces are each multiplied by 12, giving the product, 72 lb. 168 oz. To reduce 168 oz. to pounds and ounces, it is divided by 16, as there are 16 oz. in 1 lb. Thus, 168 oz.÷16=10 lb. 8 oz., and 72 lb. +10 lb.=82 lb., making the whole weight equal to 82 lb. 8 oz.

lb.	oz.	
6	14	
	12	
	28	
	14	
72 lb.	168 oz.	
=82 lb.	8 oz.	Ans.

Example 3.—The circumference of a flywheel, which also serves as a driving pulley, is 47 feet 3 inches. If the belt is in contact with the rim of the flywheel over $\frac{3}{5}$ of its circumference, what is the length of that portion of the belt that makes contact with the pulley?

ft.	in.
47	3
	$\frac{3}{5}$
$28\frac{1}{5}$ ft.	$1\frac{4}{5}$ in.
=28 ft.	$4\frac{1}{5}$ in.

Solution.—Multiplying 47 ft. 3 in. by $\frac{3}{5}$ gives $28\frac{1}{5}$ ft. $1\frac{4}{5}$ in. The fraction $\frac{1}{5}$ ft. must be reduced to inches and added to $1\frac{4}{5}$ in. Thus $12\times\frac{1}{5}=2\frac{2}{5}$ in.; $2\frac{2}{5}+1\frac{4}{5}=3\frac{6}{5}=4\frac{1}{5}$ in. The required length is, then, 28 ft. $4\frac{1}{5}$ in. Ans.

EXAMPLES FOR PRACTICE

1. Twelve pieces each 1 foot $3\frac{1}{2}$ inches long are sawed from a bar of iron. What is the total length sawed off, neglecting the width of the saw cut? Ans. 15 ft. 6 in.

2. If 18 boxes each 2 feet $4\frac{1}{2}$ inches wide are set side by side, what will be the total width? Ans. 42 ft. 9 in.

3. Five loads of pig iron, each weighing 4 tons 650 pounds, are purchased. What is the total amount of iron purchased, if the weight is in long tons? Ans. 21 T. 1,010 lb.

4. Each cell of an electric battery requires 2 pounds 12 ounces (av.) of copper sulphate for a charge. How much will 23 cells require? Ans. 63 lb. 4 oz.

5. Multiply 42 degrees 31 minutes 24 seconds by $\frac{5}{7}$. Ans. 30° 22′ $25\frac{5}{7}$″

DIVISION OF COMPOUND NUMBERS

61. Rule for Dividing Compound Numbers.—The rule to be used in dividing a compound number by another number is as follows:

Rule.—*Find how many times the divisor is contained in the first or higher denomination of the dividend; reduce the remainder, if any, to the lower denomination, and add to it the number in the given dividend expressing that denomination; divide this new dividend by the divisor, and the quotient will be the next denomination in the quotient required. Or, reduce the dividend to units of the lower denomination and divide by the given divisor.*

Example 1.—A piece of bar iron 20 feet 2 inches long is used in making 16 bolts, all of the same length. What is the length used for each bolt?

ft. in.
16) 20 2 (1 ft. $3\frac{1}{8}$ in. Ans.
16
4 ft. rem.
12
48 in.
2 in.
16) 50 in. ($3\frac{1}{8}$ in.
48
$\frac{2}{16}=\frac{1}{8}$

Solution 1.—The numbers are arranged and the division is performed as in the accompanying solution. The total length of the bar, or 20 ft. 2 in., is divided by 16. First, 16 is contained once in 20 ft., with 4 ft. as a remainder. This 4 ft. is now reduced to inches by multiplying it by 12, which gives 48 in. Adding the 2 in. in the original number, gives a sum of 50 in. Then, $50 \div 16 = 3\frac{2}{16} = 3\frac{1}{8}$ in. Thus, the amount used for each bolt is 1 ft. $3\frac{1}{8}$ in. Ans.

Solution 2.—The example may also be solved by reducing the compound number to its lowest denomination and then dividing by the given divisor. Thus, 20 ft. 2 in. is reduced as follows: $20 \times 12 = 240$; $240 + 2 = 242$ in. Then, $242 \div 16 = 15\frac{1}{8}$ in., or 1 ft. $3\frac{1}{8}$ in. Ans.

Example 2.—Divide 28 feet 5 inches by 8.

Solution.— 28 ft. divided by 8 gives a quotient of 3 ft. and 4 ft. as a remainder. The 4 ft. is now reduced to inches by multiplying by 12, which gives 48 in. 48+5=53 in. and 53÷8=6$\frac{5}{8}$ in., making the answer 3 ft. 6$\frac{5}{8}$ in.

```
  ft.  in.
8)28   5   (3 ft. 6⅝ in.  Ans.
  24
   4 ft. rem.
  12
  48 in.
   5 in.
8)53 in. (6⅝ in.
  48
   ⅝
```

Example 3.—Divide 85 tons 1,843 pounds by 36.

Solution.— 85 T. divided by 36=2 T. with a remainder of 13 T.=26,000 lb.; but, 26,000 lb.+1,843 lb.=27,843 lb., and 27,843 lb.÷36=773.4 lb., making the complete answer 2 T. 773.4 lb.

```
36)85 T.    1843 lb. (2 T.  773.4 lb. approx.  Ans.
   72
   13 T.
    2000
   26000 lb.
    1843 lb.
36)27843 lb. (773.4 lb.
   252
    264
    252
     123
     108
      150
      144
        6
```

EXAMPLES FOR PRACTICE

1. If a bar 5 feet 3 inches long is divided into four equal parts, what is the length of each? Ans. 1 ft. 3$\frac{3}{4}$ in.

2. If a man requires 23 hours 50 minutes to make 11 articles of merchandise, how long will it take to make one? Ans. 2 hr. 10 min.

3. How many armature coils weighing 1 lb. 3 oz. each can be made from a reel of copper wire weighing 200 pounds, net? Ans. 168 coils and 8 oz. of wire remaining

4. How long would it take a cannon ball traveling at the rate of 1,950 feet per second to pass over a distance of 10 miles? Ans. 27 sec., nearly

5. How long will 40 gallons of lubricating oil last, if 7 pints are used each day? Ans. 45$\frac{5}{7}$ da.

RATIO AND PROPORTION

RATIO

SIMPLE RATIO

EXPRESSING AND FINDING RATIOS

1. Comparison of Numbers.—In practice there are many cases in which it is more useful to know how many times one number is larger than another number, than to know the actual values of these numbers. For example, in calculating the relative speeds of shafts driven by gear wheels, it is more convenient to state how many times more teeth one wheel has than another, than to consider the actual number of teeth on each wheel. These comparative values of the number of teeth are generally stated in the form of a fraction, as $\frac{3}{1}$, the fraction meaning that the number of teeth on the driving wheel is three times as great as that on the driven wheel. The actual number of teeth on the two wheels is left out of consideration for the time being; they may be 90 and 30, 36 and 12, or any other combination in which one wheel has three times as many teeth as the other.

2. Another example showing the advantage of giving the relative values of numbers may be found in government health reports. One of those may state, for instance, that of each 1,000 inhabitants in a certain city 25 persons suffered from influenza, 1.24 from typhoid, etc. Reports given in this form

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are much more instructive than those which simply state that in the same city 1,250 persons suffered from influenza, 62 from typhoid, etc. By means of the first report it is possible to institute comparisons with other cities, but with reports of the latter kind comparisons are difficult, unless the total number of inhabitants is known in each case.

Similarly, when speed of operation or production is considered, it is much easier to make comparison between various machines, when their relative speeds are known. For example, on comparing the speeds of two railway trains, it is found that a train *A* makes 90 miles in 2 hours 15 minutes and that another train *B* makes 108 miles in 2 hours 24 minutes. From those statements it is difficult quickly to form an idea of the relative speeds of these trains. But, if it is said that the train *A* made 40 miles an hour and the train *B* 45 miles an hour, the comparison is easier, and is still more facilitated if the relative speeds are stated in the form of a fraction, as for instance, the speed of A is $\frac{8}{9}$ of the speed of B. This expression means that while the train *A* makes 8 miles, the train *B* makes 9 miles.

It is the purpose of this Section to show by what means the relative values of two numbers may be found, and how the existing relation between one pair of numbers may be used for finding another pair, similarly related. This Section deals with one of the most interesting and useful subjects to be found in arithmetic.

3. Finding the Ratio of Two Numbers.—Two numbers may be compared in one of two ways. Suppose, for instance, that the diameters of two belt pulleys are to be compared, one being 30 inches and the other 6 inches. If it is necessary to know how many times 30 is larger than 6, then 30 is divided by 6 giving 5 as the quotient; thus, $30 \div 6 = 5$. Hence, it may be said that 30 inches is 5 times as large as 6 inches, or that the 30-inch diameter contains 5 times as many inches as the 6-inch diameter. Or, the numbers 30 and 6 may be compared by ascertaining what part 6 inches is of 30 inches.

Then, 6 is divided by 30, giving the quotient $\frac{1}{5}$ or .2. Hence, 6 inches is $\frac{1}{5}$, or .2, of 30 inches.

As a practical example of this kind of comparison may be mentioned the case of two pulleys that are to be connected by a belt. It is here necessary to know the relation between the diameters of the two pulleys in order that their relative speeds may be calculated.

4. The two numbers compared must be units of the same denomination. For instance, if one number is given in inches, the other number must also be in inches; thus, 30 inches may be compared with 6 inches, as in the preceding example; but, 30 inches cannot be compared with 6 pounds. From the preceding remarks it will be seen that the term **ratio** means a comparison of two numbers of the same denomination or kind. The operation of comparing two numbers is called *finding the ratio* of the numbers.

5. Expressing a Ratio.—A ratio may be written in two different ways, both of which are correct. Thus, the ratio of 20 to 4, or the value of 20 compared to the value of 4, may be written $20:4$ or $\frac{20}{4}$. Each of these expressions is read *the ratio of 20 to 4*. The ratio of 4 to 20 would be written either $4:20$ or $\frac{4}{20}$. The method most commonly used in writing ratios is the first one shown; that is, the two numbers are separated by the sign (:), which is really an abbreviation of ÷, the sign of division, and has the same meaning, division being practically a method of indicating ratio. Hence, $20:4$ is equal to $20 \div 4 = 5$. In calculations, ratios are frequently written in the form of fractions; thus, the ratio of 20 to 4 may be written $\frac{20}{4}$.

6. Terms of Ratio.—The two numbers to be compared are known as the **terms** of the ratio; thus, in the ratio $30:6$, 30 and 6 are the two terms. When both terms are considered together they are called a **couplet;** when considered separately, the first term is called the **antecedent,** and the second term the **consequent.** Thus, in the ratio 30 : 6, 30 and 6 form a couplet, in which 30 is the antecedent and 6 the consequent.

7. Simple Ratio.—When a ratio has only one antecedent and one consequent, it is known as a **simple ratio.** Thus, the ratio 30 : 6 is a simple ratio.

8. Direct and Inverse Ratio.—When it is desired to compare two denominate numbers of the same kind, the comparison is usually made by finding the ratio of the first number to the second. This is known as a **direct ratio.** For instance, the direct ratio of 22 feet to 9 feet is 22 : 9. If the given terms are interchanged the ratio becomes an **inverse ratio.** Thus, the inverse ratio of 22 feet to 9 feet is 9 : 22. The direct ratio of 5 pounds to 11 pounds is 5 : 11, and the inverse ratio is 11 : 5.

Every ratio is understood to be a *direct* ratio unless otherwise stated.

9. Value of a Ratio.—Distinction must be made between a *ratio* and its *value*. A ratio is represented by its two terms. The **value** of a ratio is the quotient obtained by dividing the first term by the second. Thus, the ratio of 20 to 4 is 20 : 4; the value of this ratio is $20 \div 4 = 5$.

10. Ratio Considered as a Fraction.—By expressing the ratio in the fractional form, for example, the ratio of 18 to 3 as $\frac{18}{3}$, it follows from the laws of fractions that both terms may be multiplied or both divided by the same number, without altering the value of the ratio. Thus,

$$\frac{18}{3} = \frac{18 \times 4}{3 \times 4} = \frac{72}{12}; \quad \frac{18}{3} = \frac{18 \div 3}{3 \div 3} = \frac{6}{1}$$

In each case the value of the ratio is 6.

In the ratio 6 : 10, in which the antecedent is the smaller one of the two terms, the ratio, when expressed in the fractional form, becomes $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$. Hence, the value of the ratio

$$6 : 10 \text{ is } \frac{3}{5}.$$

When a ratio is given in the fractional form a direct ratio may be changed to an inverse ratio by simply inverting the fraction. For instance, the direct ratio of 21 to 7 is $\frac{21}{7}$, which has a value of 3. The inverse ratio of 21 to 7 is $\frac{7}{21}$; in this case the value of the ratio is $\frac{1}{3}$, which is the reciprocal of 3. An inverse ratio is, therefore, also called a **reciprocal ratio,** as its value is the reciprocal of the value of the direct ratio. The *reciprocal* of a number is 1 divided by that number; the reciprocal of a fraction is the fraction inverted.

11. Reducing a Ratio to Its Lowest Terms.—It is preferable that a ratio be reduced to its lowest terms and, if possible, that one of its terms be made unity, or 1. Thus, instead of saying that the ratio of the resistances of two electrical conductors is $\frac{5}{50}$, the ratio may be reduced to $\frac{1}{10}$; such an expression is often used instead of giving the actual values of the respective resistances.

A ratio is reduced to its lowest terms by dividing both terms by the same number until no number except 1 can be found that will divide both terms without a remainder. Thus, the ratio $\frac{84}{48}$, when reduced to its lowest terms, becomes $\frac{7}{4}$, which is obtained by dividing each number by 12. The operation may be written $\frac{84 \div 12}{48 \div 12} = \frac{7}{4}$.

12. Rules for Finding Direct and Indirect Ratio of Two Numbers.—The following rules may be used in cases where the direct or the indirect ratio of two numbers is to be found:

Rule I.—*The direct ratio of two numbers is found by making the first one of the given numbers the antecedent and the second one the consequent of the ratio.*

Rule II.—*The inverse ratio of two numbers is found by making the second number of those given the antecedent and the first one the consequent of the ratio.*

Example 1.—(*a*) What is the direct ratio of 9 to 3? (*b*) What is the value of the ratio?

Solution.—(*a*) Applying rule I, the first number, 9, is made the antecedent and the second number, 3, the consequent of the ratio; thus, the direct ratio of 9 to 3 is 9 : 3, or $\frac{9}{3}=\frac{3}{1}$. Ans.

(*b*) From Art. **9** the value of the ratio is $3 \div 1 = 3$. Ans.

Example 2.—(*a*) What is the inverse ratio of 10 to 5? (*b*) What is the value of the ratio?

Solution.—(*a*) Applying rule II, the second number, 5, is placed as the antecedent of the ratio, and 10 as the consequent; thus, the inverse ratio of 10 to 5 is 5 : 10, or $\frac{5}{10}=\frac{1}{2}$. Ans.

(*b*) From Art. **9** the value of the ratio is $1 \div 2 = .5$, or $\frac{1}{2}$. Ans.

Example 3.—(*a*) Reduce the ratio 19 : 76 to its lowest terms. (*b*) What is the value of the ratio?

Solution.—(*a*) According to Art. **11** a ratio is reduced to its lowest terms by dividing both terms by the same number, continuing the process until no number except 1 can be found that will be contained in each term without a remainder. In this case the terms of the ratio may be divided by 19; thus $\frac{19 \div 19}{76 \div 19}=\frac{1}{4}$. Ans.

(*b*) The value of the ratio is $\frac{1}{4}$, or .25. Ans.

Example 4.—What is the inverse ratio of 8 and 72, and what is the value of the ratio?

Solution.—Applying rule II, the inverse ratio of 8 to 72 is 72 : 8, which reduced to its lowest terms is 9 : 1. The value of the ratio is $9 \div 1 = 9$. Ans.

Example 5.—What is the direct ratio of 72 to 8, and what is the value of the ratio?

Solution.—Applying rule I, the direct ratio of 72 to 8 is 72 : 8 = 9 : 1. The value of the ratio is $9 \div 1 = 9$. Ans.

Note.—Examples 4 and 5 show that the direct and the inverse ratio of two numbers may be equal, depending on their relative positions in the statement contained in the example; for instance, whether 8 precedes or follows 72.

Example 6.—A pair of gear-wheels contain 60 and 35 teeth, respectively. (*a*) What is the ratio of the number of teeth in the larger to the number of teeth in the smaller? (*b*) What is the ratio of the smaller number to the larger?

SOLUTION.—(*a*) The ratio of the larger number of teeth to the smaller is the ratio of 60 to 35, or 60 : 35, and reducing this to its lowest terms by dividing both terms by 5, the ratio becomes 12 : 7. Ans.

(*b*) The ratio of the smaller number to the larger is 35 : 60, which, reduced to its lowest terms by dividing both terms by 5, is equal to 7 : 12. Ans.

EXAMPLES FOR PRACTICE

1. What is the ratio of 126 to 18, reduced to its lowest terms? Ans. 7 : 1

2. Two gears have 39 and 54 teeth, respectively. What is the value of the ratio of the larger number to the smaller? Ans. $1\frac{5}{13}$

3. What is the value of the ratio of 6.25 to .75? Ans. $8\frac{1}{3}$

4. One pulley is 24 inches in diameter and another is 60 inches in diameter. What is the inverse ratio of the diameter of the smaller pulley to that of the larger? Ans. 5 : 2

5. A man traveled 250 miles, partly by rail and partly by boat. If he traveled 150 miles by rail, state (*a*) the distance traveled by boat; (*b*) the ratio of the distance traveled by rail to that traveled by boat.

Ans. (*a*) 100 mi.
(*b*) 3 : 2

6. During one year A and B invest 5 dollars and 4 dollars, respectively, every month. (*a*) What is the ratio of A's investment to that of B? (*b*) What is the ratio of B's investment to that of A? (*c*) What are the values of the two ratios?

Ans. (*a*) 5 : 4
(*b*) 4 : 5
(*c*) $1\frac{1}{4}$ and $\frac{4}{5}$

PROPORTION

SIMPLE PROPORTION

DIRECT AND INVERSE PROPORTIONS

13. Elements of a Proportion.—A **simple proportion** consists of two simple ratios of the same value connected by an equality sign (=) or a double colon (: :). For example, the ratios 8 : 6 and 12 : 9 are of the same value, and a proportion may be formed by them, as $8 : 6 = 12 : 9$, or 8 : 6 : : 12 : 9. The equality sign is used more frequently than the double colon, so the proportions in this and other Sections will be written with the equality sign. The proportion $8 : 6 = 12 : 9$ is read *8 is to 6 as 12 is to 9*, or *the ratio of 8 to 6 is equal to the ratio of 12 to 9*. This same proportion can also be written $\frac{8}{6}=\frac{12}{9}$, each of the two ratios being given as a fraction.

The term **couplet** used in connection with a ratio is sometimes applied to the elements of a proportion. Each ratio, or couplet, has two terms and the terms of a proportion are called *first*, *second*, *third*, and *fourth*, numbering from left to right. The first and fourth terms are the **extremes**; the second and third terms are the **means.**

The following table gives the proportion $25 : 10 = 40 : 16$ and the terms applied to its various elements, or parts.

Ratios, or couplets	First				Second		
Terms..............	First		Second		Third		Fourth
Quantities..........	25	:	10	=	40	:	16
	Extreme		*Mean*		*Mean*		*Extreme*

It is seen that 25 : 10 is the first couplet or ratio, and that 40 : 16 is the second couplet or ratio. Numbering from left to right, 25 is the first and 16 the fourth term. It is also seen that the numbers 25 and 16 form the two extremes and the numbers 10 and 40 the two means of the proportion.

14. Use of Proportions.—In practice there are numerous cases in which one ratio is given and it is required to find an equal ratio of which one term is already known; this is the purpose of a proportion, as may be made clearer by an example. Let it be supposed that the top of a table is 6 feet long and 3 feet wide and that it is required to make another table the top of which is 8 feet long, the ratio of length to width being the same as in the smaller table. Here the first ratio is $6:3$ and the second ratio $8:x$, the missing term being indicated by the symbol x. The value of the first ratio is $6 \div 3 = 2$ and if the value of the second ratio is to be 2, also, it is evident that the missing term, x, must be $8 \div 2 = 4$, and the new table top must, therefore, be 4 feet wide. Hence, the complete proportion is $6:3 = 8:4$.

15. A problem frequently met with in practice may be given as another example. The ratio of the weights of the ingredients in a certain mixture is $7:3$. Of the first ingredient there is on hand 91 pounds, and it is required to know what weight of the second ingredient will be required in order that the ratio of the two weights may be as $7:3$. The first ratio is $7:3$ and the second is $91:x$. In this case the relation between the two ratios is so simple that the problem may be solved by inspection in the following manner: It is seen that 7 is contained 13 times in 91; according to Art. **10** both terms of a ratio may be multiplied by the same number without altering the value of the ratio. Hence, on multiplying the terms of the ratio $7:3$ by 13, the result is $7 \times 13 : 3 \times 13 = 91:39$. The numbers 91 and 39 are, therefore, the terms of the second ratio, showing that x, or the weight of the second ingredient, is equal to 39 pounds.

These examples will suffice to show some of the simpler cases in which the principle of proportion is applied in practice. Further explanations will show that the subject of proportion forms one of the most useful sections of arithmetic.

16. Direct and Inverse Proportions.—A **direct proportion** is one in which both couplets are direct ratios. A proportion is always understood to be direct unless the statement of a problem clearly indicates otherwise. An **inverse proportion** is one that requires one of the couplets to be expressed as an inverse ratio. Thus, if 8 is to 4 inversely as 3 is to x, one of the ratios must be reversed (it does not matter which one) and the proportion may be written in either of the following ways: $8 : 4 = x : 3$, or $4 : 8 = 3 : x$. In the first proportion the second couplet is reversed; in the second proportion the first couplet is reversed.

17. Directly Proportional and Inversely Proportional Quantities.—In technical literature, one quantity is said to be *proportional* to another, or to *vary with* it, or to *increase with* it, or to *decrease with* it; any one of these expressions means that a change in one of the quantities causes, or is accompanied by, a corresponding change in the other. If the word *inversely* is used with one of these expressions, the meaning is that a change in one quantity causes an opposite change in the other. Sometimes the word *directly* is used when it is desirable to make sure that the meaning will not be understood to be inverse. For example, the resistance of an electric wire is directly proportional to its length, or varies with, or increases with, its length; that is, the longer the wire, the greater is its resistance. But, if a number of men are engaged in building a fence, the time required to finish the fence is *inversely* proportional to the number of men working on it; that is, the more men, the shorter the time.

18. Rules for Finding Unknown Terms of Proportions.—In any proportion, *the product of the extremes is equal to the product of the means.* For example, in the proportion $17 : 51 = 14 : 42$, the extremes are 17 and 42 and the means are 51 and 14. According to the preceding statement the product 17×42 must be equal to 51×14, which is true because $17 \times 42 = 714$ and $51 \times 14 = 714$. This important principle makes it possible to find an unknown term of a proportion when the three other terms are known. The following rules are based on this principle:

Rule I.—*To find an unknown extreme, divide the product of the means by the given extreme.*

Rule II.—*To find an unknown mean, divide the product of the extremes by the given mean.*

19. Direct Proportion.—In forming the two ratios of a proportion it is important to see that both terms of each ratio are of the *same* kind. For instance, if one term is given in pounds, the other must also be in pounds. But the four terms of a proportion need not be of the same kind; in one ratio both terms may be in pounds, and in the other both in feet, and so forth.

Another point to be noted in forming a direct proportion is that the terms are arranged so that the first term of each ratio refers to *one* of the things compared and that the second term in each ratio refers to the *other* thing compared. For instance, an example states that the weights of two parcels of sugar are 2 and 5 pounds, respectively, and that the cost of the smaller parcel is 12 cents. It is required to find the cost of the larger parcel. Here, one ratio is $2:5$, and the other must be $12:x$, as the term 2 in the first ratio refers to the parcel that costs 12 cents; hence, 12 must be the first term of the second ratio, and the proportion is written

$$2:5=12:x$$

20. Arrangement and Solution of Direct Proportions.—In the following method for solving proportions, it is found convenient always to let the unknown term, x, occupy the position of the *fourth* term in the proportion, and for the *third* term to write the number that is of the same kind as the required fourth term.

The conditions given in the example are now examined to ascertain whether the fourth term, x, will be larger or smaller than the third one. If *larger*, the larger number of those that are to form the first ratio is written as its *second* term, and the remaining number is written as the *first* term.

The preceding method of procedure is embodied in the following rule:

Rule.—*Make the unknown term, x, the fourth term of the proportion, and for the third term write the number that is of a* *similar kind. If the fourth term will be larger than the third, the second term must be larger than the first; or if the fourth term will be smaller than the third, the second term must be smaller than the first.*

EXAMPLE 1.—If 7 pounds of putty costs 56 cents, what will be the cost of 36 pounds?

SOLUTION.—In this example the cost of 36 pounds of putty is the required term, and is, therefore, written as the fourth term, x. The other number of the same kind is 56 cents. So, the second ratio of the proportion must be written $56 : x$.

As 36 pounds of putty must cost more than 7 pounds, it follows that the term x will be larger than the third term, 56. Hence, according to the rule, the second term must be larger than the first. The proportion is, therefore, written

$$7 : 36 = 56 : x$$

In this case one of the extremes is not known. Hence, by rule I, Art. **18**, the unknown extreme, x, is equal to the product of the means divided by the known extreme, or

$$x = \frac{36 \times 56}{7} = 288 \text{ cents, or \$2.88.} \quad \text{Ans.}$$

EXAMPLE 2.—The weights of two patterns, made of pine, are in the ratio of 2 to 3, and the iron casting made from the smaller pattern weighs 42 pounds. What is the weight of the iron casting made from the larger pattern?

SOLUTION.—The weight of the casting made from the larger pattern is unknown and is therefore indicated by x. The other number of the same kind is 42 pounds. So, the second couplet of the proportion must be written $42 : x$.

The example states that the weights of the patterns are as $2 : 3$, and the weight of the casting made from the lighter pattern is 42 pounds; it follows that the casting made from the other pattern must be heavier. According to the rule, the second term must be larger than the first and the proportion is written

$$2 : 3 = 42 : x$$

By rule I, Art. **18**, the unknown extreme, x, is equal to the product of the means divided by the known extreme, or

$$x = \frac{3 \times 42}{2} = \frac{126}{2} = 63$$

That is, the casting made from the larger pattern weighs 63 lb. Ans.

EXAMPLE 3.—The ratio of the numbers of teeth in two gear-wheels is as 2 to 5. If the number of teeth in the second or larger wheel is 90, how many teeth are there in the smaller wheel?

Solution.—The unknown term, x, is here the number of teeth on the smaller wheel and is put down as the fourth term in the proportion. The other number of the same kind is 90 teeth, which is made the third term in the proportion. So the second ratio is written

$$90 : x$$

It is seen from the example that the term 2 refers to the unknown number of teeth. As 2 is smaller than 5, it follows that x must be smaller than 90. Hence, the second term must be smaller than the first and the proportion is written

$$5 : 2 = 90 : x$$

By rule I, Art. **18**,

$$x = \frac{2\times 90}{5} = \frac{180}{5} = 36 \text{ teeth.} \quad \text{Ans.}$$

Note.—In all problems involving fractions cancelation should be resorted to wherever possible so as to simplify operations. If in these and other examples cancelation is omitted it is for the purpose of concentrating attention on the one process under consideration.

21. Arrangement and Solution of Inverse Proportions.—Many problems in proportion are so stated that the first and third terms in the complete proportion refer to the same thing. This is the case with the preceding examples. For instance, in the proportion given as a solution of example 1, Art. **20**, the first and the third terms, 7 and 56, stand for 7 pounds of putty at a cost of 56 cents. The second and the fourth terms refer to the cost of 36 pounds of putty. Such proportions are known as *direct* proportions.

But the conditions stated in the problems are not always such as to give proportions of this kind. Sometimes, it is found, on examination of the complete proportion, that the first and the third terms do *not* refer to the same thing. Such proportions are known as *inverse* proportions. It must, however, be clearly understood that whether a proportion is direct or inverse does not in any way interfere with the application of the rule in Art. **20**, as will be seen from the following examples in which the solution is in every case found by means of inverse proportion.

Example 1.—If 3 men can do a certain job in 20 days, how long will it take 12 men to do a similar job, working at the same rate?

Solution.—The unknown term is the number of days required to do a certain job; it is written as the fourth term. The number of a similar kind is 20 days, which is written as the third term. The second ratio of the proportion is, therefore, $20 : x$.

It is seen from the example that the time required by 12 men must be less than that needed by 3 men. It follows that x must be smaller than 20. Hence, the second term must be smaller than the first and the first ratio is written 12 : 3. The complete proportion is

$$12 : 3 = 20 : x$$

from which $$x = \frac{3 \times 20}{12} = 5 \text{ days.} \quad \text{Ans.}$$

In the complete proportion, 12 : 3 = 20 : 5, the first and the third terms do not refer to the same thing; the term 12 refers to the *larger* number of men, while the term 20 refers to the number of days required by the *smaller* number of men to finish the job. It follows that the proportion is an inverse proportion.

Example 2.—A mining camp has provisions enough to sustain 120 men for 28 days. If the working force is increased to 210 men how long will the provisions last?

Solution.—The unknown term is the number of days through which the provisions will last for 210 men; it is written as the fourth term. The number of a similar kind is 28 days, which is written as the third term. The second ratio of the proportion is, therefore, 28 : x.

It is seen from the example that the provisions will last a shorter time with an increase in the number of men. It follows that x must be smaller than 28. According to the rule, the second term of the proportion must be smaller than the first. The proportion is, therefore, written

$$210 : 120 = 28 : x$$

from which according to Art. **18**,

$$x = \frac{120 \times 28}{210} = 16 \text{ days.} \quad \text{Ans.}$$

Example 3.—It requires 36 hours to fill a tank with water by means of a pump discharging 9 gallons per minute. If the pump is displaced by one that will discharge 16 gallons per minute, how long will it take to fill the tank?

Solution.—The unknown term is the number of hours required to fill the tank; it is written as the fourth term. The number of a similar kind is 36 hours, which is written as the third term. The second ratio of the proportion is, therefore, 36 : x.

It is obvious that the time required by the pump of the greater capacity must be shorter than that required by the pump discharging only 9 gallons per minute. Hence, x must be smaller than 36. As, according to the rule, the second term under the circumstances must be smaller than the first, it follows that the proportion must be written8

$$16 : 9 = 36 : x$$

from which $$x = \frac{9 \times 36}{16} = 20\tfrac{1}{4} \text{ hours.} \quad \text{Ans.}$$

22. Denominate Numbers in Proportions.—It was stated in Art. 4 that the two terms of a ratio must be of the same denomination. Evidently, this requirement must also be complied with in a proportion. When the first and second terms are given in different denominations, they must be reduced to the same denomination. Also, if they are compound numbers, they must be reduced to the same denomination, either the highest or the lowest denomination mentioned in either term. In case the third term is a compound number it must be reduced to one denomination in terms of either its lowest or its highest denomination. The fourth term, x, will be in a denomination corresponding to that of the third term.

For instance, if the first and second terms are 9 inches and 4 feet 3 inches, respectively, the second term may be reduced to inches, as $4 \times 12 = 48$ inches; $48 + 3 = 51$ inches. Or, both terms may be reduced to feet and fractions of a foot. Thus, 9 inches $= \frac{3}{4}$ foot $= .75$ foot, and 4 feet 3 inches $= 4\frac{1}{4}$ feet $= 4.25$ feet. The ratio may, therefore, have one of the following forms: $9 : 51$; $.75 : 4.25$; $\frac{3}{4} : 4\frac{1}{4}$.

Example.—If a rod 9 inches long weighs 1 pound 3 ounces, what will be the weight of a rod of the same kind, 3 feet 7 inches long?

Solution.—The unknown term is the weight of the longer bar, and the third term is the weight of the short bar. The second ratio of the proportion will therefore be

$$1 \text{ lb. } 3 \text{ oz} : x$$

As x will be greater than the third term, the second term must be greater than the first, and the complete porportion will be

$$9 \text{ in.} : 3 \text{ ft. } 7 \text{ in.} = 1 \text{ lb. } 3 \text{ oz.} : x$$

As the second term is in feet and inches, it will be reduced to inches to correspond with the first term; thus, 3 ft. 7 in. = 43 in. The third term will be reduced to its lowest denomination; thus, 1 lb. 3 oz. = 19 oz. On substituting the reduced numbers, the proportion will be

$$9 : 43 = 19 : x$$

from which $x = \frac{43 \times 19}{9} = 90.78$ oz. $= 5$ lb. $10\frac{3}{4}$ oz., nearly. Ans.

EXAMPLES FOR PRACTICE

1. If a pump discharging 6 gallons of water per minute can fill a tank in 20 hours, how long will it take a pump discharging 15 gallons per minute to fill it? Ans. 8 hr.

2. If 75 pounds of lead costs $6.75, how much will 125 pounds cost at the same rate? Ans. $11.25

3. The circular seam of a boiler requires 50 rivets when the pitch, or distance between centers of rivets, is $2\frac{1}{2}$ inches; how many would be required if the pitch were $3\frac{1}{8}$ inches? Ans. 40

4. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 da.

5. If an electric car runs 12 miles in 35 minutes, how long will it take to run 30 miles at the same rate?
Ans. $87\frac{1}{2}$ min., or 1 hr. 27 min. 30 sec.

UNIT METHOD

PRINCIPLE AND APPLICATION

23. Solving Problems by the Unit Method.—Problems dealing with two ratios, as in proportion, may also be solved by what is known as the **unit method.** By means of this method it is possible also to solve problems dealing with more than two ratios, as will be shown in some of the succeeding examples. The general procedure in the unit method may be seen from the following example.

It is supposed that 4 bags of cement costs $1.36 and it is required to find the price of 55 bags.

Since the cost of 4 bags of cement = $1.36,

$$\text{the cost of 1 bag of cement} = \frac{\$1.36}{4} = \$.34$$

Therefore, the cost of 55 bags of cement = $.34 × 55 = $18.70.

It is seen that the principal number in the problem, here the price of 4 bags of cement, is reduced to the value of a unit; that is, to the price of 1 bag. This unit price may then serve as a basis for finding the value of any other number of units called for in the problem. In this example the price of 1 bag was multiplied by 55 to find the price of 55 bags.

24. In the preceding example the price of 1 unit is actually calculated, but in practice this is not done. The successive steps, that is, the operations of multiplication and division, are merely indicated by the respective signs, and no multiplication or division is performed until the very last, as then the answer may be obtained more easily by cancelation. This method of procedure should generally be followed in all arithmetical calculations. The following examples show the general method of procedure in solving problems by the unit method.

EXAMPLE 1.—If a pump discharging 6 gallons of water per minute can fill a tank in 18 hours, how long will it take a pump discharging 14 gallons per minute to fill the tank?

SOLUTION.—The problem is solved by the unit method, as follows:

6 gal. per min. requires 18 hr. for filling tank
1 gal. per min. requires 6×18 hr. for filling tank

Therefore, 14 gal. per min. requires $\frac{6\times 18}{14}$ hr. for filling tank.

$$\frac{6\times 18}{14}=7\tfrac{5}{7}\text{ hr.}\quad\text{Ans.}$$

NOTE.—It is seen in this example that the time required for filling the tank at the rate of 1 gal. per min. is not calculated, but simply indicated by the product 6×18 hr. Evidently this time must be *divided* by 14 to find the time required for a rate of discharge of 14 gal. per min., as the time must be one-fourteenth of that required by a 1-gal. discharge.

EXAMPLE 2.—If 4 men earn $65.80 in 7 days, how much can 14 men, paid at the same rate, earn in 12 days?

SOLUTION.— 4 men in 7 days earn $65.80. Therefore, 1 man in 7 days earns $\frac{\$65.80}{4}$, and 1 man in 1 day earns $\frac{\$65.80}{4\times 7}$. Therefore, 1 man in 12 days earns $\frac{\$65.80\times 12}{4\times 7}$, and 14 men in 12 days earn

$$\frac{14\times\$65.80\times 12}{4\times 7}=\frac{\overset{2}{\cancel{14}}\times\$65.80\times\overset{3}{\cancel{12}}}{\cancel{4}\times\cancel{7}}=\$65.80\times 2\times 3=\$394.80.\quad\text{Ans.}$$

EXAMPLE 3.—If 3 men can dig a trench in 20 days, how long will it take 12 men to dig a similar trench at the same rate?

SOLUTION.—As 3 men dig a trench in 20 days, 1 man can dig it in 3×20 days, and 12 men can dig it in $\frac{3\times 20}{12}$ days = 5 days. Ans.

Example 4.—If a block of granite 8 feet long, 5 feet wide, and 3 feet thick weighs 7,200 pounds, what is the weight of a block of granite 12 feet long, 8 feet wide, and 5 feet thick?

Solution 1.—A block 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb. Therefore, a block 1 ft. long, 5 ft. wide, 3 ft. thick weighs $\frac{7{,}200}{8}$ lb.; and a block 1 ft. long, 1 ft. wide, 3 ft. thick weighs $\frac{7{,}200}{8\times5}$ lb.; and a block 1 ft. long, 1 ft. wide, 1 ft. thick weighs $\frac{7{,}200}{8\times5\times3}$ lb. Therefore, a block 12 ft. long, 8 ft. wide, 5 ft. thick weighs

$$\frac{7{,}200\times12\times8\times5}{8\times5\times3}=\frac{7{,}200\times\overset{4}{\cancel{12}}\times\cancel{8}\times\cancel{5}}{\cancel{8}\times\cancel{5}\times\cancel{3}}=28{,}800 \text{ lb.} \quad \text{Ans.}$$

Solution 2.—The contents of the blocks, in cubic feet, may be compared.

Contents of first block = 8 ft. × 5 ft. × 3 ft. = 120 cu. ft.
Contents of second block = 12 ft. × 8 ft. × 5 ft. = 480 cu. ft.

If 120 cu. ft. weighs 7,200 lb., then 1 cu. ft. weighs $\frac{7{,}200}{120}$ lb.

Therefore, 480 cu. ft. weighs $\frac{480\times7{,}200}{120}=28{,}800$ lb. Ans.

Example 5.—If 12 horses can plow 96 acres in 6 days, how many horses will be required to plow 64 acres in 8 days?

Solution.—In 6 days 96 acres can be plowed by 12 horses; in 1 day 96 acres can be plowed by 6×12 horses; and in 1 day 1 acre can be plowed by $\frac{6\times12}{96}$ horses. Therefore, in 8 days, 1 acre can be plowed by $\frac{6\times12}{96\times8}$ horses, it being evident that fewer horses are required to do the same work in 8 days than in 1 day. In 8 days 64 acres can be plowed by

$$\frac{64\times6\times12}{96\times8}=6 \text{ horses.} \quad \text{Ans.}$$

Example 6.—If 7 horses require 35 bushels of oats in 20 days, for how many days will 96 bushels be sufficient for 18 horses?

Solution.— 35 bu. lasts 7 horses for 20 days. Therefore, 1 bu. lasts 7 horses for $\frac{20}{35}$ days, and 1 bu. lasts 1 horse for $\frac{7\times20}{35}$ days. Therefore, 96 bu. lasts 1 horse for $\frac{96\times7\times20}{35}$ days, and 96 bu. lasts 18 horses for

$$\frac{96\times7\times20}{35\times18}=\frac{64}{3}=21\tfrac{1}{3} \text{ days.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. If one pump discharges 90,000 gallons of water in 20 hours, in what time will it discharge 144,000 gallons? Ans. 32 hr.

2. If 25 men can finish a certain job in 30 days, how many days will be required by 35 men to do the same job? Ans. $21\frac{3}{7}$ da.

3. If $63 is paid for 15 tons of coal, how much must be paid for 27 tons? Ans. $113.40

4. A pump making 30 strokes per minute discharges 450 gallons of water per minute. If the speed is reduced to 28 strokes per minute, what will be the discharge in gallons per minute? Ans. 420 gal.

5. A wheel makes 248 revolutions in 5 minutes. How many revolutions will it make in 1 hour and 20 minutes? Ans. 3,968 rev.

6. A locomotive runs 2.8 miles in 4 minutes. How far will it go in 1 hour? Ans. 42 mi.

7. If 7 barrels of sugar costs $104.30, what is the cost of 42 barrels? Ans. $625.80

8. If 42 yards of goods are made from 9 pounds of yarn, how many yards can be made from 165 pounds of a similar yarn? Ans. 770 yd.

9. If 17 men, paid at the same rate, receive in all $357 for one week, what is the sum that must be paid to 24 men for 1 week, if paid at the same rate? Ans. $504

10. Twelve men can do a certain piece of work in 15 days. How many days will be required by 36 men to do the same work? Ans. 5 da.

POWERS AND ROOTS

INVOLUTION, OR FINDING POWERS OF NUMBERS

PERFECT AND IMPERFECT POWERS

DEFINITIONS AND RULES

1. Factors.—The **factors** of a number are those numbers which, when multiplied together, will equal that number. Thus, 5 and 3 are the factors of 15, since 5×3=15.

In engineering calculations it is often found necessary to multiply a number by itself one or more times; thus, 5×5×5=125. In this case the number 125 consists of three equal factors, each of which is 5.

2. Powers.—A product obtained from several *equal* factors is called a **power** of the number that is used as the factor. The power is named according to the number of equal factors in the product. Thus, 9 is the **second power,** or **square,** of 3, as 9 is equal to the product of the *two* equal factors 3 and 3. A product is called the **third power,** or **cube,** of a number, if it contains *three* equal factors; thus, 64 is the third power of 4, as 64 is equal to the product of the three factors 4, 4, and 4, or 4×4×4=64.

3. Squares and Cubes.—The term **square** is used for the second power of a number, because the area of a square is equal to the product of two equal numbers, each of which

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represents the length of one side. Thus, the area of a square is equal to the *second power* of a number that represents the length of one of its sides. For example: The side of a square is 4 feet; its area $=4\times4=16$ square feet.

The term **cube** is used for the third power of a number, because the volume of a cube is equal to the product found by using the length of one edge three times as a factor; that is, its volume is equal to the *third power* of a number representing the length of one of its edges. For example, the edge of a cube is 5 inches; its volume is $5\times5\times5=125$ cubic inches.

4. Involution.—The process of finding powers of quantities is called **involution.** The term *raise* is generally used in connection with this process. Thus, it is said that 7 is *raised* to the third power by using it as a factor three times, or $7\times7\times7=343$.

5. Exponents.—It is not sufficient to say that a *power* of a given number is to be found; one must also know *which* power is required, whether it is to be the second, the third, etc. For the purpose of indicating the required power of a number, a small number, called an **exponent,** is written to the right and near the top of the number. This number indicates the power to which a quantity is to be raised, or the number of times the quantity is to be used as a factor. Thus, in the expression 3^6, the number ^{6} is the exponent, and shows that 3 is to be used as a factor six times, or that 3^6 is a contraction of

$$3\times3\times3\times3\times3\times3$$

When an exponent is attached to a number it is read as in the following examples:

4×4 is written 4^2, and is read **four square,** or *four exponent two;*

$5\times5\times5$ is written 5^3, and is read **five cube,** or *five exponent three;*

$8\times8\times8\times8$ is written 8^4, and is read **eight to the fourth power,** or *eight exponent four.*

6. Use of Parenthesis.—When several numbers are connected by any of the arithmetical signs and the result is to be raised to a given power, the numbers must be written inside a parenthesis and the exponent outside it, as in the following example: $(2+5)^2=7^2=49$. Without the parenthesis the result would be: $2+5^2=2+25=27$.

The same rule applies if a fraction is to be raised to a given power. For instance, if the square of $\frac{3}{4}$ is to be found, it must be written $(\frac{3}{4})^2$. By omitting the parenthesis it will appear as if the exponent refers to the numerator alone, instead of applying to the numerator and the denominator. The effect produced by omitting the parenthesis may be seen from the following example: $\left(\frac{3}{4}\right)^2=\frac{3^2}{4^2}=\frac{9}{16}$. Omitting the parenthesis, the result is $\frac{3^2}{4}=\frac{9}{4}$.

7. Perfect and Imperfect Powers.—There are comparatively few numbers that can be separated into equal factors; these numbers are called **perfect powers.** Thus, 16 is a perfect power of 4, because $16=4\times4$; 216 is a perfect power of 6, because $216=6\times6\times6$. Numbers that cannot be separated into exactly equal factors are called **imperfect powers.** Thus, 10, 12, 15, and 20 are imperfect powers, because none of them is the product of equal factors. In the numbers from 1 to 1,000, inclusive, there are only 50 perfect powers, not counting 1, and of these only 30 are perfect squares and 9 perfect cubes.

TABLE I

PERFECT SQUARES AND CUBES

n	n^2	n^3	n	n^2	n^3
1	1	1	6	36	216
2	4	8	7	49	343
3	9	27	8	64	512
4	16	64	9	81	729
5	25	125	10	100	1,000

Table I contains the squares and cubes of numbers from 1 to 10, inclusive. The column of numbers is headed by the letter *n*, which is an abbreviation of the term *number*. The squares and cubes of the numbers found in the first and fourth columns are given in the other columns, headed by n^2 and n^3, respectively. Thus, the square of 8 is 64 and the cube of 5 is 125.

8. Rule for Raising a Number to Any Power.—To find any power of a number, the following rule should be used:

Rule.—I. *To raise a whole number or a decimal to any power, use the number as a factor as many times as the power requires or as the exponent indicates.*

II. *To raise a fraction to any power, use the numerator and the denominator as factors as many times as the power requires or as the exponent indicates, and write these products as numerator and denominator, respectively.*

Example 1.—What is the third power, or cube, of 35?

Solution.—From Art. **3**, the expression *cube of a number* is equivalent to the number with 3 as an exponent. Applying the rule, $35^3 = 35 \times 35 \times 35$, or

$$\begin{array}{r}
35 \\
35 \\
\hline
175 \\
105 \\
\hline
1225 \\
35 \\
\hline
6125 \\
3675 \\
\hline
\textit{cube} = 42{,}875
\end{array}\quad \text{Ans.}$$

Example 2.—What is the value of 1.2^2?

Solution.—According to the rule, the number must be used as a factor the number of times indicated by the exponent, 2. Hence, $1.2^2 = 1.2 \times 1.2 = 1.44$. Ans.

Example 3.—What is the value of $(\frac{5}{8})^2$?

Solution.—The exponent is 2, so the numerator and the denominator must each be used twice as a factor. Then,

$$\left(\frac{5}{8}\right)^2 = \frac{5^2}{8^2} = \frac{5 \times 5}{8 \times 8} = \frac{25}{64}. \quad \text{Ans.}$$

Example 4.—What is the fourth power of 15?

SOLUTION.—From Art. **5**, the fourth power of 15 is equivalent to 15^4. Applying the rule, $15^4=15\times15\times15\times15$, or

$$\begin{array}{r}
15\\
15\\ \hline
75\\
15\\ \hline
225\\
15\\ \hline
1125\\
225\\ \hline
3375\\
15\\ \hline
16875\\
3375\\ \hline
\textit{fourth power}=50{,}625
\end{array}\quad\text{Ans.}$$

EXAMPLE 5.—What is the cube of .12?

SOLUTION.—The cube is found by using the number three times as a factor; therefore, the cube of .12 is

$$.12\times.12\times.12=.001728.\quad\text{Ans.}$$

EXAMPLES FOR PRACTICE

Raise the following to the powers indicated:

(*a*)	85^2.	Ans.	(*a*)	7,225
(*b*)	$(\frac{12}{13})^2$.		(*b*)	$\frac{144}{169}$
(*c*)	6.5^2.		(*c*)	42.25
(*d*)	14^4.		(*d*)	38,416
(*e*)	$(\frac{3}{4})^3$.		(*e*)	$\frac{27}{64}$
(*f*)	$(\frac{5}{6})^3$.		(*f*)	$\frac{125}{216}$
(*g*)	$(\frac{7}{2})^3$.		(*g*)	$\frac{343}{8}$
(*h*)	1.4^5.		(*h*)	5.37824

EVOLUTION, OR FINDING ROOTS OF NUMBERS

SQUARE ROOT

DEFINITIONS

9. Roots of Numbers.—It was stated in Art. **2** that the product obtained from a number of equal factors is called a power of the number. If the process is reversed and the number of equal factors into which a number may be separated is found, then any one of these factors is known as the **root** of the number. For instance, if the number 27 is divided into the three factors 3, 3, and 3, then any one of these factors is known as a root of this number, as $27=3\times3\times3$. This process of finding a root of a number is known as **evolution;** it is the reverse of *involution.* The term *extract* is generally used in connection with this process, and it is said that the root of the number is *extracted.*

10. Classification of Roots.—If a number is separated into *two* equal factors, one of these factors is known as the **square root** of the number. Thus, if 25 is separated into two equal factors 5 and 5, then 5 is the square root of 25, because $5\times5=25$. The square root of 49 is 7, because $7\times7=49$; the square root of 1.21 is 1.1, because $1.1\times1.1=1.21$.

If a number is separated into *three* equal factors, one of the factors is known as the **cube root** of the number. Thus, 3 is the cube root of 27, since $3\times3\times3=27$.

The **fourth root** of a number is one of the *four* equal factors into which the number may be separated. Thus, the fourth root of 256 is 4, because $4\times4\times4\times4=256$.

The **fifth root** of a number is one of the *five* equal factors into which the number may be separated. Thus, 7 is the fifth root of 16,807, since $7\times7\times7\times7\times7=16{,}807$.

11. Derivation of Terms Square Root and Cube Root.—The terms *square* and *cube,* when applied to roots, are derived from the same source as similar terms applied to powers of numbers. Reversing the conditions stated in Art. **3,** it follows that, if a number represents the area of a square, the *square root* of the number must be equal to the length of one side of the square. For instance, if the area of a square floor is 81 square yards, the square root of 81, or 9, represents, in yards, the length of one side of the room.

If a given number represents the contents of a cube, the *cube root* of the number must give the length of one of the edges. Thus, if the contents of a cubical box is 27 cubic feet, the cube root of the number, or 3, gives the length of one of the edges, in feet.

12. Radical Sign.—The fact that the root of a number is to be extracted is usually indicated by placing the *radical sign* $\sqrt{}$ in front of it. The term **radical** is derived from the Latin word *radix,* meaning *root.* A *vinculum* $\overline{\quad}$ is connected with the radical sign and placed over the quantity to which the radical sign applies. Thus, $\sqrt{4{,}574{,}300}$ indicates that the root of the number following the radical sign is to be extracted.

13. Index.—Although the radical sign shows that a root is to be extracted, it must also show what root is required, and for this purpose an **index** is used. This is a small figure placed *above* the radical sign; thus, $\sqrt[2]{100}$ indicates the *square root* of 100, and $\sqrt[3]{1{,}728}$ indicates the *cube root* of 1,728. However, when the square root of a number is to be extracted, the index is usually omitted. Thus, $\sqrt{81}$ means the square root of 81; also, $\sqrt{22.48}$ means the square root of 22.48.

CALCULATION OF SQUARE ROOT

SQUARE ROOTS OF WHOLE NUMBERS AND OF DECIMALS

14. Introduction.—In calculating, or extracting, the square root of a number, it is necessary to perform a number of separate operations in a certain order. If the nature of each operation is kept in mind and if the operations are performed successively in the correct order, the process of extracting a square root should not offer any particular difficulties.

To facilitate the first part of the calculation, it is well to memorize the squares of the first twelve integers, or whole numbers, given herewith. The first line gives the numbers and the second line the corresponding squares.

Integers:	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12
Squares:	1,	4,	9,	16,	25,	36,	49,	64,	81,	100,	121,	144

15. Example of Extracting Square Root.—The method of finding the square root can best be explained by using an actual example and describing each step of the work. The first step is to point off, or separate, the number into periods, or parts, each containing two figures. Thus, suppose the square root of the whole number 31,505,769 is to be extracted. Ignore the commas that are used to divide the number and write it without them, thus: 31505769. Now, beginning at the right-hand figure, point off, or separate, the number into periods of two figures each, proceeding toward the left and using the mark ′ to separate the periods. The number will then appear as follows: 31′50′57′69. In case the whole number contains an odd number of figures, there will be only *one* figure in the period at the left. For instance, take the number 53,361. When this is pointed off it becomes 5′33′61, in which there are three periods. Here the 5 forms the first period, even though it consists of only one figure.

The reason for thus pointing off the number is to find out how many figures there will be in the root of the number. It is always true that the number of figures in the root is equal to the number of periods into which the number is divided. Thus, as 31′50′57′69 contains four periods, it is known at once, before any calculations are made, that there will be four figures in the square root of that number. Similarly, as 5′33′61 has three periods, the square root must contain three figures; and this is the case, as the square root of 53,361 is 231. After the number has been properly pointed off, the square root is extracted in the manner shown in the following example:

Example.—Extract the square root of 31,505,769.

Solution.—

	steps	*number*	*root*
(*a*)	5	31′50′57′69(	5613
	20	$5^2=25$	
first trial divisor	100	650	*first dividend*
	6	636	
first complete divisor	106	1457	*second dividena*
(*b*)	56	1121	
	20	33669	*third dividend*
second trial divisor	1120	33669	
	1		
second complete divisor	1121		
(*c*)	561		
	20		
third trial divisor	11220		
	3		
third complete divisor	11223		

Explanation.—Beginning at the right, the number 31,505,769 is pointed off into periods of two figures each, as already explained. The largest single number whose square is less than 31, the first period, is now found. This is evidently 5, since the square of 6, or 36, is greater than 31. This number, 5, is written to the right of the number, as in long division; it is also written to the left, as at (*a*). The square of this first figure of the root, or $5^2=25$, is written under the first period, as shown, and is subtracted from it, leaving 6 as a remainder. The second period of the number is annexed to this remainder, giving 650 as the **first dividend.**

The first figure of the root, written at (*a*), is now multiplied by 20, giving a product 100, which is called the **first trial divisor.** The first dividend, 650, is now divided by this first trial divisor, 100, and the quotient 6 is obtained, which is *probably* the second figure of the root. This figure is written in the root, as shown, and is also added to 100, the first trial divisor, giving the sum 106, which is called the **first complete divisor.**

The first complete divisor, 106, is multiplied by 6, the second figure in the root, giving the product 636, which is subtracted from the first dividend; the remainder is 14, to which the two figures, 57, in the third period of the number are annexed, giving 1457 as the **second dividend.** The two figures of the root, 56, are now written at (*b*) and multiplied by 20, thus giving 1120, which is the **second trial divisor.** Dividing 1457 by the second trial divisor, 1120, the quotient 1 is obtained as the third figure of the root. Adding this figure to the second trial divisor, the result is 1121, which is the **second complete divisor.** Multiplying this divisor by 1, the third figure in the root, gives the product 1121, which is written under the second dividend, 1457. Then, subtracting it from the second dividend, the remainder is 336, to which the fourth period, 69, of the number is annexed, giving 33669 as the **third dividend.**

The three figures 561 in the root are now placed at (*c*) and multiplied by 20, giving the product 11220, which is the **third trial divisor.** Dividing 33669 by the third trial divisor, 11220, the quotient, 3, is obtained as the fourth figure of the root. Adding this figure to 11220, the result is 11223, the **third complete divisor.** Multiplying this divisor by 3, the fourth figure of the root, the product is 33669, which is written under the third dividend and subtracted from it, leaving no remainder. It follows that $\sqrt{31{,}505{,}769}=5{,}613$, and that $31{,}505{,}769=5{,}613\times5{,}613$.

16. Choosing Figures for the Root.—In the preceding explanation, where reference was made to finding the second figure, 6, of the root, the statement was made that *probably* the figure 6 would be the one required. The word *probably* is used here, because the various figures that are successively selected for the root are, at first, only trial figures. There can be no certainty that each of these figures will be the correct one, and not too large, until it is multiplied by the corresponding complete divisor, thus allowing the resulting product to be compared with the dividend. Referring to the foregoing example, it is not certain that 6, the second figure in the root, is correct, before it has been multiplied by the first complete divisor, 106, and the product found to be not greater than the first dividend, 650. In this case the product 636 is less than 650; but, suppose that the first dividend were 620, as in the example that follows. The product 636 would be too large, thus indicating that the figure 6 in the root is too large. It would then be necessary to substitute, in place of the 6, the figure 5 as the second figure in the root. The remaining part of the solution is simply a repetition of that already described.

	(*a*)	5
		20
first trial divisor		100
		5
first complete divisor		105
	(*b*)	55
		20
second trial divisor		1100
		8
second complete divisor		1108
	(*c*)	558
		20
third trial divisor		11160
		6
third complete divisor		11166

root

31'20'3.3'96 (5586

$5^2 = 25$

620 *first dividend*

525

9533 *second dividend*

8864

66996 *third dividend*

66996

17. Main Features of Extracting Square Root.—In order that the main features of calculating a square root may be remembered, it should be noted that the various trial divisors are obtained by multiplying the existing figures in the root by 20. Thus, the *first* figure in the root multiplied by 20 gives the *first* trial divisor; the *first two* figures of the root multiplied by 20 gives the *second* trial divisor, and so forth. Considering this feature as the framework of the process, the remainder will easily be remembered.

18. Method of Procedure When Trial Divisor is Larger Than Its Dividend.—Sometimes, in extracting a root, it may happen that a trial divisor is larger than the corresponding dividend. In such a case the method adopted is similar to that used in long division; that is, a cipher is annexed to the root and to the trial divisor and the next period is annexed to the last remainder to form a new dividend, larger than the new trial divisor. The following example shows how to proceed:

EXAMPLE.—Find the square root of 255,025.

SOLUTION.—

		root	
(*a*)	5	25'50'25(505	
	20	$5^2=25$	
first trial divisor	100	5025	*second dividend*
second trial divisor	1000	5025	
	5		
second complete divisor	1005		

EXPLANATION.—After the number is separated into periods, it is seen that the root of the first period must be 5. This figure is written to the right of the given number and also at (*a*). The square of 5, or 25, is subtracted from the first period, leaving no remainder. The second period, 50, is brought down as the first dividend and the number 5 at (*a*) is multiplied by 20, giving 100 as the first trial divisor. As this trial divisor, 100, is not contained even once in the first dividend, 50, a cipher is annexed to the root and to the divisor, making the root 50 and the new, or second, trial divisor 1000. The third period of the number, 25, is now brought down and annexed to the remainder 50, making 5025, which is the second dividend. Dividing by the second divisor, 1000, the quotient, 5, is placed as the last figure of the root. Adding 5 to 1000 gives 1005 as the second complete divisor. Multiplying by 5 gives 5025, which is equal to the second dividend; hence, there will be no remainder.

In ordinary practice, instead of writing a separate second trial divisor, as in this example, the cipher would be annexed directly to the first trial divisor. Thus, a cipher could have been annexed to the first trial divisor, 100, giving the second trial divisor, 1000.

19. Extracting Square Roots of Decimals.—The square of any number, wholly decimal, always contains twice as many decimal places as the number squared. For example, $.1^2=.01$, $.13^2=.0169$, $.751^2=.564001$, etc. Conversely, it follows that the square root of a decimal contains only one half the number of decimal places found in the decimal itself. Thus, $\sqrt{12.723489}\doteq 3.567$. If the decimal contains an uneven number of figures, a cipher must be annexed to give an even number, as will now be explained.

When the square root of a decimal is to be extracted, the decimal is pointed off into periods of two figures each, *begin-*

ning at the decimal point and going to the *right.* Then, if the last period contains but one figure, a cipher is annexed to complete the period. Thus, the decimal .62371 is pointed off as follows: .62′37′10. Annexing ciphers to the right of a decimal does not change its value, as was explained in a preceding Section.

If the decimal is a portion of a mixed number, as 142.716, the whole-number part is pointed off to the *left* and the decimal part to the *right,* beginning at the decimal point in both cases, and considering the decimal point as a mark of separation. Thus, the number 142.716 is pointed off as follows: 1′42.71′60.

The operation of finding the square root in all these cases is similar to that previously described, except that when the decimal point is reached, a decimal point is placed in the answer. There will be as many decimal places in the root as there are periods in the decimal part of the number.

20. The following examples will show the method of extracting roots of decimals:

EXAMPLE 1.—Find the square root of 606.6369.

SOLUTION.—

		root	
	2	6′06.63′69(24.63	
	20	$2^2 =$ 4	
first trial divisor	40	206	*first dividend*
	4	176	
first complete divisor	44	3063	*second dividend*
		2916	
	24		
	20	14769	*third dividend*
		14769	
second trial divisor	480		
	6		
second complete divisor	486		
	246		
	20		
third trial divisor	4920		
	3		
third complete divisor	4923		

EXAMPLE 2.—What is the square root of .000576?

SOLUTION.—

		root
	2	.00'05'76(.024
	20	2^2= 4
trial divisor	40	176
	4	176
complete divisor	44	

EXPLANATION.—Beginning at the decimal point and separating the decimal into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure in the root must be a cipher. The decimal is now treated as if 5 were the first figure and the calculation is continued in the manner previously explained.

21. Perfect and Imperfect Powers.—Comparatively few numbers are exact squares; consequently, it is only in a small number of cases that the exact square root can be found. A number that has an exact root is called a **perfect power;** in the case of an exact square root the number from which it is extracted is termed a **perfect square**. The factors of the perfect powers are called **rational factors.** An exact square root of a number represents one of the two equal rational factors into which it is possible to separate the perfect square. For instance, $\sqrt{81}=9$, and $9\times9=81$; hence, the two numbers 9 are rational factors.

Numbers that cannot be separated into exactly equal factors are called **imperfect powers,** and the factors are called **irrational factors.** Any number, that cannot be divided into as many rational factors as there are units in the index of the root, will have a root with an unending decimal. For example, 20 lies between 16 ($=4^2$) and 25 ($=5^2$); hence, the square root of 20, or $\sqrt{20}$, is greater than 4 and less than 5, and is therefore equal to 4 plus an unending decimal. In other words, no matter to how many figures the square root of 20 may be calculated, the root will never be found exactly. Numbers ending in 2, 3, 7, or 8 are imperfect squares.

22. Extracting Root of Imperfect Power.—Although the square root of an imperfect power cannot be found exactly, as close an approximation may be obtained as is desired. The root may be carried to any required number of decimal places by annexing periods of two ciphers each to the number. In practice, five significant figures are all that are likely to be required, and four are generally sufficient.

Example 1.—What is the square root of 3? Find the result to five decimal places.

```
                           1                                root
                          20      3.00'00'00'00'00 ( 1.73205+
                          --  1²=1
                          20      ---
                           7      200  first dividend
                          --      189
first divisor             27      ---
                                   1100  second dividend
                         17        1029
                          20       ----
                         ---         7100  third dividend
                         340         6924
                           3         ----
                         ---          1760000  fifth dividend
second divisor           343          1732025
                                      -------
                        173             27975
                         20
                        ----
                        3460
                           2
                        ----
third divisor           3462

                      1732
                        20
                      ------
                      346400
                           5
                      ------
fifth divisor         346405
```

Explanation.—As five decimal places are required, it is necessary to annex five periods of ciphers to the right of the decimal point. The method employed in extracting the square root is the same as explained in the preceding examples. Attention is called only to the omission of the fourth divisor and dividend. After the fourth period is annexed to the remainder, 176, making the fourth dividend 17600, it is found that this dividend does not contain the divisor 34640. A cipher is therefore placed as the next figure in the root and a cipher annexed to the divisor, changing it into a fifth trial divisor. Another period of two ciphers is now annexed to the fourth dividend, changing it into a fifth dividend, and the calculation is continued, 5 being obtained as the next figure of the root. The required five decimal places have now been obtained and the operation could stop at this point; but in the case of an unending decimal, it is advisable to continue the operation one decimal place further, to ascertain whether the next figure in the decimal is 5 or greater. If so, the fifth figure is increased by 1. It will be found that the next figure is a cipher; hence the figure 5 will remain. The square root of 3 is, then, 1.73205+.

PROOF.—*To prove the square root, it is squared. If the number is an exact square, the square of the root will equal it; if it is not an exact square, the square of the root, plus the remainder, will equal it.*

EXAMPLE 2.—What is the square root of .3 to five decimal places?

SOLUTION.—
```
      5              .3 0'0 0'0 0'0 0'0 0 (.5 4 7 7 2 2
     20               25
    ---               --
    100                500
      4                416
    ---                ---
    104                 8400
                        7609
     54                 ----
     20                  79100
    ----                 76629
   1080                  -----
      7                   247100
   ----                   219084
   1087                   ------
                           2801600
   547                     2190884
    20                     -------
  -----                     610716
  10940
      7
  -----
  10947

  5477
    20
 ------
 109540
      2
 ------
 109542

 54772
    20
-------
1095440
      2
-------
1095442
```

EXPLANATION.—In this example a cipher is annexed to .3, making the first period .30, since every period in a decimal, as was mentioned in Art. **19**, must have two figures in it. The operation is carried on to six decimal places, to find out whether the sixth figure is greater than 5. It is seen that the sixth figure is 2; hence, the preceding figure is not altered, and the square root of .3 is .54772+ to five decimal places.

23. Abbreviated Method of Extracting Square Root.—When some proficiency is gained in extracting square root, it is possible to perform a portion of the calculation mentally. The preceding solution may then be found as follows:

		.30'00'00'00'00(.54772
		25
	100	500
complete divisor	104	416
	1080	8400
complete divisor	1087	7609
	10940	79100
complete divisor	10947	76629
	109540	247100
complete divisor	109542	219084
		28016

EXPLANATION.—The first figure, 5, in the root is squared, as before, and subtracted from 30. The figure 5 in the root is now mentally multiplied by 20 and the product, 100, is written to the left, as before. The next figure in the root, or 4, is mentally added to 100 and the sum, 104, is multiplied by 4; the product 416 is subtracted from 500, leaving 84. The next period is brought down and the two figures 54 in the root are mentally multiplied by 20; the product, 1080, is written in the usual place. Mentally, the quotient 7 is added to 1080 and the sum, 1087, multiplied by 7. The product, 7609, is subtracted. The calculation is continued in this manner until the required number of figures is obtained in the root. After long practice it is possible to omit the writing of products, such as 100, 1080, etc., used to obtain trial divisors and simply write the divisors, as 104, 1087, etc.

24. Rule for Extracting Square Root.—The instructions given in the preceding explanations have been collected and combined in the form of a rule, as follows:

Rule.—*To extract the square root of a number, first point off, or separate, the number into periods of two figures each, commencing at the right; or, if the number contains a decimal, begin at the decimal point and separate the number on either side into periods.*

Find the largest number whose square is contained in the first, or left-hand, period and write this number as the first *figure of the root. Write its square under the first period and subtract. To the remainder annex the next period for a first dividend.*

Place the first figure of the root also to the left of the given number; multiply this figure by 20, and use the product as the first trial divisor.

Divide the dividend by the divisor for the second figure in the root and add this figure to the trial divisor to form the complete divisor. Multiply the latter by the second figure of the root and subtract the product from the dividend. (If this product is larger than the dividend, a smaller number must be tried for the second figure of the root.)

Bring down the third period and annex it to the last remainder. Place the first two figures of the root to the left of the given number and multiply them by 20; the product is the second trial divisor.

Continue in this manner to the last period. Should any additional places in the root be required, annex cipher periods of two figures each to the remainders and continue the operation.

If at any time a trial divisor is larger than the corresponding dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.

If the root is an unending decimal and it is desired to terminate the operation at some point, as the fourth decimal place, continue the operation to one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1.

EXAMPLES FOR PRACTICE

Find the square root of:

(*a*) 186,624		(*a*) 432
(*b*) 2,050,624		(*b*) 1,432
(*c*) 29,855,296		(*c*) 5,464
(*d*) .0116964 to five decimal places		(*d*) .10815
(*e*) 198.1369 to four decimal places	Ans.	(*e*) 14.0761
(*f*) 994,009		(*f*) 997
(*g*) 2.375 to four decimal places		(*g*) 1.5411
(*h*) .571428 to five decimal places		(*h*) .75593

SQUARE ROOTS OF FRACTIONS

25. Rule for Extraction.—If the given number is in the form of a fraction, and it is required to find the square root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the square root of the decimal. If, however, the numerator and denominator of the fraction are perfect squares, extract the square root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

Rule.—*Extract the square root of the numerator and denominator separately; or, reduce the fraction to a decimal, and extract the square root of the decimal.*

EXAMPLE 1.—What is the square root of $\frac{9}{64}$?

SOLUTION.—Applying the rule, the roots of the numerator and the denominator are extracted separately, giving 3 and 8, respectively, as the numerator and denominator in the new fraction. Thus,

$$\sqrt{\frac{9}{64}}=\frac{\sqrt{9}}{\sqrt{64}}=\frac{3}{8}. \text{ Ans.}$$

EXAMPLE 2.—What is the square root of $\frac{5}{8}$?

SOLUTION.—Reduce the fraction to a decimal; thus, $\frac{5}{8}=.625$. Then the square root of $\frac{5}{8}$ is the square root of .625.

$$\sqrt{.625}=.79057. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

Find the square root of:

(*a*) $\frac{25}{256}$	Ans.	(*a*) $\frac{5}{16}$
(*b*) $\frac{9}{16}$		(*b*) $\frac{3}{4}$
(*c*) $\frac{3}{4}$ to five decimal places		(*c*) .86603
(*d*) $\frac{5}{12}$ to four decimal places		(*d*) .6455

APPLICATIONS OF SQUARES AND SQUARE ROOTS

INTRODUCTION

26. Plane Surfaces.—The process of finding the square root of a number is very useful in many varieties of calculations, as, for example, in finding certain dimensions of squares, triangles, circles, and other surfaces, when their areas are given. Before entering into a description of the methods to be applied for these purposes, it is necessary to give a short description of the principal properties of these surfaces.

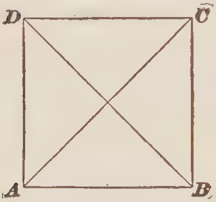

FIG. 1

The following definitions refer only to *plane* surfaces: A **plane surface** is one on which straight lines drawn in any direction lie wholly in the surface. For instance, the surface of a table is a plane surface. A **plane figure** is a plane surface bounded by straight or curved lines. A **polygon** is a plane figure bounded by any number of straight lines. Squares and rectangles, Art. **27,** and triangles, Art. **28,** are examples of polygons. The **area** of a plane figure is the number of square units included within its boundary lines.

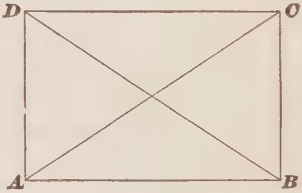

FIG. 2

27. Squares and Rectangles.—A **square,** Fig. 1, is a plane figure bounded by four straight lines AB, BC, CD, and DA of equal lengths, the adjacent lines being at right angles to each other.

A **rectangle,** Fig. 2, is a plane figure bounded by four straight lines, the adjacent lines being at right angles to each other. Only opposite sides are of equal lengths; thus, $AB = CD$, and $AD = CB$.

28. Triangles.—A **triangle,** Fig. 3, is a plane figure bounded by three straight lines. A **right-angled triangle,**

Fig. 4, also known as a **right triangle,** is a triangle that has one right angle. The side opposite the right angle is known as the **hypotenuse,** and is the longest one of the three sides. The **base** is the side on which the triangle is shown to rest. In the right triangle, Fig. 4, the right angle is at *C* and *A B* is the hypotenuse. The side *A C* is the base, as the triangle is shown resting on this side. The shortest distance between the base and the point at which the other sides meet is known as the **altitude** of a triangle. In Fig. 4 the side *B C* is also the altitude, as *B C* is the shortest distance from the side *A C* to the point *B* at which the sides *A B* and *B C* meet. If one of the two sides that form the right angle serves as a base, the other side is the altitude. Thus, if *B C* is the base, *A C* is the altitude.

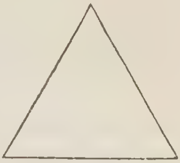

FIG. 3

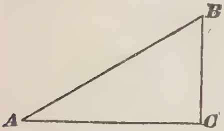

FIG. 4

29. Dividing Surfaces Into Triangles.—A straight line drawn between opposite corners of a square or a rectangle is known as a **diagonal.** In Figs. 1 and 2 the lines *A C* and *B D* are diagonals. Each diagonal divides the square and rectangle into two equal right triangles. For instance, in Figs. 1 and 2, the diagonal *A C* separates each figure into the two triangles *A B C* and *C D A,* and the diagonal *B D* in each produces the triangles *B C D* and *D A B.* When some of the dimensions of rectangles are to be obtained, it often simplifies the calculations to consider the rectangle as composed of two equal right triangles and to calculate the dimensions of one of these, according to the rules given farther on.

NOTE.—In practice, when referring to the diagonal of a square, this dimension is sometimes called the width *across corners.*

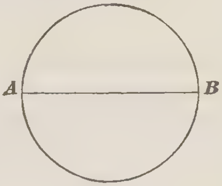

FIG. 5

30. Circles.—A **circle,** Fig. 5, is a plane figure bounded by a curved line called the **circumference.** A **diameter** is a line passing through the center and terminating at the circumference, as *A B.*

CALCULATIONS RELATING TO PLANE SURFACES

31. Finding Side of a Square.—It was shown in Art. **3** that the area of a square is equal to the second power, or square, of the number representing one of its sides. It follows that, if the area of the square is given, the length of one side is found by reversing the process; that is, by extracting the square root of the number that represents the area of the square. In examples of this class the following rule is applied:

Rule.—*To find the length of one side of a square of a given area, extract the square root of the area.*

EXAMPLE.—The area of a square is 1,764 square inches. What is the length of one side of the square?

SOLUTION.—Applying the rule, the side is equal to $\sqrt{1{,}764}=42$ in. Ans.

32. Principle of Right Triangle.—If a square is constructed on each of the three sides of a right triangle, as on *A B C*, Fig. 6, it can be shown that the area of the square erected on the hypotenuse *A C* is equal to the combined areas of the squares erected on the sides *A B* and *B C*. For instance, let it be assumed that the sides *A B*, *B C*, and *A C* are 3, 4, and 5 inches long, respectively. If the side *A B* is divided into three parts, each 1 inch long, and the square *A B I K* is erected on *A B*, as one of its sides, the square will contain 9 equal squares, or 9 square inches. Similarly, the square *B C H F* on the side *B C* will contain 16 square inches, and the square *A C E D* will contain 25 square inches, as shown. But, $9+16=25$.

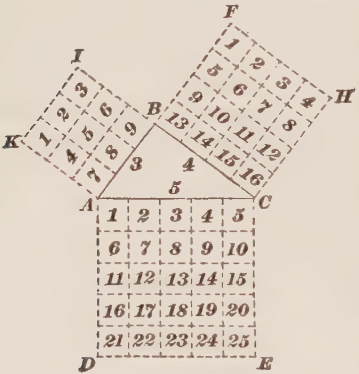

FIG. 6

It was stated in Art. **3** that the area of a square is equal to the square of the number representing the length of one side. The principle illustrated in Fig. 6 may therefore be stated in the following two forms:

I. *The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two sides.*

II. *In a right triangle, the square of one of the sides is equal to the square of the hypotenuse minus the square of the other side.*

On these statements are based the following rules:

Rule I.—*To find the hypotenuse of a right triangle, square the length of each side, add the squares, and take the square root of the sum.*

Rule II.—*To find the length of either side of a right triangle, square the length of the hypotenuse, subtract from it the square of the length of the known side, and take the square root of the remainder.*

33. Calculating Sides of Right Triangles.—The application of the rules in Art. **32** for finding the sides and the hypotenuse of a right triangle is shown in the following examples:

Example 1.—The sides $A\,C$ and $C\,B$, Fig. 4, are 8 and 6 inches long, respectively. What is the length of the hypotenuse $A\,B$?

Solution.—By rule I, the lengths of the sides are squared. Thus, $8^2=64$, and $6^2=36$, and $64+36=100$; then, extract the square root, $\sqrt{100}=10$. Therefore, $A\,B=10$ in. Ans.

Example 2.—If it is assumed that the hypotenuse $A\,B$, Fig. 4, is 25 inches long, and that the side $A\,C$ measures 20 inches, what is the length of the side $B\,C$?

Solution.—By rule II, the square of the length of $A\,B$ is $25^2=625$, and the square of the length of $A\,C$ is $20^2=400$. $625-400=225$; then, the square root is $\sqrt{225}=15$. Therefore, $B\,C=15$ in. Ans.

Example 3.—In a right triangle the hypotenuse is 12 inches long and one side is 4 inches long. What is the length of the other side?

Solution.—By rule II, the square of the length of the hypotenuse is $12^2=144$, and the square of the length of the known side is $4^2=16$. Then $144-16=128$; and, $\sqrt{128}=11.31$. Therefore, the length of the unknown side is 11.31 in. Ans.

Example 4.—A ladder is leaning against the side of a house with its lower end 8 feet from the side. If the upper end of the ladder is 35 feet above the ground, what is the length of the ladder?

Solution.—The ladder may in this case be considered as the hypotenuse of a right triangle in which the sides are 8 ft. and 35 ft. long, respectively. By rule I, the lengths of the sides are squared. Thus. $35^2=1{,}225$ and $8^2=64$, and $1{,}225+64=1{,}289$. Then $\sqrt{1{,}289}=35.9$. Therefore, the length of the ladder is 35.9 ft. Ans.

34. Finding Diameter of a Circle.—The ratio of the circumference of a circle to its diameter is as 3.1416 to 1. Then if the diameter of a circle is known, its circumference can be found by multiplying the diameter by 3.1416.

The area of a circle is equal to the product 3.1416 $\times\frac{(\text{diameter})^2}{2^2}=3.1416\times\frac{(\text{diameter})^2}{4}=.7854\times(\text{diameter})^2$. Or, the area of a circle is equal to the product obtained by multiplying .7854 by the square of the diameter. Conversely, if the area of a circle is known the diameter may be found by applying the following rule:

Rule.—*To find the diameter of a circle whose area is known, divide the area by .7854 and extract the square root of the quotient.*

Example 1.—The area of a circle is 38.4846 square inches. What is its diameter?

Solution.—According to the rule, the value of the area, or 38.4846, is divided by .7854 and the square root of the quotient is extracted. Or,

$$\sqrt{\frac{38.4846}{.7854}}=\sqrt{49}=7$$

Therefore, the diameter is 7 in. Ans.

Example 2.—What is the diameter of a circle whose area is 1.7671 square feet?

Solution.—According to the rule, the diameter is

$$\sqrt{\frac{1.7671}{.7854}}=\sqrt{2.25}=1.5$$

The diameter of the circle is 1.5 ft., or $1\frac{1}{2}$ ft. Ans.

EXAMPLES FOR PRACTICE

1. What must be the length of a ladder to reach to the top of a wall 40 feet high, if the foot of the ladder is 9 feet from the wall?
Ans. 41 ft.

2. 5,776 barrels standing on end are to be arranged in the form of a square. How many barrels will there be on each side of this square? Ans. 76 bbl.

3. Two vessels sail from the same port. At a given time one vessel is at a distance of 360 miles due north from the port; the other is at a distance of 450 miles due east from the port. What is the distance between the two vessels? Ans. 576.28 mi.

4. The floor of a room is 24 feet long and 18 feet wide; what is the length of the diagonal? Ans. 30 ft.

5. A light rope, drawn from the top of a flag pole to a point at a distance of 43 feet from the center of the pole support and at the same level as its base, is 110 feet long; what is the height of the pole? Ans. 101.2 ft.

6. The opening of a safety valve on a steam boiler has an area of 12.566 square inches. What is the diameter of the opening? Ans. 4 in.

7. The base of a right triangle, as in Fig. 4, is 20 feet, and the altitude is 18 feet; what is the length of the hypotenuse? Ans. 26.9 ft.

8. The square nut for a bolt is $3\frac{1}{8}$ inches on each side. What is the width across corners? Ans. 4.42 in.

9. A circular plate of brass has an area of 100 square inches. What is its diameter? Ans. 11.28 in.

MENSURATION

LINES, SURFACES, AND SOLIDS

LINES AND ANGLES

LINES

1. Geometry.—That branch of mathematics which deals with the relations, properties, and measurements of lines, angles, surfaces, and solids is known as **geometry.** The particular branch of geometry that treats only of the measurement of lines, angles, surfaces, and solids is known as **mensuration.**

In laying out work the mechanic will frequently make use of lines and angles in constructing various surfaces. The general property of these elements, as well as some methods of constructing them, will be explained in this Section, together with the calculation of surface areas and the contents of solids.

2. Points.—In mathematics a **point** is supposed to be without any dimensions; that is, it has no length, breadth, or thickness. In accurate work the position of a point is indicated by two lines crossing each other at the place where the point is intended to be located.

3. Lines.—From a mathematical point of view a **line** has only one dimension—*length;* it serves to connect two points. On paper a line is represented by means of a mark, that may vary in thickness, depending on the means used to make it.

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A **straight line,** Fig. 1 (*a*), is one that does not change direction throughout its whole length. A straight line is frequently called a *right line.* Lines shown in illustrations are usually referred to by letters at their ends, as *a b,* Fig. 1 (*a*).

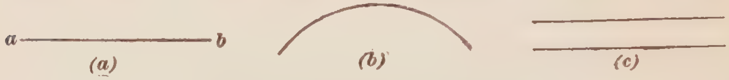


FIG. 1

A **curved line,** Fig. 1 (*b*), changes its direction at every point, no part of the line being straight.

4. Lines Named According to Relative Positions. When two straight lines are equally distant from each other throughout their length, as in Fig. 1 (*c*), they are said to be **parallel lines.** Either one of two parallel lines is said to be *parallel to* the other.

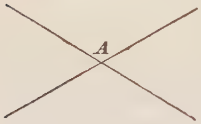


FIG. 2

5. When two lines cross or cut each other, as in Fig. 2, they are said to be **intersecting lines,** and the point *A,* at which they intersect, or cut each other, is called the **point of intersection.**

6. A line is said to be **perpendicular** to another line, when it meets that line so as not to incline toward it on either side. Thus, in Fig. 3 (*a*), the line *A B* is perpendicular to the line *C D*; also, *C D* is perpendicular to *A B.* Either line is also said to be **at right angles** to the other.

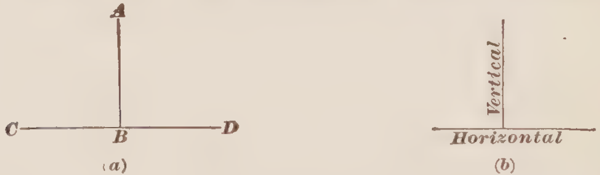


FIG. 3

7. A line that is parallel to the horizon, or water level, as *C D* in Fig. 3 (*a*) or the corresponding line in Fig. 3 (*b*), is known as a **horizontal line.**

8. A line that is perpendicular to a horizontal line, as in Fig. 3 (*b*), is called a **vertical line**; it has the direction of a *plumb-line*.

9. Drawing Perpendicular Lines.—In laying out some classes of work, it is necessary to draw one line perpendicular, or at right angles, to another; therefore, methods of doing this will be explained. In the example, Fig. 4, a line *A B* is drawn, and it is desired to draw a line perpendicular to it at the point *P*. To do this, the legs of a pair of compasses are separated to any convenient distance, the pointed leg is set at the point *P* while with the other leg, which carries a pencil, two marks are made at *C* and *D* on the line *A B*. These marks will be at equal distances from the point *P* and on opposite sides of it. The legs of the compasses are now brought farther apart, so that they will span a distance greater than *P C*, the pointed leg is set at *C*, and a short arc (or part of a circle) is struck with the pencil above *P*, as at *F*. The leg situated at *C* is now removed to *D*, with the legs of the compasses the same distance apart as before, and another arc is struck at *F*. The points at which these arcs intersect is the point required, as a line drawn from it to *P* will be perpendicular to *A B*.

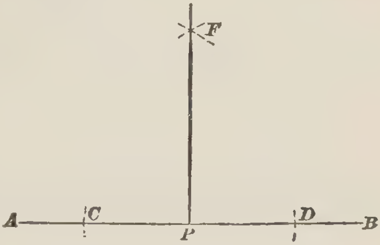


Fig. 4

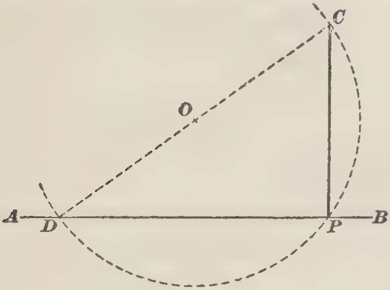


Fig. 5

10. If the point *P*, Fig. 4, at which the perpendicular is to be drawn, lies near the end of the line, as, for instance, at the edge of a piece of work, the foregoing method cannot be used. In such cases, the construction shown in Fig. 5 may be applied.

where *A B* is the line and *P* the point at which the perpendicular is to be drawn. The legs of the compasses are separated to some convenient distance, one leg is placed at a point above *A B,* as at *O,* and with the other the greater part of a circle is drawn so as to pass through the point *P,* as shown by the dotted curve. This circular arc will also cross the line *A B* at *D.* From the latter point a line is drawn through the center *O* and continued till it intersects the arc at *C.* A line drawn from the point *C* to the point *P* will then be perpendicular to *A B.*

11. Sometime a perpendicular has to be drawn to a line from a given point outside the line. The method to be employed in this case is shown in Fig. 6, in which *A B* is the line and *P* the given point. One leg of the compasses is placed at *P* and with the other leg an arc is described, so that it will intersect the line *A B* in any two points, *C* and *D,* as shown by dotted curve. One leg of the compasses is placed successively at the points *C* and *D,* and short arcs are described with the other leg at *E.* From their point of intersection, *E,* a line is drawn to *P*; then *E P* is perpendicular to *A B.*

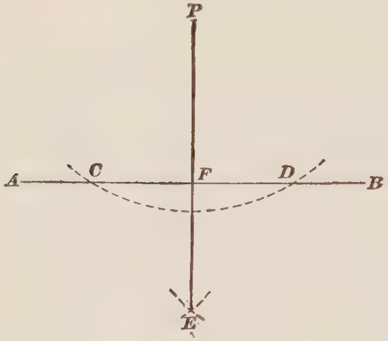


FIG. 6

A perpendicular, drawn from a point outside a straight line, is the **shortest distance** from the point to the line. Thus, in Fig. 6, the perpendicular *P F* is the shortest distance from *P* to *A B.*

ANGLES

12. Definition of an Angle.—When two lines, as *a b* and *b c,* Fig. 7, meet or intersect at a point *b,* the opening between the lines is known as an **angle.** The lines are known as the **sides** and the point *b* as the **vertex** of the angle.

An angle may also be defined as the difference in direction between two straight lines. This may be made clearer by means of a practical illustration. If the lines *a b* and *b c*, Fig. 7, are supposed to represent the legs of a pair of compasses, the arc *d* will represent the path through which a point on the leg *a b* will have to move in order that this point may coincide with another point on the leg *b c*, at the same distance from *b*. The length of the arc *d*, in degrees, etc., will, therefore, serve to indicate the size of the angle.

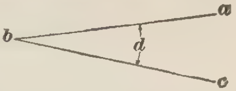
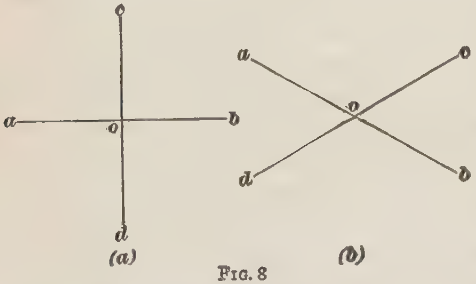


FIG. 7

In referring to an angle, three letters are commonly required, one letter at a point on each leg and one at the vertex. In naming the angle, the letter at the vertex is always placed between the other two. Thus, the angle, Fig. 7, is referred to as the angle *a b c*. If the angle stands by itself, as in this case, it may be referred to simply by the letter at the vertex, as angle *b*. Angles are sometimes referred to by means of a short arc and a letter, as in Fig. 7, where *d* indicates the angle.

13. Classification of Angles.—When two lines cross, as in Fig. 8 (*a*) and (*b*), they form four angles *a o c*, *c o b*, *b o d*, and *d o a*. Angles that have the same vertex, and one side in common, are **adjacent angles**, as *a o c* and *c o b*, in which *o* is the vertex of each and the side *o c* is common to both angles.



(a) (b)

FIG. 8

When two lines intersect, the angles formed on opposite sides of the vertex are **opposite angles.** For instance, the angles made by a pair of scissors blades and by the handles, respectively, are opposite angles. In Fig. 8, *a o c* and *b o d* are opposite angles; so are *a o d* and *b o c*.

14. When one straight line meets another so that the adjacent angles formed are equal, as *a o c* and *c o b*, Fig. 8 (*a*), the angles are called **right angles,** and the lines are said to be **at right angles,** or **normal,** or perpendicular, to each other. When a line is perpendicular to another line, as in Fig. 8 (*a*), the adjacent angles must be right angles.

The sum of all the angles that are on one side of a straight line and have a common vertex is always equal to two right angles. In Fig. 8 (*a*) and (*b*) the sum of the angles *a o c* and *c o b* is equal to two right angles. If additional straight lines are drawn through the vertex *o* the number of angles will increase, but the sum of the angles on the *same* side of the line *a b* will remain equal to two right angles.

If there are two right angles on one side of a straight line, as in the case of the line *a b,* Fig. 8 (*a*), there must also be two right angles on the other side. Hence, the sum of all angles about a common vertex is equal to four right angles.

15. An angle greater than a right angle, as the angle *a o c,* Fig. 8 (*b*), is an **obtuse angle,** and an angle less than a right angle, as the angle *c o b,* is an **acute angle.**

16. Complements and Supplements.—An angle is said to be the complement of another, when the sum of the two angles is *one* right angle. In Fig. 9, if *b o* is at right angles to *a d,* the angle *b o c* is the complement of the angle *c o d,* because the sum of the two angles is a right angle, *b o d.* Two angles that together make one right angle are said to be **complementary.** Thus, *b o c* and *c o d* are complementary angles.

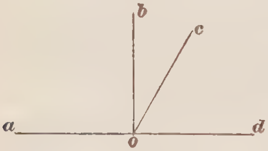


FIG. 9

When the sum of two angles is equal to *two* right angles, the angles are said to be **supplementary,** and each is the **supplement** of the other. In Fig. 9, the angle *c o d* is the supplement of the angle *c o a,* and *c o a* is the supplement of *c o d.* Also, the angles *a o b* and *b o d* are supplementary angles.

17. Angular Measure.—If the length of the arc *d,* Fig. 7, is to serve as a measure of the size of an angle, it is necessary to select a suitable unit. For this purpose the circumference of a circle is divided into 360 equal parts called **degrees,** and each degree is divided into 60 equal parts known as **minutes.** A minute is divided into 60 equal parts called **seconds,** which are used where greater accuracy is required than can be expressed in degrees and minutes.

If two lines are drawn at right angles to each other, as in Fig. 8 (*a*), the point of intersection, *o*, may be used as the center of a circle. This circle will then be divided by the intersecting lines into four equal parts, known as **quadrants,** each representing an angle of $\frac{360}{4}=90$ degrees, usually written 90°, as shown in Fig. 10. One-half of a right angle is $\frac{90}{2}=45°$, one-fourth is $22\frac{1}{2}°$, one-third is 30°, two-thirds is 60°, etc., as indicated.

If the radius of the circle, drawn with the vertex of the angle as a center, is increased in length, the circumference of the circle will also become greater, and, consequently, also the length of each division. But the angle, or amount of opening, included between two intersecting lines will not be affected. Thus, the angle marked 90° in Fig. 10 will include $\frac{1}{4}$ of 360°, as before.

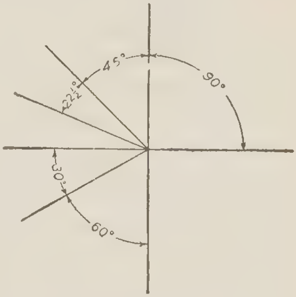


FIG. 10

This system of measuring angles is called **angular measure;** its units and abbreviations are given in the following table. The signs for minutes and seconds are ′ and ″, the same as for feet and inches; but as they usually occur in connection with the sign for degrees, there is little danger of their being misread for feet and inches. The expression 27° 13′ 45″ is read *27 degrees 13 minutes 45 seconds.*

Usually the mechanic will not have to deal with smaller divisions of a degree than minutes; that is, the values of the angles he may have to use in calculations will ordinarily be given in degrees and minutes, and not in degrees, minutes, and seconds.

ANGULAR MEASURE

60 seconds (″)	=1 minute	′
60 minutes	=1 degree	°
360 degrees	=1 circle	cir.

18. Calculations in Angular Measure.—In some calculations it may be necessary to reduce degrees and minutes to degrees and a fraction of a degree; that is, to reduce a denominate number to a higher denomination, as explained in *Weights and Measures*. To do so, the number of minutes is written as the numerator of a fraction with 60 as the denominator, and added to the whole number of degrees. For example, 12° 45′ is equal to $12\frac{45}{60}$ degrees, or $12\frac{3}{4}$ degrees, which may be expressed decimally as 12.75°. If the expression contains both minutes and seconds, the minutes and seconds are reduced to seconds and divided by 3,600, the number of seconds in one degree, and the fraction is added to the whole number of degrees. For example, if 39° 16′ 15″ is to be reduced to degrees, the expression 16′ 15″, reduced to seconds, becomes $(16\times 60)+15=975$ seconds. Dividing this product by 3,600, the fraction is $\frac{975}{3600}=\frac{13}{48}=.271$. Therefore, 39° 16′ 15″ may be written $39\frac{13}{48}$ degrees, or 39.271°.

19. Angles may be added and subtracted in the same way as other compound numbers. For example, if the angles of 45° and $22\frac{1}{2}$°, Fig. 10, are to be added, their sum is $45°+22\frac{1}{2}° =67\frac{1}{2}°$.

The following examples show how to proceed in calculations dealing with angular measure.

Example 1.—Find the sum of 12° 34′, 7° 48′, and 36° 11′.

Solution.—The compound numbers are arranged as shown, with like units in the same column. The sum of the right-hand column is 93′, which is equal to 1° 33′. The number 33 is written under the minutes, the 1° is carried over and added to the degree column, giving a total of 56°. The sum is then 56° 33′.

$$\begin{array}{rr}
12^\circ & 34' \\
7 & 48 \\
36 & 11 \\
\hline
56^\circ & 33'
\end{array} \quad \text{Ans.}$$

Example 2.—Subtract 12° 25′ from 45° 40′.

Solution.—The smaller value is placed under the larger and subtracted in the usual manner.

$$\begin{array}{rr}
45^\circ & 40' \\
12 & 25 \\
\hline
33^\circ & 15'
\end{array} \quad \text{Ans.}$$

Example 3.—From 39° 4′ subtract 27° 56′.

Solution.—As 56′ cannot be taken from 4′, 1°, or 60′, is taken from the 39°, leaving 38°, and added to the 4′, making 64′. The minuend will then assume the form as shown below, and the subtraction may now be carried out in the usual manner.

$$\begin{array}{rr} 39° & 4′ \\ \underline{27} & \underline{56} \end{array}$$

$$\begin{array}{rrl} 38° & 64′ & \\ \underline{27} & \underline{56} & \\ 11° & 8′ & \text{Ans.} \end{array}$$

The minuend is not usually rewritten as shown here, but the process of taking 1° from 39° and adding its equivalent, or 60′, to the minutes is done mentally.

20. Means for Measuring Angles.—Angles are commonly measured by the use of a **protractor,** a form of which is shown in Fig. 11. It consists of a piece of celluloid or of metal of a semicircular shape and very thin. Along the curved edge are a number of divisions, the smallest representing $\frac{1}{2}$°, or 30′, the next larger representing 1°, and the larger ones representing 5° and 10°, respectively. To use the protractor, it is laid flat on the angle to be measured, with the point *O* directly on the vertex of the angle and the line *A B* directly over one side of the angle. The point where the other side of the angle crosses the scale shows the size of the angle.

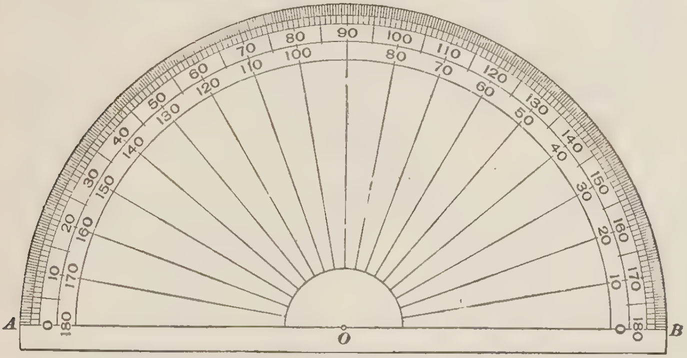


Fig. 11

21. Laying Off an Angle.—If a line is given and it is required to draw another line that inclines toward it at a given angle, the required line may be drawn in the following manner: Let *C F*, Fig. 12, be the given line and *C* the point from which a line is to be drawn at an angle of 54° with the line *C F*. The protractor, Fig. 11, is laid on the line *C F* so that its center *O* comes directly over the vertex *C* of the angle and so that the line *A B* coincides with the line *C F*, Fig. 12. Then 54° is counted off from the lower end *B* of the scale on the protractor. This means 5 large divisions, each of which represents 10°, and 4 small divisions, each of which represents 1°. Opposite the end of the 54° mark the point *D* is located with a sharp pencil or a *scriber*, a pointed steel tool used for *scribing*, or scratching, lines on metal surfaces. Then the protractor is removed and a straight line is drawn or scribed so as to pass through *C* and *D*. This line *C D* will make an angle of 54° with the line *C F*; that is, the angle *D C F* will be 54°.

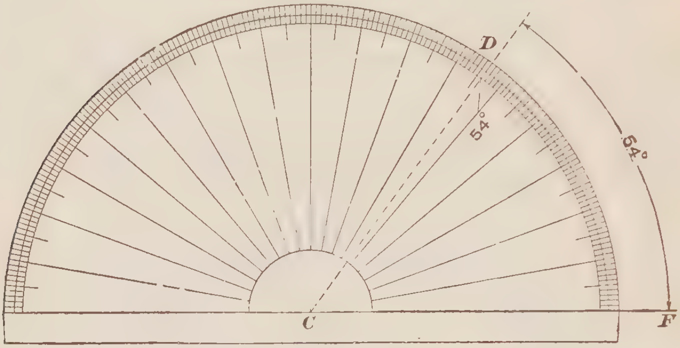


FIG. 12

EXAMPLES FOR PRACTICE

1. Find the sum of 26° 18′ and 18° 42′.	Ans. 45°
2. What is the difference between 88° 28′ and 42° 12′?	Ans. 46° 16′
3. What is the complement of 36° 32′?	Ans. 53° 28′
4. What is the supplement of 87° 29′?	Ans. 92° 31′
5. Find the difference between the complement of 18° 30′ and the supplement of the same angle.	Ans. 90°
6. Subtract 82° 48′ from 112° 23′.	Ans. 29° 35′
7. How many seconds are there in 32° 14′ 6″?	Ans. 116,046″
8. How many degrees, minutes, and seconds do 38,582 seconds amount to?	Ans. 10° 43′ 2″

9. In a pulley with five arms, what part of a right angle is included between the center lines of any two adjacent arms?

Ans. $\frac{4}{5}$ of a right angle

10. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°

SURFACES

DEFINITIONS AND CLASSIFICATION

22. Plane Surface.—A **plane surface,** usually called a **plane,** is a surface upon which straight lines may be drawn in any direction. A practical example of a plane is the surface of a *surface plate,* a steel plate ground perfectly flat on its top surface. If a straightedge is laid on its surface, every point along the edge of the straightedge will touch the surface, no matter in what direction it is laid.

23. Plane Figures.—Any part of a plane surface bounded by any number of straight or curved lines or a combination of the two is known as a **plane figure.**

24. Polygons.—When a plane figure is bounded by straight lines only, it is called a **polygon.** The bounding lines are called **sides,** and the combined length of the sides is known as the **perimeter** of the polygon. Polygons include figures with three, four, or more sides, but it is customary to divide polygons into three classes, calling those with three sides *triangles,* those with four sides *quadrilaterals,* and those with more than four sides *polygons.*

TRIANGLES

25. Classification.—A **triangle** is a polygon with three sides. Triangles are named according to the relative lengths of their sides as *isosceles, equilateral,* and *scalene triangles,* and according to the nature of the angles as *right-angled* and *oblique triangles.*

26. When two sides of a triangle are of equal lengths, as in Fig. 13, it is known as an **isosceles** (i-sos′se-leez) **triangle.** This word *isosceles* means *equal legs.*

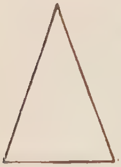


FIG. 13

27. When the three sides of a triangle are of equal lengths, as in Fig. 14, it is called an **equilateral triangle.**

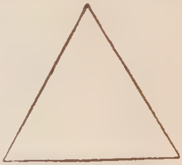


FIG. 14

28. A triangle in which all the sides are of different lengths, as Fig. 15, is called a **scalene triangle.**

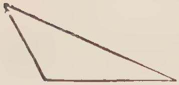


FIG. 15

29. If one of the angles in a triangle is a right angle, the triangle is a **right-angled triangle.** The side opposite the right angle is called the **hypotenuse.** In Fig. 16 the side $A\,B$ is the hypotenuse because it is opposite the right angle C. For brevity, a right-angled triangle is generally termed a **right triangle.**

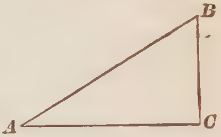


FIG. 16

30. Base and Altitude.—That side of a triangle on which it is supposed to stand is known as the **base;** any side may be considered as the base. In Figs. 16, 17, and 18, $A\,C$ is shown as the base.

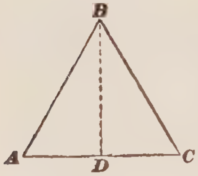


FIG. 17

The **altitude,** or **height,** of any triangle is represented by a line drawn from the vertex of the angle opposite the base perpendicular to the base or to an extension of the base. Thus, in Figs. 17 and 18, $B\,D$ is the altitude of the triangle $A\,B\,C$. In Fig. 18 the perpendicular falls outside the triangle, and the base $A\,C$ is shown extended to meet the line $B\,D$.

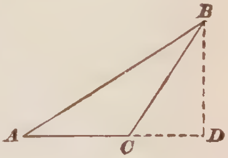


FIG. 18

31. Similar and Equal Triangles.—Two triangles are **similar** when the *angles* of one are equal to the angles of the other. If in the triangle *a b c*, Fig. 19, a line *d e* is drawn parallel to the side *b c*, the triangle *a d e* is similar to the triangle *a b c*, as their angles are equal, each to each.

Two triangles are **equal** when the *sides* of one are equal to the *sides* of the other. It is seen that triangles may be similar without being equal.

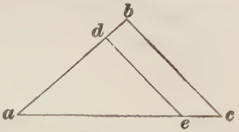


Fig. 19

QUADRILATERALS

32. Parallelograms.—A **parallelogram** is a quadrilateral with opposite sides parallel and opposite angles equal. There are four kinds of parallelograms: the *rectangle*, the *square*, the *rhomboid*, and the *rhombus.*

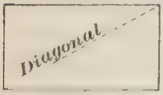


Fig. 20

Fig. 21

33. A **rectangle,** Fig. 20, is a parallelogram with sides at right angles to one another. If the sides are equal, as in Fig. 21, the rectangle is known as a **square.**

34. A **rhomboid,** Fig. 22, is a parallelogram in which there are no right angles and in which the length of one pair of opposite sides differs from that of the other pair.

Fig. 22

A **rhombus,** Fig. 23, is a parallelogram without any right angles, and with sides of equal lengths.

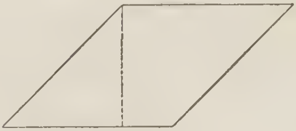


Fig. 23

35. Trapezoid and Trapezium.—A **trapezoid,** Fig. 24, is a quadrilateral which has only two of its sides parallel.

A **trapezium,** Fig. 25, is a quadrilateral without any parallel sides.

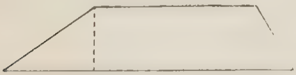


Fig. 24

36. Altitudes and Diagonals of Quadrilaterals. Any side of a quadrilateral may be considered as its **base,** but in a trapezoid one of the two parallel sides is usually considered as its base. The **altitude** of a parallelogram or of a trapezoid is the perpendicular distance between the base and the opposite side, as indicated by the dotted line in Figs. 22, 23, and 24.

37. A **diagonal** is a straight line joining two opposite corners of a quadrilateral, as indicated by the dotted line in Fig. 20. Each quadrilateral has two diagonals, and either diagonal divides the quadrilateral into two triangles. In a parallelogram, these triangles are equal; for example, either diagonal of the rectangle, Fig. 20, divides it into two equal triangles. It is important to remember that the two diagonals of any parallelogram **bisect** each other; that is, they divide each other into two equal parts.

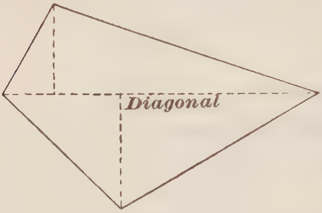


FIG. 25

REGULAR AND IRREGULAR POLYGONS

38. Regular Polygons. — Excluding, according to Art. **24,** any polygons with less than five sides, a **regular polygon** may be defined as one with five or more sides of equal lengths and with equal angles. Fig. 26 shows some common regular polygons with more than four sides. At (*a*) is shown the *pentagon,* or five-sided figure; at (*b*) the *hexagon,* or six-sided figure; at (*c*) the *heptagon,* or seven-sided figure; and at (*d*) the *octagon,* or eight-sided figure.

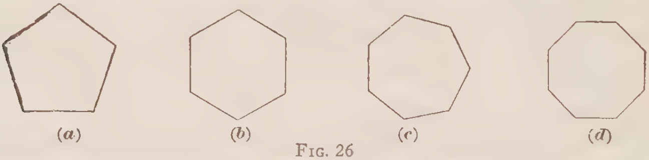


FIG. 26

It is true that equilateral triangles and quadrilaterals are regular polygons of three and four sides, respectively, but they are not commonly called polygons.

39. Constructing Regular Polygons.—If perpendiculars are erected to all the sides of a regular polygon at their middle points, these perpendiculars will meet in a common point, which is called the **center** of the polygon. In the hexagon, Fig. 27, the perpendiculars *P O, Q O,* etc. meet at the center *O* and are of equal lengths. Lines drawn from the vertexes to the center *O,* as *B O* and *C O,* are also equal. It follows that a circle with *O* as center and *O A* as radius will pass through each of the vertexes *A, B, C, D, E,* and *F*; and a circle with *O* as center and *O P* as radius will pass through each of the points *P, Q, R, S, T,* and *U.* A polygon is said to be **inscribed** in a circle when the vertexes of the polygon lie on the circumference of the circle. A polygon is **circumscribed** about a circle when the circumference of the circle touches the center of each side, as at points *P, Q, R,* etc., Fig. 27.

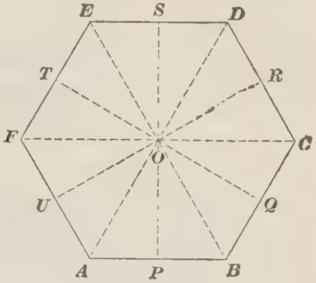


FIG. 27

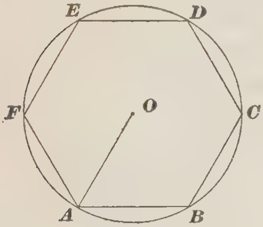


FIG. 28

40. A regular polygon may, therefore, be constructed by inscribing it in a circle. For example, to construct a hexagon, a circle is described from a center *O,* Fig. 28, with a radius *A O* corresponding to the line *A O,* Fig. 27, and divided into six equal parts. The points of division are connected by the straight lines *A B, B C,* etc., thus obtaining the hexagon *A B C D E F.* In a hexagon the sides are equal to the radius of the surrounding circle; hence the length *A O,* Fig. 28, is used for setting out the points of division on the circle.

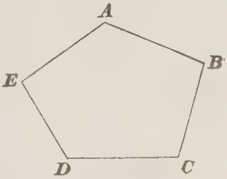


FIG. 29

41. Irregular Polygon.—A polygon in which neither the sides nor the angles are equal, as Fig. 29, is known as an **irregular polygon.**

CIRCLE

42. Arc, Chord, and Radius.—A circle, Fig. 30, is a plane figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

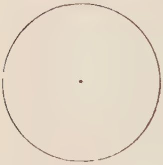


FIG. 30

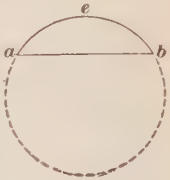


FIG. 31

43. Any part of the circumference, as *a e b,* Fig. 31, is called an **arc.**

44. A straight line joining any two points in the circumference is a **chord.** Or, a chord is a straight line joining the extremities of an arc. Thus, in Fig. 31, *a b* is the chord of the arc *a e b.* Every chord divides the circumference into two arcs, and the chord is said to *subtend,* or to be opposite, either arc. If a chord passes through the center, as *A B,* Fig. 32, it is called **a diameter.**

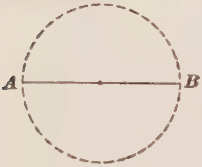


FIG. 32

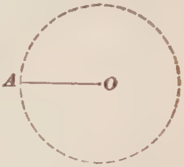


FIG. 33

45. A straight line joining the center with any point in the circumference, as *O A,* Fig. 33, is a **radius** (plural, *radii*) ; a radius is one-half of a diameter.

46. Segment and Semicircle.—A **segment** of a circle is that portion of its area that is included between an arc and its chord. Each chord divides a circle into two segments; thus, in Fig. 31, one segment is between the chord *a b* and the arc *e* in full lines. The other segment is between the chord *a b* and the arc shown in dotted lines.

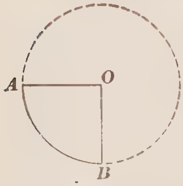


FIG. 34

If the chord of a segment is a diameter, each segment is a **semicircle** and each arc a **semi-circumference.**

47. Sector and Quadrant.—A **sector** of a circle is that portion included between an arc and two radii drawn to the ends of the arc. Thus, in Fig. 34, the portion included between the arc $A B$ and the radii $O A$ and $O B$ is a sector of the circle.

48. If two diameters are drawn at right angles to each other, as $A B$, $C D$ in Fig. 35, the circumference, as well as the area, of the circle is divided into four equal parts. Each of the four equal arcs and sectors is called a **quadrant.** Thus, the arcs $A D$, $D B$, $B C$, and $C A$, as well as the sectors $A O D$, $D O B$, $B O C$, and $C O A$, are all known as quadrants.

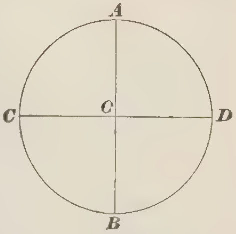


FIG. 35

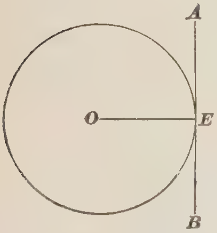


FIG. 36

49. Tangents.—A straight line that meets a circle at one point only is a **tangent** to the circle; thus, in Fig. 36, $A B$ is a tangent touching the circle at the point E. The latter point is called the **point of contact,** or the **point of tangency.**

A tangent to a circle is at right angles to the radius drawn to the point of contact. Thus, if O is the center of the circle, the tangent $A B$ is at right angles to the radius $O E$.

50. Tangent Circles. One circle is said to be **tangent** to another circle if they touch each other at one point only. This point is called the **point of tangency,** or the **point of contact.** In Fig. 37 the points of contact of the circle A and the circles B and C are at a and b, respectively. A straight line passing through the centers of the tangent circles also passes through the point of contact; thus, the point a, at which the circles A and B are tangent, lies on an extension of the line connecting the centers c and c_1. Similarly,

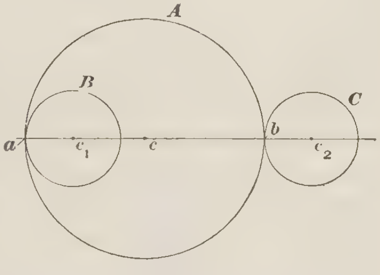


FIG. 37

the point *b*, at which the circles *A* and *C* are tangent, lies on the line $c\ c_2$.

51. Concentric and Eccentric Circles.—When two or more circles are described from the same center, as in Fig. 38, they are called **concentric circles.**

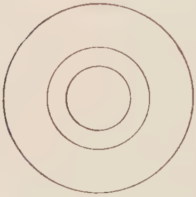


FIG. 38

A circle described inside another circle, but from a different center, is said to be **eccentric** to the larger circle, and the two circles are termed **eccentric circles.** In Fig. 39 the circle *B* is eccentric to the circle *A*, as the centers *b* and *a* are not at the same point.

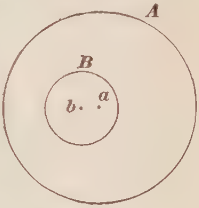


FIG. 39

52. Dividing Circumference of Circle Into Equal Parts.—The usual way of dividing the circumference of a circle into a number of equal parts is to step a pair of dividers around the circle, adjusting the distance between its points until a distance is found that is used the required number of times in going around the circumference. For instance, if the circle, Fig. 40, is to be divided into five equal parts, the dividers are adjusted until they will step off five equal chords *A B*, *B C*, *C D*, *D E*, and *E A*.

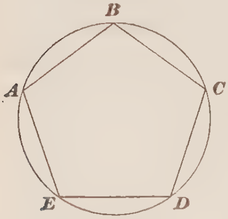


FIG. 40

53. To save the time required to find the exact length of the parts into which a circle is to be divided, Table I is provided, which gives the length of the chord, per inch of diameter, for all divisions from 3 to 100, inclusive. The length of the chord, or the distance between the points of the dividers, is found by multiplying the diameter of the circle by the multiplier corresponding to the number of divisions desired.

The application of the table is shown by means of the following examples:

EXAMPLE 1.—A circle 14 inches in diameter is to have 24 holes spaced equally around it. In setting out the centers of these holes by dividers, what must be the distance between the points of the dividers?

SOLUTION.—According to Table I, the multiplier corresponding to 24 divisions is .13053; but as this applies to a circle 1 in. in diameter, it must be multiplied by 14 for a circle of 14 in. diameter. Therefore, the dividers must be set a distance apart equal to

$$14 \times .13053 = 1.82742, \text{ or } 1.83 \text{ in., nearly.} \quad \text{Ans.}$$

TABLE I

LENGTHS OF CHORDS

Number of Divisions	Multiplier	Number of Divisions	Multiplier	Number of Divisions	Multiplier	Number of Divisions	Multiplier
3	.86603	28	.11197	53	.05924	77	.04079
4	.70711	29	.10812	54	.05815	78	.04027
5	.58779	30	.10453	55	.05709	79	.03976
6	.50000	31	.10117	56	.05607	80	.03926
7	.43388	32	.09802	57	.05509	81	.03878
8	.38268	33	.09506	58	.05414	82	.03830
9	.34202	34	.09227	59	.05322	83	.03784
10	.30902	35	.08964	60	.05234	84	.03739
11	.28173	36	.08716	61	.05148	85	.03695
12	.25882	37	.08480	62	.05065	86	.03652
13	.23932	38	.08258	63	.04985	87	.03610
14	.22252	39	.08047	64	.04907	88	.03569
15	.20791	40	.07846	65	.04831	89	.03529
16	.19509	41	.07655	66	.04758	90	.03490
17	.18375	42	.07473	67	.04687	91	.03452
18	.17365	43	.07300	68	.04618	92	.03414
19	.16460	44	.07134	69	.04552	93	.03377
20	.15643	45	.06976	70	.04487	94	.03342
21	.14904	46	.06824	71	.04423	95	.03306
22	.14232	47	.06679	72	.04362	96	.03272
23	.13617	48	.06540	73	.04302	97	.03238
24	.13053	49	.06407	74	.04244	98	.03205
25	.12533	50	.06279	75	.04188	99	.03173
26	.12054	51	.06156	76	.04133	100	.03141
27	.11609	52	.06038				

Example 2.—A pulley 26 inches in diameter has 5 spokes. What must be the distance between the ends of their center lines, situated along the pulley rim?

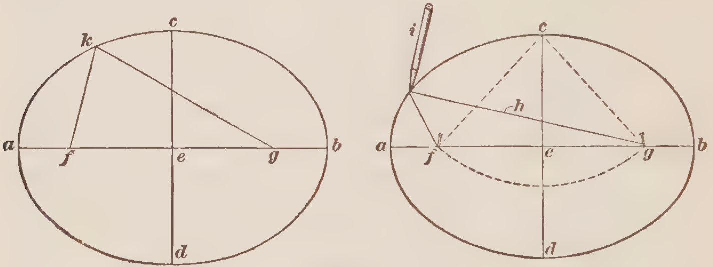
Solution.—According to the table, the multiplier is .58779. Therefore, the dividers must be set to a distance of

$$26 \times .58779 = 15.28 \text{ in.} \quad \text{Ans.}$$

ELLIPSE

54. Axes and Foci of Ellipse.—An **ellipse** is a plane figure of oblong outline, as shown in Fig. 41 (*a*). The line *a b* is called the **major axis** or **long diameter** of the ellipse and the line *c d* is the **minor axis** or **short diameter.**

55. Constructing an Ellipse.—The method of constructing an ellipse may be explained by reference to Fig. 41 (*b*). It is assumed that the lengths of the long and short diameters *a b* and *c d* are given. The construction is begun by drawing these diameters at right angles to each other, so as to intersect at the middle point *e* of each diameter. Then, with *c* as a center and a radius equal to *a e,* an arc is described which cuts the major axis at *f* and *g*. Each of these points *f* and *g* is known as a **focus** (plural, *foci*). As will be seen by the method of construction, the distance from each focus to either extremity of the minor axis is equal to one-half of the major axis.



(*a*) Fig. 41 (*b*)

Pins are inserted at the foci, as shown; a pencil *i* is placed at one end of the major axis, the point *a* in this case, and a cord is looped around the pencil point and fastened tautly to the pins

f and g at the foci. The pencil is now moved from the point a toward the position c and beyond to b, keeping the cord stretched taut, thus tracing one-half of an ellipse that will pass through the points a, c, and b. Trace the other half similarly.

It will be noted that the length of cord between the foci equals the length of the major axis, because one position of the pencil is at c, Fig. 41 (b), and $fc+cg=ab$. Therefore, the sum of the distances from any point in an ellipse to the foci equals the length of the major axis; for example, in Fig. 41 (a), $fk+kg=ab$.

MEASUREMENT OF LINES AND AREAS

LENGTHS OF LINES

56. Introduction.—As stated in Art. **1**, calculations dealing with lengths of lines, areas of surfaces, and volumes of solids belong to that branch of mathematics called **mensuration.** The lines whose lengths will be considered in this Section are those that constitute the outlines of triangles, circles, portions of circles, and ellipses.

57. Perimeter of Triangles.—If the lengths of two sides of a right triangle are given it is always possible to find the length of the third side and, thus, the perimeter, or the combined length of the three sides. The following rules, which were given in a preceding Section, apply:

Rule I.—*To find the hypotenuse of a right triangle, extract the square root of the sum of the squares of the other two sides.*

Rule II.—*To find one of the sides about the right angle, subtract from the square of the hypotenuse the square of the other side, and extract the square root of the remainder.*

Example 1.—Find the perimeter of a right triangle, if the sides about the right angle are 9.5 and 7 feet, respectively.

Solution.—By rule I, the hypotenuse is equal to

$$\sqrt{9.5^2+7^2}=\sqrt{90.25+49}=11.8 \text{ ft.}$$

The perimeter is:

$$9.5+7+11.8=28.3 \text{ ft. Ans.}$$

Example 2.—A piece of land $abcd$, Fig. 42, has the form of a trapezium, and has right angles at b and d. A fence is to be built around the land and it is required to know which pair of sides is the longer, $ab+bc$ or $cd+da$. The diagonal ac is 65 feet, bc is 51 feet, and ad is 28.5 feet.

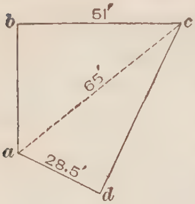


Fig. 42

Solution.—By rule II, $ab=\sqrt{65^2-51^2}=40.3$ ft. and $cd=\sqrt{65^2-28.5^2}=58.4$ ft. It follows that $ab+bc=40.3+51=91.3$ ft. and that $ad+dc=28.5+58.4=86.9$ ft. Therefore, there is a difference of $91.3-86.9=4.4$ ft. between the two pairs of sides, the pair $ab+bc$ being the longer of the two. Ans.

58. Circumference, Diameter, and Radius of Circles.—To find the circumference, the diameter, or the radius of a circle, the following rules are used:

Rule I.—*The circumference of a circle equals the diameter multiplied by 3.1416.*

Rule II.—*The diameter of a circle equals the circumference divided by 3.1416.*

Rule III.—*The radius of a circle equals the circumference divided by 2×3.1416.*

Example 1.—What is the circumference of a circle whose diameter is 15 inches?

Solution.—By rule I, the circumference is $15\times3.1416=47.12$ in. Ans.

Example 2.—(a) What is the diameter of a circle whose circumference is 65.9736 inches? (b) What is the radius?

Solution.—(a) By rule II, the diameter is

$$\frac{65.9736}{3.1416}=21 \text{ in. Ans.}$$

(b) By rule III, the radius is

$$\frac{65.9736}{2\times3.1416}=10\tfrac{1}{2} \text{ in. Ans.}$$

Or, the radius is equal to the diameter divided by 2. Thus, $\frac{21}{2}=10\frac{1}{2}$ in.

59. Ratio of Circumference to Diameter.—The number 3.1416 used in the preceding rules is the ratio of the circumference of a circle to its diameter; it is represented very often by the Greek letter π (pronounced "pi"). It is an unending decimal and its value has been calculated to over 700 decimal places, but the value here given is the one most generally used, four decimal places being sufficient for all practical purposes. The values $\frac{1}{4}\pi$, or .7854, and $\frac{1}{6}\pi$, or .5236, are frequently used. In case the calculation does not need to be very accurate, the value of π may be taken as $\frac{22}{7}$, which is equal to 3.1429, a fairly close approximation.

60. Length of Arc of Circle.—The length of an arc of a circle may be found by the following rule:

Rule.—*The length of an arc of a circle equals the circumference of the circle of which the arc is a part, multiplied by the number of degrees in the arc, and the product divided by 360.*

EXAMPLE.—What is the length of an arc of 24°, the radius of the circle being 18 inches?

SOLUTION.—The diameter of the circle is $2\times18=36$ in., and the circumference is, according to rule I, Art. **58**, 36×3.1416. By the preceding rule, the length of the arc is

$$\frac{36\times3.1416\times24}{360}=7.54 \text{ in. Ans.}$$

61. Approximate Method for Finding Length of Arc.—The following method, if applied to arcs less than 120°, is correct to three figures. It may be used for arcs up to 180°, but when the arc exceeds 120° the results are less accurate.

If the length of the arc ACB, Fig. 43, is to be found, the middle point D of the chord AB is located and DC is drawn at right angles to AB. On drawing AC, this line is the chord of half the arc. After the lengths of these chords are measured, the length of the arc is found by the following rule:

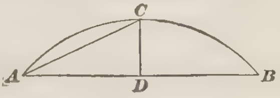


FIG. 43

Rule.—*From eight times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by 3. The quotient is the length of the arc, approximately.*

Referring to Fig. 43, this rule may also be expressed in the following form:

$$\text{Length of arc } AB=\frac{8\times AC-AB}{3}$$

Example.—Find the length of the arc *A C B*, Fig. 44, if the length of the chord *A B* is 72 inches and *D C* is 8 inches.

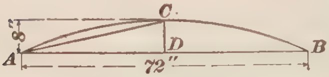


Fig. 44

Solution.—Before the rule can be applied it is necessary to find the length of the chord *A C* of half the arc. It is known that $AD = \frac{1}{2}$ of $AB = \frac{1}{2} \times 72 = 36$ in.

From the rules for right triangles, given in Art. **57**, it is known that the hypotenuse $AC = \sqrt{\overline{AD}^2 + \overline{DC}^2} = \sqrt{36^2+8^2} = 36.88$ in.

Hence, by the rule,

$$\text{length of arc } AB = \frac{8 \times AC - AB}{3} = \frac{8 \times 36.88 - 72}{3} = 74.35 \text{ in. Ans.}$$

62. Graphic Method for Finding Length of Arc. If the circular arc does not exceed one-sixth of the circumference, its length may be found accurately enough for practical purposes by means of the construction shown in Fig. 45.

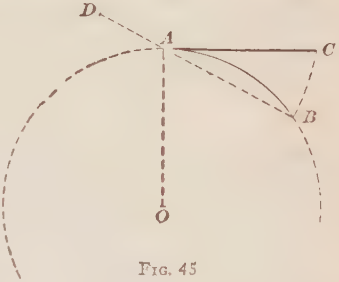


Fig. 45

For example, it is required to find the length of the arc *A B* of a circle described with the radius *O A* from the center *O*. At *A* a line *A C* is drawn at right angles to the radius *O A*; then, the chord *A B* is drawn and extended beyond the point *A* to the point *D*, making *A D* half the length of the chord *A B*. With *D* as a center, and with a radius equal to *D B*, the arc *B C* is described, cutting *A C* at *C*. The length of *A C* will then be very nearly equal to the length of the arc *A B*.

63. Circumference of Ellipse.—The circumference, or length of the curved outline, of an ellipse may be found, approximately, by means of the following rule:

Rule.—*The circumference of an ellipse is approximately equal to the product obtained by multiplying 3.1416 by the square root of half the sum of the squares of the two diameters.*

Or, referring to Fig. 41 (*a*), the rule is stated as follows:

$$\text{Circumference of ellipse} = 3.1416\sqrt{\frac{\overline{ab}^2 + \overline{cd}^2}{2}}$$

EXAMPLE.—Find the circumference of an ellipse whose diameters are 14 inches and 10 inches, respectively.

SOLUTION.—When the known values are substituted in the formal statement of the rule, the circumference is equal to $3.1416\sqrt{\frac{14^2+10^2}{2}}$ $=3.1416\sqrt{\frac{196+100}{2}}=3.1416\times 12.17=38.233$ in. Therefore, correct to three figures, the circumference is 38.2 in. Ans.

AREAS OF PLANE SURFACES

TRIANGLES, QUADRILATERALS, POLYGONS, CIRCLES, AND ELLIPSES

64. Area of Triangle.—The area of a triangle may be found by the following rule, if the length of the base and the altitude are known:

Rule.—*The area of any triangle equals one-half the product of the base and the altitude.*

If the triangle is a right triangle, one of the short sides may be taken as the base, and the other short side as the altitude; hence, *the area of a right triangle is equal to one-half the product of the two short sides.*

EXAMPLE.—What is the area of a triangle whose base is 18 inches and whose altitude is $7\frac{3}{4}$ inches?

SOLUTION.—Applying the rule, the area is

$$\tfrac{1}{2}\times(18\times 7\tfrac{3}{4})=69\tfrac{3}{4} \text{ sq. in.} \quad \text{Ans.}$$

65. The area of any triangle may be found, when the length of each side is known, by means of the following rule:

Rule.—*From half the sum of the three sides, subtract each side separately; find the product of these remainders and of half the sum of the sides. The square root of the product is equal to the area of the triangle.*

If the various values in this rule are expressed by means of symbols it may be given in a more condensed form. First, the sum of the three sides *a*, *b*, and *c*, Fig. 46, is found and divided by 2, as $\frac{a+b+c}{2}$ This quotient is indicated by the letter *s*; that is, $s=\frac{a+b+c}{2}$. The difference is then found between *s* and each of the sides, as $s-a$, $s-b$, and $s-c$. These differences are multiplied together and by *s*, and the square root of the product is extracted.

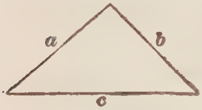


FIG. 46

When these successive calculations are expressed collectively, the whole will appear as follows:

$$\text{Area of triangle} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

EXAMPLE.—What is the area of the triangle, Fig. 46, if the sides, *a*, *b*, and *c*, are 18, 20, and 28 feet, respectively?

SOLUTION.—First, it is necessary to find the value of *s*, or $\frac{a+b+c}{2}$ $=\frac{18+20+28}{2}=33$. Then, the differences between *s* and each of the sides are found, as,

$$s-a=33-18=15$$
$$s-b=33-20=13$$
$$s-c=33-28=5$$

Then,

$$\text{area}=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{33\times15\times13\times5}=179.37 \text{ sq. ft.} \quad \text{Ans.}$$

66. Area of Quadrilateral.—To find the area of a parallelogram the following rule may be used:

Rule.—*The area of any parallelogram equals the product of the base and the altitude.*

EXAMPLE.—What is the area of a parallelogram whose base is 12 inches and whose altitude is 7½ inches?

SOLUTION.—By the rule, the area is 12×7½=90 sq. in. Ans.

If the area and one dimension are given, the other dimension may be found by dividing the area by the known dimension. If the parallelogram is a square, and its area is given, the length of a side is found by extracting the square root of the area.

67. To find the area of a trapezoid the following rule may be used:

Rule.—*The area of a trapezoid equals one-half the sum of the parallel sides multiplied by the altitude.*

Example.—What is the area of a trapezoid whose parallel sides are 9 inches and 15 inches, and whose altitude is 6 inches?

Solution.—The sum of the parallel sides is $9+15=24$ in., and half of this is 12 in. Then,

$$12\times6=72 \text{ sq. in.} \quad \text{Ans.}$$

68. Area of Regular Polygon.—The area of a regular polygon may be found by the following rule:

Rule.—*To find the area of a regular polygon, square the length of a side and multiply this by the proper multiplier taken from Table II.*

Example.—A regular octagon has sides 8 inches long. What is the area?

Solution.—The multiplier for an octagon, taken from Table II, is 4.8284. By the rule, the area is

$$8^2\times4.8284=64\times4.8284=309.02 \text{ sq. in.} \quad \text{Ans.}$$

TABLE II

MULTIPLIERS FOR FINDING AREAS OF REGULAR POLYGONS

Name	Number of Sides	Multiplier	Name	Number of Sides	Multiplier
Equilateral triangle....	3	0.4330	Octagon...	8	4.8284
Square	4	1.0000	Nonagon ..	9	6.1818
Pentagon....	5	1.7205	Decagon...	10	7.6942
Hexagon.....	6	2.5981	Undecagon	11	9.3656
Heptagon....	7	3.6339	Dodecagon	12	11.1960

69. Area of Irregular Polygon.—The foregoing rules apply for finding areas of such figures as triangles, squares, rectangles, and regular polygons. If the figure or surface is of irregular shape, it can generally be divided by means of diagonals into simpler forms, such as squares, triangles, etc. The areas of these parts may then be calculated by the rules given, and the sum of the various areas will be the total area of the figure.

Example.—It is required to find the area of the irregular polygon $ABCDEF$, Fig. 47, the dimensions being as indicated and the angles FCD and CDE being right angles.

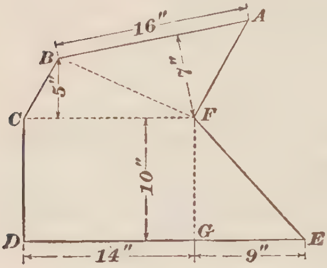


Fig. 47

Solution.—The diagonals BF and CF are drawn and also the line FG perpendicular to DE, thus dividing the polygon into the triangles ABF, BCF, and FGE, and the rectangle $FCDG$. With the dimensions indicated in Fig. 47 on the lines representing altitudes, namely, 7, 5, and 10, and on the sides AB, DG, and GE, namely, 16, 14, and 9, the areas are as follows:

$$\text{area } ABF = \frac{16\times 7}{2} = 56 \text{ sq. in.}$$

$$\text{area } BCF = \frac{14\times 5}{2} = 35 \text{ sq. in.}$$

$$\text{area } FCDG = 14\times 10 = 140 \text{ sq. in.}$$

$$\text{area } FGE = \frac{9\times 10}{2} = 45 \text{ sq. in.}$$

Total area $= 56+35+140+45 = 276$ sq. in. Ans.

70. Area of Circle.—The area of a circle may be found by the following rule:

Rule.—*The area of a circle is equal to .7854 times the square of the diameter, or 3.1416 times the square of the radius.*

Example.—What is the area of a circle whose diameter is 15 inches?

Solution.—By the rule, the area is

$$.7854\times 15^2 = .7854\times 225 = 176.72 \text{ sq. in.} \quad \text{Ans.}$$

In practice it is found very convenient to use tables giving the diameters, circumferences, and areas of circles, as they make it unnecessary to perform the calculation just described, and thus save much time.

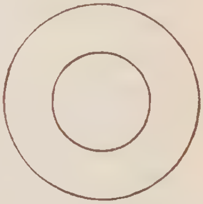


Fig. 48

71. Area of Flat Circular Ring.—The area of a flat circular ring like that in Fig. 48 may be found by the following rule:

Rule.—*The area of a flat circular ring is equal to the area of the outer circle minus the area of the inner circle.*

EXAMPLE.—The diameters of the outer and inner circles of a flat ring are 6½ inches and 4 inches, respectively. What is the area of the ring?

SOLUTION.—The area of the outer circle is $.7854\times(6\frac{1}{2})^2=33.183$ sq. in. and of the inner circle is $.7854\times4^2=12.566$ sq. in. Then, the area of the ring is $33.183-12.566=20.617$ sq. in. Ans.

72. Area of Sector.—The area of a sector may be found by the following rule:

Rule.—*Divide the number of degrees in the arc of the sector by 360, and multiply the result by the area of the circle of which the sector is a part. The product is the area of the sector.*

EXAMPLE.—The angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75° and the diameter of the circle is 12 inches. What is the area of the sector?

SOLUTION.—The area of a circle 12 in. in diameter is $.7854\times12^2=113.1$ sq. in., nearly. By the rule, the area of the sector is

$$\tfrac{75}{360}\times113.1=23.56 \text{ sq. in.} \quad \text{Ans.}$$

73. If the length of the arc and the radius of a sector are given, the following rule may be used to find the area:

Rule.—*The area of a sector is equal to one-half the product of the radius and the length of the arc.*

EXAMPLE.—If the radius of an arc is 5 inches, and the length of arc is 4 inches, what is the area of the sector?

SOLUTION.—By the rule, the area is

$$\tfrac{1}{2}\times(5\times4)=10 \text{ sq. in.} \quad \text{Ans.}$$

74. Area of Segment.—To find the area of the segment *A B C*, Fig. 49, lines are drawn from the center *D* of the circle to the points *A* and *B*. Then, the area of the segment is equal to the area of the sector *A D B C* minus the area of the triangle *A B D*. Hence, the following rule is used to find the area of a segment of a circle:

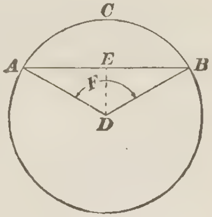


FIG. 49

Rule.—*Draw radii from the center of the circle to the ends of the arc of the segment; find the area of the sector thus* *formed, subtract from this the area of the triangle formed by the radii and the chord of the arc of the segment; the result is the area of the segment.*

In problems where the area of a segment is to be found the angle and the diameter or radius of the circle are usually given. The area of the sector can be found by the rule in Art. **72.** A drawing is then made as in Fig. 49, the known angle *F* being laid off by means of a protractor. The radii are then drawn to scale, as at *D A* and *D B*, the chord is drawn as at *A B*, and the perpendicular distance from the vertex of the angle to the chord is measured, as at *D E*. This distance is the altitude of the triangle *A B D*, and the area of this triangle is one-half the product of the base, or the chord *A B*, and the altitude *D E*.

Example.—Find the area of the segment *A B C*, Fig. 49, if the chord *A B* is 8.66 inches, *E D* is 2½ inches, the radius *A D* is 5 inches, and the angle *F* is 120°. (Arc *A C B* is 120°. See Art. **31,** Serial 1978.)

Solution.—By the rule of Art. **72,** the area of the sector is

$$\tfrac{120}{360}\times(3.1416\times 5^2)=\tfrac{1}{3}\times 78.54=26.18 \text{ sq. in.}$$

The area of the triangle is $\frac{1}{2}\times(8.66\times 2\frac{1}{2})=10.83$ sq. in. Therefore, by the foregoing rule, the area of the segment is

$$26.18-10.83=15.35 \text{ sq. in. Ans.}$$

75. Approximate Method for Finding Area of a Segment.—If the conditions are such that the position of a chord, as *A C*, Fig. 50, is known, but not its length, the following rule may be employed for finding the area of a segment. This rule is not exact, but gives results sufficiently correct for practical purposes.

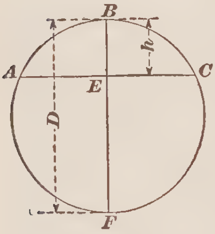


Fig. 50

Rule.—*Divide the diameter by the height of the segment; subtract .608 from the quotient and extract the square root of the remainder. This root is multiplied by 4 and by the square of the height of the segment and then divided by 3. The quotient gives the area, very nearly.*

Example.—In the circle shown in Fig. 50, what is the area of the segment *A B C E A*, if the diameter *D* is 54 inches and the height *h* of the segment is 20 inches?

SOLUTION.—The application of the rule is shown by the following operations, which are performed in the order given in the rule:

$$54 \div 20 = 2.7$$
$$2.7 - .608 = 2.092$$
$$\sqrt{2.092} = 1.446$$
$$4 \times 1.446 = 5.784$$
$$5.784 \times 20^2 = 5.784 \times 400 = 2{,}313.6$$
$$2{,}313.6 \div 3 = 771.2 \text{ sq. in.} \quad \text{Ans.}$$

76. The method explained in Art. **75** requires that the diameter be given. Should, however, the chord be given, instead of the diameter, the length of the latter may be found by a method based on the principle that if from any point on the circumference of a circle, as from *A*, Fig. 51, a line *A E* is drawn perpendicular to a diameter, as *B F*, it will divide the diameter into two parts, one of which will be in the same ratio to the perpendicular as the perpendicular is to the other part. That is, the perpendicular will be what is called a *mean proportional* between the two parts of the diameter. For example, in Fig. 51 the perpendicular *A E* divides the diameter into the two parts *B E* and *E F* such that the relation between *A E* and these parts may be stated by the proportion,

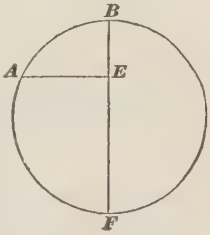


FIG. 51

$$BE : AE = AE : EF$$

The perpendicular *A E*, which is one-half of a chord, is the mean proportional between the two parts of the diameter. The application of this principle will now be shown by means of Fig. 50 and the following example:

EXAMPLE.—If *A C*, Fig. 50, is 30 feet long and the height *B E* is 8 feet, what is the diameter of the circle, the chord *A C* being at right angles to the diameter *B F*?

SOLUTION.—*A E* is one-half of the chord or 30÷2=15 ft. According to the preceding explanations the following proportion applies: $BE : AE = AE : EF$; or, $8 : 15 = 15 : EF$.

Therefore, $EF = \frac{15 \times 15}{8} = \frac{225}{8} = 28\frac{1}{8}$ ft. The diameter is equal to $EF + EB = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ ft. Ans.

77. Area of Ellipse.—The area of an ellipse may be found by the following rule:

Rule.—*The area of an ellipse is equal to .7854 times the product of its long and short diameters.*

Example.—The link shown in Fig. 52 has a central part that is elliptical in *cross-section*. That is, if the link is cut in two along the line shown under the section *a*, the end surfaces will each be shaped as shown at *a*. This cross-section, *a*, is an ellipse 4 inches long and 2½ inches wide. What is the area of this cross-section?

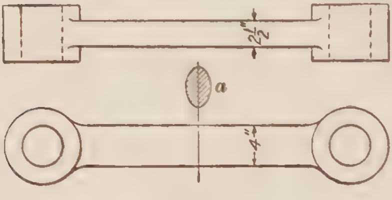


Fig. 52

Solution.—The long diameter is 4 in. and the short diameter is 2½ in. and, by the rule, the area is

$$.7854 \times 4 \times 2\tfrac{1}{2} = 7.854 \text{ sq. in. Ans.}$$

78. Areas of Irregular Curvilinear Figures.—A method was shown in Art. **69** by which the area of an irregular polygon could be calculated. But, if the outline of an irregular figure is made up of curved lines, instead of being composed of straight lines, there is no general rule by which the exact area may be found. In such a case it is necessary to find the area by an approximate method. There are several methods of this kind, all of which will give fairly accurate results. The aim of the following method is to find the dimensions of a rectangle that has an area equivalent to that of the figure.

If the area of an irregular figure, similar to that shown in Fig. 53, is to be found, it is necessary to draw a straight line,

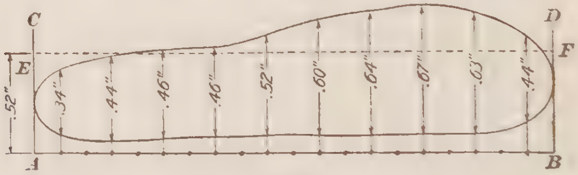


Fig. 53

as *A B*, below the figure. Then, the perpendiculars *C A* and *D B* are drawn tangent to the curved ends of the figure. The distance *A B*, which is equal to the length of the figure, is now divided into a number of equal parts, in this case 20. This can be done by using a pair of dividers, setting their points a certain distance apart, and stepping off from *A* to *B*. The approximate distance between the points of the dividers may be found by measuring the length of *A B* and dividing it by 20. For instance, if the figure is 2.7 inches long, the length of each division is $2.7 \div 20 = .135$ inch. The distance between the points should be adjusted at each trial, until a setting is found that will give exactly 20 equal spaces on *A B*. Each division should be marked with a dot made with a sharp pencil point, as shown.

79. Beginning at the dot next to one end of the figure, Fig. 53, a perpendicular is drawn through it, so as to cross the figure. Other perpendiculars are drawn from each alternate point, that is, skipping every second point. There will then be 10 lines drawn across the figure from top to bottom, as shown. The lengths of those portions of the 10 lines that are included between the arrow heads are now measured carefully and the lengths are marked in decimals of an inch. When all the lengths are measured, they are added and their sum divided by the number of lines, in this case 10. The result is the *average* length of the lines, or the average height of the figure. In this example the sum of the lengths is $.34 + .44 + .46 + .46 + .52 + .60 + .64 + .67 + .63 + .44 = 5.2$ inches, and $5.2 \div 10 = .52$ inch; that is, the average height of the figure is .52 inch. The length of the figure is equal to the length of *A B*, which measures 2.7 inches. The area is equal to the length times the average height, or $2.7 \times .52 = 1.4$ square inches.

It is seen from Fig. 53 that the area of the figure is assumed to be equal to the area of a rectangle *A E F B* whose length *A B* is equal to the length of the figure and whose height *A E* is equal to the average height of the figure.

NOTE.—It is to be observed that the line *A B* is divided into 20 parts while only 10 perpendiculars are drawn. The purpose of this procedure is to make it possible to locate the first and the last perpendicular at a distance from the respective ends equal to one-half the length of a division. Since the outline of the figure shows the greatest curvature at the ends, greater accuracy in the calculation is secured by placing these perpendiculars nearer the ends.

80. Approximate Area of Any Polygon.—Although the method just described for finding areas is most commonly applied to irregular figures, it can also be used for finding the approximate area of any regular figure, in case the formula for finding such area cannot be remembered. Also, it is not absolutely necessary to divide the base line of the figure into 20 equal parts. Any number of equal parts may be used; but the greater the number, the greater will be the accuracy of the result. Not less than 20 divisions should be used, so that 10 lines of measurement will be obtained, if fairly accurate results are expected.

It is not necessary to measure each of the 10 lines separately, as was done in the preceding article. A narrow strip of paper with one straight edge may be laid across the figure and the lengths of the 10 lines may be marked off on it, end to end, beginning at the left-hand end of the strip. The distance between this end and the last mark made is then measured, which distance is equivalent to the sum of the ten lengths set off. The required calculation is then performed as explained in Art. **79.**

EXAMPLES FOR PRACTICE

1. What is the area in square feet of a parallelogram whose base is 84 inches, and whose altitude is 3 feet? Ans. 21 sq. ft.

2. One side of a shop is 16 feet long. If the floor space is 240 square feet, what is the length of the other side? Ans. 15 ft.

3. How many square feet in one face of a board 12 feet long, 18 inches wide at one end and 12 inches wide at the other end? Ans. 15 sq. ft.

4. What is the area of a triangle whose base is 10 feet 6 inches long, and whose altitude is 18 feet? Ans. 94.5 sq. ft.

5. A wagon wheel is adjusted into a position so that the center lines of two opposite spokes are horizontal. If there are five equally spaced spokes above them, how many degrees are there in the angle between two adjacent spokes? Ans. 30°

6. What is the length of an arc of 64°, the radius of the arc being 30 inches? Ans. 33.51 in.

7. Find the area of a circle 2 feet 3 inches in diameter. Ans. 3.976 sq. ft.

8. From a circle, 42 inches in diameter, a segment is cut off that is 11 inches high. What is the area of the segment? Ans. 289.04 sq. in.

9. Find the area of a flat circular ring whose outside diameter is 12 inches and whose inside diameter is 6 inches. Ans. 84.82 sq. in.

10. The diameter of a flywheel is 18 feet. What is the length of its circumference to the nearest 16th of an inch? Ans. 56 ft. 6$\frac{9}{16}$ in.

11. A flat roof, 46 feet by 80 feet in size, is covered by tin roofing weighing one-half pound per square foot; what is the total weight of the roofing? Ans. 1,840 lb.

12. How much would it cost to lay a sidewalk a mile long and 8 feet 6 inches wide, at the rate of 20 cents per square foot? How much at the rate of $1.80 per square yard? Ans. $8,976 in each case

VOLUMES OF SOLIDS

PRISM AND CYLINDER

81. Solids.—A **solid,** or body, has three dimensions: length, breadth, and thickness. The boundaries of a solid are surfaces. If these surfaces are planes, they are known as **faces,** and the lines along which they join or intersect are called **edges.**

82. Classification of Prisms and Cylinders.—A **prism** is a solid whose ends, or bases, are equal and parallel polygons and whose sides are parallelograms. Prisms take their names from the form of their ends. Thus, a *triangular prism* is one whose ends are triangles; a *hexagonal prism* is one whose ends are hexagons, etc. A *rectangular prism,* Fig. 54, is a prism whose ends are rectangles. A *cube,* Fig. 55, is a rectangular prism whose faces and ends are squares.

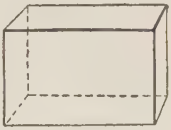


Fig. 54

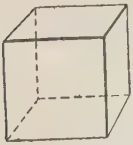


Fig. 55

A **cylinder,** Fig. 56, is a body of uniform diameter whose ends are equal, parallel circles.

The end on which a prism or cylinder is shown to rest is called its **base.**

Fig. 56

83. A **right prism** or a **right cylinder** is one whose center line, or axis, is perpendicular to its base. In this Section all of the solids will be considered as having their center lines perpendicular to their bases.

The **altitude** of a prism or a cylinder is the perpendicular distance between its two ends.

84. Volume of Right Prism or Cylinder.—The volume of a prism similar to that shown in Fig. 54 or in Fig. 55 is equal to the product of its length, width, and thickness. But, as the area of the base in either case is equal to the product of two of these dimensions, as, for instance, length and width, it follows that the volume is found by multiplying the area of the base by the altitude of the prism. The three dimensions must be in the same units. The volume of a cylinder also is equal to the product of the area of its base and the altitude. Hence, for finding the volume of a right prism or of a right cylinder the following rule applies:

Rule.—*The volume of any right prism or cylinder equals the area of the base multiplied by the altitude.*

If the given prism is a cube, the three dimensions are all equal, and the volume is equal to the cube of the length of one edge. If the volume and the area of the base are given, the altitude is equal to the volume divided by the area.

Example 1.—What is the volume of a rectangular prism whose base is 6 inches by 4 inches, and whose altitude is 12 inches?

Solution.—As the base of a rectangular prism is a rectangle, its area is equal to $6 \times 4 = 24$ sq. in. By the rule, the volume is $24 \times 12 = 288$ cu. in. Ans.

Example 2.—What is the volume of a cube whose edge is 9 inches?

Solution.—The volume is equal to the cube of one edge, or $9^3 = 9 \times 9 \times 9 = 729$ cu. in. Ans.

Example 3.—What is the volume of a cylinder whose base is 7 inches in diameter, and whose altitude is 11 inches?

Solution.—From Art. **70** the area of a circle is equal to .7854 times the square of the diameter. Hence, the area of the base is $.7854 \times 7^2 = 38.48$ sq. in. By the preceding rule, the volume of the cylinder is $38.48 \times 11 = 423.28$ cu. in. Ans.

EXAMPLE 4.—A cylindrical tank is to contain 140 cubic feet of water. If the area of the base is 28 square feet, what must be the height of the tank?

SOLUTION.—As it was stated that the altitude of a cylinder is equal to the volume divided by the area, it follows that the height of the tank is 140÷28=5 ft. Ans.

85. Volume of Hollow Right Prism or Cylinder.

If the cylinder or prism is *hollow,* the volume is equal to the actual area of the base multiplied by the altitude.

EXAMPLE.—The outside diameter of a hollow cylinder, Fig. 57, is 14 inches and the diameter of the interior opening is 12 inches. If the altitude of the cylinder is 9 inches, what is its volume?

SOLUTION.—The base may be considered as a flat circular ring, similar to Fig. 48, and its area is found by using the rule in Art. **71.** The area of the outer circle is $.7854\times14^2=153.938$ sq. in., and of the inner circle it is $.7854\times12^2=113.098$ sq. in. Then, the area of the base is $153.938-113.098=40.84$ sq. in. Then by the rule, the volume is $40.84\times9=367.56$ cu. in. Ans.

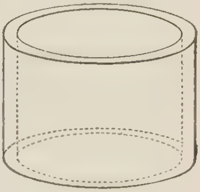


FIG. 57

PYRAMID, CONE, AND SPHERE

86. Pyramid and Cone.—A **pyramid,** Fig. 58, is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex.**

A **cone,** Fig. 59, is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.**

The **altitude** of a pyramid or a cone is the perpendicular distance from the vertex to the base, as shown by the dotted line in Figs. 58 and 59. If this line passes through the center of the base, the solid is called a *right pyramid* or a *right cone.*

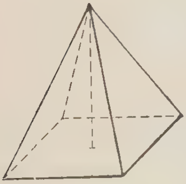


FIG. 58

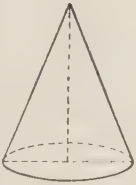


FIG. 59

87. Volume of Right Pyramid or Right Cone.

The volume of a right pyramid or of a right cone may be found by the following rule:

Rule.—*The volume of a right pyramid or a right cone equals the area of the base multiplied by one-third of the altitude.*

If the base of the pyramid is a regular polygon, its area may be found by the rule of Art. **68.**

Example 1.—What is the volume of a triangular right pyramid, the edges of whose base each measure 6 inches, and whose altitude is 8 inches?

Solution.—The base is an equilateral triangle; hence, by the rule of Art. **68,** its area is $6^2 \times .433 = 15.59$ sq. in. Applying the rule, the volume is

$$15.59 \times \frac{8}{3} = 41.57 \text{ cu. in.} \quad \text{Ans.}$$

Example 2.—What is the volume of a right cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

Solution.— $.7854 \times 14^2 = 153.94$ sq. in., the area of the base. By the rule, the volume is

$$153.94 \times \frac{18}{3} = 923.64 \text{ cu. in.} \quad \text{Ans.}$$

88. Convex Area of Pyramid, Cone, or Cylinder. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height corresponds, therefore, with the altitude of any one of the triangular faces. The slant height of a *cone* is any straight line drawn from the vertex to a point on the circumference of the base.

To find the convex area of a pyramid or cone (which is the area of the outside surface, not including the base) apply the following rule:

Rule I.—*To find the convex area of a pyramid or of a cone, multiply the perimeter of the base by one-half the slant height.*

The convex area of a cylinder is found by the following rule:

Rule II.—*To find the convex area of a cylinder, multiply the perimeter of the base by the altitude.*

Example 1.—What is the convex area of a pentagonal pyramid if one side of the base measures 6 inches and the slant height is 14 inches?

Solution.—The base of a pentagonal pyramid is a pentagon with five equal sides. The perimeter of the base is $6 \times 5 = 30$ in. and, by rule I, the convex area is $30 \times \frac{14}{2} = 210$ sq. in. Ans.

Example 2.—*What is the entire area of a right cone whose slant height is 17 inches and whose base is 8 inches in diameter?*

Solution.—The perimeter, or circumference, of the base is 3.1416 × 8 = 25.1328 in.; the area of the base is .7854 × 8^2 = 50.27 sq. in.; the convex area is 25.1328 × $\frac{17}{2}$ = 213.63 sq. in., and the total area of cone is 50.27 + 213.63 = 263.90 sq. in. Ans.

Example 3.—*What is the convex surface of a cylinder 4 inches in diameter and 75 inches high?*

Solution.—The perimeter of the base is 3.1416 × 4 = 12.57 in. Hence, by rule II, the convex area is 12.57 × 75 = 942.75 sq. in. Ans.

89. Frustums.—If a pyramid is cut off parallel to the base, as in Fig. 60, so as to form two parts, the lower part is called the **frustum** of the pyramid. If a cone is cut in a similar manner, as in Fig. 61, the lower part is called the **frustum** of the cone. The upper end of the frustum of a pyramid or cone is called the **upper base**, and the lower end the **lower base**. The **altitude** of a frustum is the perpendicular distance between the bases, indicated by the dotted lines in Figs. 60 and 61.

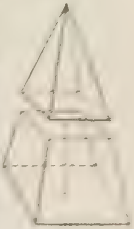


Fig. 60

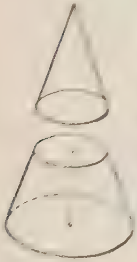


Fig. 61

90. Volume of Frustum of Pyramid Having Regular Polygon for Base.—The volume of a frustum of a pyramid may be found by the following rule:

Rule.—*Multiply together the lengths of one side of the upper and lower bases and the corresponding multiplier taken from Table II, and to the product add the sum of the areas of the upper and lower bases; then multiply the total sum by one-third the altitude of the frustum. The result will be the volume of the frustum.*

Example.—*In a frustum of a pyramid, whose bases are regular hexagons, each edge of the upper base is 5 inches long and of the lower base is 8 inches long. If the altitude is 14 inches, what is the volume?*

Solution.—The area of the upper base, by the rule of Art. **68**, is 5^2 × 2.5981 = 64.95 sq. in. By the same rule the area of the lower base is

$8^2\times2.5981=166.28$ sq. in. The sum of these areas is $64.95+166.28=231.23$ sq. in. Then, by the rule, the volume of the frustum is

$$[(5\times8\times2.5981)+231.23]\times\tfrac{14}{3}=(103.92+231.23)\times\tfrac{14}{3}$$
$$=335.15\times\tfrac{14}{3}=1{,}564\text{ cu. in.}\quad\text{Ans.}$$

91. Volume of Frustum of Cone.—The volume of the frustum of a cone may be found by using the following rule:

Rule.—*Multiply together the diameters of the two bases and .7854 and to the product add the sum of the areas of the upper and lower bases; then multiply the total sum by one-third the altitude of the frustum. The result will be the volume of the conical frustum.*

Example.—What is the volume of a frustum of a cone whose upper base is 8 inches in diameter, whose lower base is 12 inches in diameter, and whose altitude is 15 inches?

Solution.—The area of the upper base is $.7854\times8^2=50.27$ sq. in., and that of the lower base is $.7854\times12^2=113.1$ sq. in. Their sum is $50.27+113.1=163.37$ sq. in. By the rule, the volume of the frustum is

$$[(8\times12\times.7854)+163.37]\times\tfrac{15}{3}=(75.4+163.37)\times\tfrac{15}{3}$$
$$=238.77\times5=1{,}193.85\text{ cu. in.}\quad\text{Ans.}$$

92. Volume of Sphere.—A **sphere,** Fig. 62, is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center. The word *ball* is commonly used instead of sphere. The volume of a sphere may be found as follows:

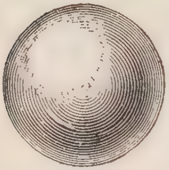


Fig. 62

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by .5236.*

Example.—What is the volume of a cannon ball 12 inches in diameter?

Solution.—By the rule, the volume is $12^3\times.5236=12\times12\times12\times.5236=904.78$ cu. in. Ans.

The volume of a spherical shell, or hollow sphere, is equal to the difference in volume between two spheres having the outer and inner diameters of the shell.

93. Area of Sphere.—The surface area of a sphere is found by the following rule:

Rule.—*Square the diameter and multiply the result by 3.1416.*

Example.—What is the surface area of a sphere 14 inches in diameter?

Solution.—According to the rule, the area is

$$14^2 \times 3.1416 = 14 \times 14 \times 3.1416 = 615.75 \text{ sq. in. Ans.}$$

PRISMOIDS.

94. Definition.—A **prismoid** is a solid with two faces, or bases, that are parallel polygons and the remaining faces triangles or quadrilaterals. Thus, the solid shown in Fig. 63 is a prismoid; its bases are the irregular pentagon *A B C D E* and the trapezoid *F G H I*, which are parallel. The other faces of the prismoid are the triangle *G B C* and the quadrilaterals *G C D H*, *H D E I*, *I E A F*, and *F A B G*. The perpendicular distance between the bases of a prismoid is called its **altitude.**

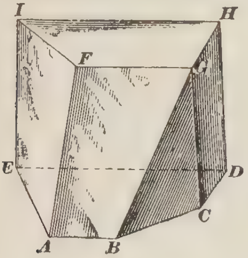


Fig. 63

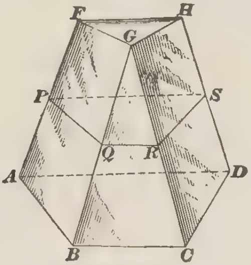


Fig. 64

95. Middle Section of Prismoid.—If an imaginary cut is made through a prismoid, as at *P Q R S*, Fig. 64, parallel to the two bases and at a point equidistant between them, the polygon so formed is known as the **middle section** of the prismoid.

96. Volume of Prismoid.—In calculating the volume of a prismoid the perimeter of the middle section is an important factor. Its length is found by means of the following rule:

Rule I.—*Find one-half the sum of each pair of corresponding sides of the bases. The sum of these results gives the length of the perimeter of the middle section.*

The meaning of this rule will be understood by referring to Fig. 64. Thus, the sides of the middle section are found in the following manner: The side $P\,Q$ is equal to $\frac{A\,B+F\,G}{2}$; the side $Q\,R=\frac{B\,C}{2}$, as the vertex of the triangle has no dimension; the side $R\,S=\frac{G\,H+C\,D}{2}$; and the side $S\,P=\frac{H\,F+D\,A}{2}$. The perimeter of the middle section is equal to the sum of the values found for the sides.

The volume of the prismoid may now be found by means of the following rule and formula, known as the **prismoidal formula:**

Rule II.—*The volume of a prismoid is equal to one-sixth of the product obtained by multiplying the altitude by the sum of the area of the lower base, the area of the upper base and four times the area of the middle section.*

The latter rule may also be expressed as follows:

$$Volume=\tfrac{1}{6}\ (altitude)\times\left(\begin{matrix}lower\\base\end{matrix}+\begin{matrix}upper\\base\end{matrix}+4\times\begin{matrix}middle\\section\end{matrix}\right).$$

Example.—Find the volume of the prismoid shown in Fig. 65, if the altitude is 14 inches and other dimensions are as indicated.

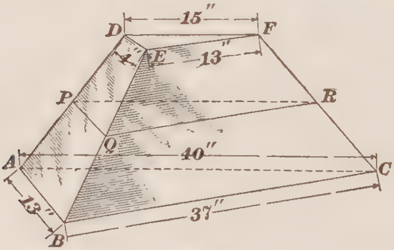


Fig. 65

Solution.—In the illustration the triangle $P\,Q\,R$ is the middle section. The sides of this triangle are found by means of rule I. Thus,

$$PQ=\frac{AB+DE}{2}=\frac{13+4}{2}=8.5 \text{ in.}$$
$$QR=\frac{BC+EF}{2}=\frac{37+13}{2}=25 \text{ in.}$$
$$RP=\frac{AC+DF}{2}=\frac{40+15}{2}=27.5 \text{ in.}$$

The areas of the triangles *A B C, D E F,* and *P Q R* are calculated by the rule of Art. **65,** which gives area of $ABC=240$ sq. in., area of $DEF=24$ sq. in., and area of $PQR=105.2$ sq. in., nearly. According to rule II of this Article, the volume of the prismoid is

$$\tfrac{1}{6}\times 14\times(240+24+4\times 105.2)=1{,}597.9 \text{ cu. in., nearly. Ans.}$$

97. General Application of Prismoidal Formula. The prismoidal formula explained in Art. **96** is used extensively in practice. It applies not alone to prismoids, but also for calculating the volume of a prism, pyramid, cylinder, cone, frustum of a pyramid, frustum of a cone, sphere, or segments of a sphere, in addition to the irregularly shaped bodies to which it is usually applied.

When the formula is applied to a pyramid and cone the dimensions and area of the upper base are equal to 0; for a sphere the areas of the upper and the lower base are 0; for a prism and a cylinder the areas of the upper and lower bases and of the middle section are all equal.

EXAMPLES FOR PRACTICE

1. A rectangular tank 12 feet long, 8 feet wide, and 10 feet deep, inside measurements, is half full of water. How many cubic feet of water does it contain? Ans. 480 cu. ft.

2. A box filled with sand has the dimensions 4 feet, 3 feet 6 inches, and 2 feet 6 inches on the inside. How many pounds of sand does it hold, if the sand weighs 96 pounds per cubic foot? Ans. 3,360 lb.

3. A forged steel shaft 9 inches in diameter and 10 feet long has a hole 4½ inches in diameter through it from end to end. If forged steel weighs .283 pound per cubic inch, what is the weight of the shaft? Ans. 1,620 lb.

4. How many cubic feet are contained in a foundation wall that is 20 feet long, 8 feet high, and 2 feet thick? Ans. 320 cu. ft.

5. What is the volume of a cylindrical oil tank 3 feet in diameter and 6 feet 6 inches high? Ans. 45 cu. ft. 1,635 cu. in.

6. What is the capacity of a tool box, which, measured on the inside, is 2 feet 3 inches long, 18 inches wide, and 1 foot deep? Ans. 3 cu. ft. 648 cu. in.

7. Considering only its outside dimensions, what is the volume of a ball float with a diameter of 22½ inches? Ans. 5,964 cu. in.

8. The diameters of the upper and lower bases of a conical frustum are 8 and 10 feet, respectively. If the altitude is 12 feet, what is the volume of the frustum? Ans. 766.55 cu. ft.

9. If a tank had the dimensions of the frustum in question 8, how many gallons of water would it hold, one gallon being equivalent to 231 cubic inches? Ans. 5,734.2 gal., nearly

10. The cavity of a mold is 20 inches long, 4 inches wide, and 4 inches deep. If a lead casting is made in this mold, what would be its weight, the weight of lead being .41 pound per cubic inch? Ans. 131.2 lb.

PROBLEMS FOR PRACTICE

CALCULATIONS OF MATERIAL, WEIGHT, AND TAPER

CALCULATING LENGTH OF STOCK

98. Forging Problems.—At times it is difficult to estimate the amount of stock required for a certain job. For instance, a blacksmith may have in stock iron bars of a certain size, that are to be used for making some bars of a smaller size. As the length of stock required will be correspondingly shorter, it will be advantageous to know what length of stock to allow.

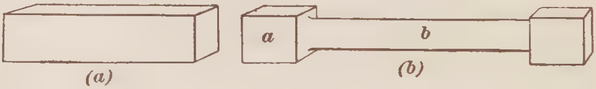


Fig. 66

In Fig. 66 (*a*) is shown a piece of square stock that is to be forged into the form shown at (*b*). The ends *a* of the finished piece are of the same size as the original square bar at (*a*), but the middle part *b* is round, instead of square, and is considerably longer and smaller than the middle part of the square stock. The problem then is to find how much square stock to allow, so that, when it is hammered out into the long, round part *b*, this part will be of the required length. The rule to be used is as follows:

Rule.—*Calculate the volume, in cubic inches, of the part that is to be made from the stock and divide it by the area of the end of the bar stock, in square inches. The quotient is the length of stock required.*

If the finished piece is to be shorter and thicker than the bar stock, the same rule can be used.

EXAMPLE 1.—A forging like that shown in Fig. 66 (*b*) is to be made from a piece of bar stock 3 inches square. If the dimensions of the forging are to be as shown in Fig. 67, what length of stock is required?

SOLUTION.—The ends of the forging are of the same size as the bar stock, that is, 3 in. square, and each is 3½ in. long; therefore, it will take 2×3½ in.=7 in. of stock for the ends. The middle part is a cylinder 2 in. in diameter and 15 in. long. The volume of this part, therefore, is, by the rule of Art. **84**,

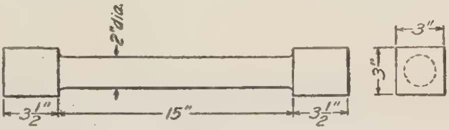


FIG. 67

$$.7854 \times 2^2 \times 15 = 47 \text{ cu. in.}$$

The area of the end of the bar stock, which is 3 in. square, is 3×3=9 sq. in. Then, by the rule of this article, the length of stock required for the middle section of the forging is

$$47 \div 9 = 5.22 \text{ in., or } 5\tfrac{1}{4} \text{ in., very nearly}$$

That is, a piece of 3-in. square stock 5¼ in. long, when hammered down to a round piece 2 in. in diameter, will be 15 in. long. The total length of bar stock required for the forging, therefore, is the sum of that needed for the ends and that needed for the middle part, or

$$7 + 5\tfrac{1}{4} = 12\tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

Example 2.—A forging is to be made, of the shape shown in Fig. 68, from a round 1-inch bar. The central portion, which is 18 inches long, is the same size as the bar, but the two ends are upset until their diameters are increased to 1¼ inches. If each of the upset parts is 6 inches long, how much stock should be allowed for?

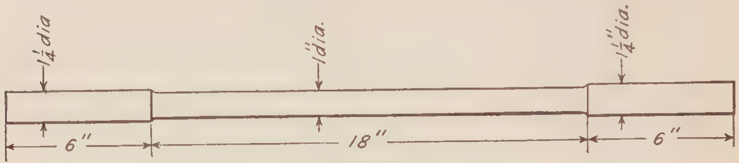


Fig. 68

Note.—When a red-hot bar of iron is hammered on one end in a longitudinal direction, it becomes shorter and thicker, and it is then said to be *upset*.

Solution.—A cylinder 1¼ in. in diameter and 6 inches long has, according to the rule in Art. **84**, a volume of $.7854 \times 1.25^2 \times 6 = 7.363$ cu. in. The volume of the two ends is $2 \times 7.363 = 14.726$ cu. in.

The area of the end of the bar stock, which has a diameter of 1 inch, is $.7854 \times 1^2 = .7854$ sq. in. Then, according to the rule of this article, the length of stock required for the end portions is $14.726 \div .7854 = 18.75$ in. The total length of bar stock required is equal to the sum of that needed for the middle part and for the ends, or

$$18 + 18.75 = 36\tfrac{3}{4} \text{ in.} \quad \text{Ans.}$$

CALCULATION OF WEIGHTS

99. Method of Estimating Weight.—It often becomes necessary to calculate the weight of a body whose dimensions are known but which cannot conveniently be put on scales to be weighed. For instance, suppose that it is desired to estimate the weight of a casting that has not been made. From the drawing it is possible to obtain all the dimensions of the casting. Then, by the principles of mensuration given previously, the volume of the casting can be found. After the volume is known, it is multiplied by the weight of a unit volume of the material, and thus the weight is obtained. Sometimes, castings or forgings have intricate shapes; but usually it is possible to consider them as made up of a combination of cylinders, spheres, rectangular prisms, cones, etc. The volume of each of these is calculated and the total is the approximate volume of the piece.

TABLE III

AVERAGE WEIGHTS OF MATERIALS

Material	Weight per Cubic Foot Pounds	Weight per Cubic Inch Pound	Material	Weight per Cubic Foot Pounds	Weight per Cubic Inch Pound
Dry Woods			*Metals—(Cont.)*		
Ash	45	.0260	Brass castings	510	.295
Beech	46	0266	Brass, rolled	518	.300
Birch	41	.0237	Bronze	527	.305
Box	70	.0405	Copper	550	.318
Cedar	39	.0226	Iron, cast	450	.260
Cherry	41	.0237	Iron, wrought	480	.278
Chestnut	35	.0203	Lead	710	.411
Cork	14	.0081	Steel castings	480	.278
Elm	38	.0220	Steel, forged	490	.283
Fir	37	.0214	Tin	458	.265
Hemlock	24	.0139	Zinc castings	432	.250
Hickory	48	.0278	Zinc sheets	450	.260
Lignum vitæ	62	.0359	*Miscellaneous*		
Mahogany	51	.0287	Cement, loose	92	
Maple	42	.0243	Cement in barrel	115	
Oak, red	46	.0266			
Oak, white	48	.0278	Concrete	120 to 155	
Pine, white	28	.0162			
Pine, yellow	38	.0220	Earth, loose	72 to 80	
Poplar	30	.0174			
Spruce	28	.0162	Earth, rammed	90 to 110	
Walnut	36	.0208			
Willow	34	.0197	Sand	90 to 110	
Metals					
Aluminum	167	.096	Sand, wet	118 to 130	
Antimony	420	.243			
Babbitt metal	466	.270			

100. Weights of Materials.—In finding the weight of a machine part, the volume is usually calculated in cubic inches. This volume must then be multiplied by the weight of the material per cubic inch, to obtain the weight of the body. If the volumes are calculated in cubic feet, the weight per cubic foot is used. To enable the weights of bodies to be calculated, the weights of a number of different materials are given in Table III, in pounds per cubic foot and per cubic inch. The method of using the table is shown in the following examples:

Example 1.—A hollow box-shaped casting has outside dimensions of 18, 10, and 6 inches, and inside dimensions of 16, 8, and 4 inches. What is its weight, if it is a steel casting?

Solution.—The first step is to calculate the volume of the casting. This is the difference between the volume represented by the outside and inside dimensions. The volume of a solid 18 in. by 10 in. by 6 in. is $18\times10\times6=1{,}080$ cu. in., and that of a solid 16 in. by 8 in. by 4 in. is $16\times8\times4=512$ cu. in. Then, the volume of the casting is $1{,}080-512=568$ cu. in. According to Table III, a cubic inch of steel casting weighs .278 lb. Therefore, the weight of the casting is

$$568\times.278=158 \text{ lb., nearly. Ans.}$$

Example 2.—Find the weight of a cast-iron bracket like that in Fig. 69 (*a*), having the dimensions shown in (*b*).

Solution.—The bracket may be considered as being made up of three parts, two of which are rectangular prisms, and the other of which is a triangular prism. One rectangular prism has a length *a b* of 8 in., a width *c d* of 7½ in., and a thickness *a e* of 1½ in. Its volume, therefore, is $8\times7\frac{1}{2}\times1\frac{1}{2}=90$ cu. in. The other rectangular prism has a width *a b* of 8 in., a height *a f* of $9\frac{1}{2}-1\frac{1}{2}=8$ in., and a thickness *g h* of 1½ in., so its

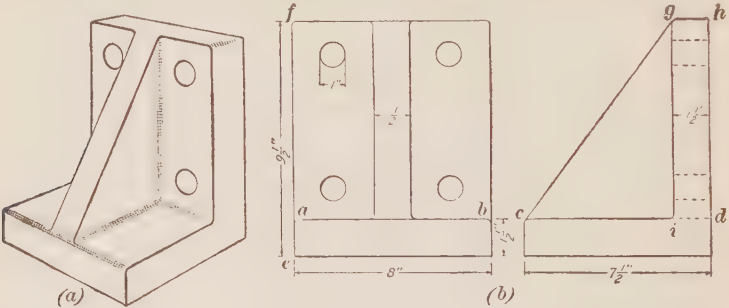


Fig. 69

volume is $8\times8\times1\frac{1}{2}=96$ cu. in. The bracing web has the shape of a right triangle whose base *c i* is $7\frac{1}{2}-1\frac{1}{2}=6$ in., and whose altitude *i g* is $9\frac{1}{2}-1\frac{1}{2}=8$ in. Its area is therefore $\frac{6\times8}{2}=24$ sq. in. As the thickness is 1½ in., the volume of the triangular prism, according to the rule of Art. **84,** is $24\times1\frac{1}{2}=36$ cu. in. There are four holes each 1 in. in diameter and 1½ in. long; so the amount of metal cut out by each is the volume of a cylinder 1 in. in diameter and 1½ in. long. The area of the base is the area of a 1-in. circle, or $.7854\times1^2=.7854$ sq. in. Then, by the rule of Art. **84,** the volume of one cylinder is $.7854\times1\frac{1}{2}=1.1781$ cu. in. As there are four holes, their volume is $4\times1.1781=4.7$ cu. in., approximately. The actual volume of the bracket, therefore, is $90+96+36-4.7=217.3$ cu. in. According to Table III, a cubic inch of cast iron weighs .260 lb. The weight of the bracket, therefore, is

$217.3\times.260=56\frac{1}{2}$ lb., very nearly. Ans.

EXAMPLE 3.—What is the weight of a forged steel shaft of the form and dimensions shown in Fig. 70, the end *d* being square.

SOLUTION.—The shaft may be considered as being made up of a cylindrical piece *a* that is 1 in. in diameter and 2½ in. long, a cylinder *b* 1½ in. in diameter and 9¾ in. long, a frustum of a cone with bases 1¼ in. and 1½ in. in diameter and 2 in. long, and a prism ¾ in. square and 1½ in. long. According to the rule of Art. **84,** volume of part *a* $=.7854\times1^2\times2\frac{1}{2}=1.96$ cu. in., and volume of part $b=.7854\times(1\frac{1}{2})^2\times9\frac{3}{4}=17.23$ cu. in.

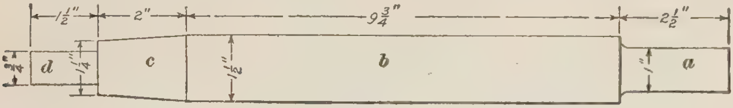


FIG. 70

The part *c* is a frustum of a cone. The area of the smaller base is $.7854\times(1\frac{1}{4})^2=1.23$ and that of the larger base is $.7854\times(1\frac{1}{2})^2=1.77$ cu in. Then, by the rule of Art. **91,** the volume of the frustum is

$$[(1\tfrac{1}{4}\times1\tfrac{1}{2}\times.7854)+1.23+1.77]\times\tfrac{2}{3}=(1.47+1.23+1.77)\times\tfrac{2}{3}=2.98 \text{ cu. in., or}$$
3 cu. in., nearly

The volume of the rectangular solid end *d,* according to the rule of Art. **84,** is $\frac{3}{4}\times\frac{3}{4}\times1\frac{1}{2}=.8$ cu. in.

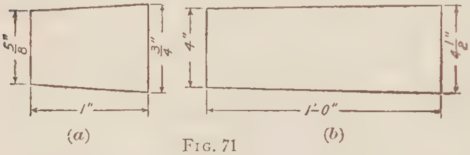
The total volume of the shaft, therefore, is $1.96+17.23+3+.8=22.99$ cu. in., or 23 cu. in., nearly.

According to Table III, the weight of a cubic inch of forged steel is .283 lb. The weight of the shaft, then, is

$23\times.283=6\frac{1}{2}$ lb., nearly. Ans.

CALCULATION OF TAPERS

101. Definition of Taper.—The term *taper* means a gradual and uniform increase or decrease in the width or diameter of a piece from one end to the other. The taper is usually expressed by stating how much the width or diameter of the piece increases or decreases per inch or per foot of length. Thus, if the piece shown in Fig. 71 (*a*) is 1 inch long, $\frac{3}{4}$ inch wide at one end, and $\frac{5}{8}$ inch wide at the other, the taper is $\frac{3}{4}-\frac{5}{8}=\frac{1}{8}$ inch in 1 inch, or $\frac{1}{8}$ inch per inch. If the piece shown in (*b*) is 1 foot long and its diameters at the ends are $4\frac{1}{2}$ inches and 4 inches, the taper is $4\frac{1}{2}-4=\frac{1}{2}$ inch per foot. The taper may be expressed in inches per inch or in inches per foot, as desired.



(*a*) (*b*)

FIG. 71

102. Finding the Taper.—If the length and the end dimensions of a piece are known, the taper may be found by the following rules:

Rule I.—*To find the taper in inches per foot, divide the difference of the end dimensions, in inches, by the length of the piece in feet.*

Rule II.—*To find the taper in inches per inch, divide the difference of the end dimensions, in inches, by the length of the piece in inches.*

EXAMPLE 1.—A tapered bar $2\frac{1}{2}$ feet long is $4\frac{5}{8}$ inches wide at one end and 4 inches wide at the other. What is the taper per foot?

SOLUTION.—By rule I, the taper is

$$(4\tfrac{5}{8}-4)\div 2\tfrac{1}{2}=\tfrac{5}{8}\times\tfrac{2}{5}=\tfrac{1}{4} \text{ in. per ft.} \quad \text{Ans.}$$

EXAMPLE 2.—A round tapered piece 8 inches long has diameters of $1\frac{5}{8}$ and $2\frac{1}{8}$ inches at the ends. What is the taper per inch?

SOLUTION.—By rule II, the taper is

$$(2\tfrac{1}{8}-1\tfrac{5}{8})\div 8=\tfrac{1}{2}\times\tfrac{1}{8}=\tfrac{1}{16} \text{ in. per in.} \quad \text{Ans.}$$

103. Finding Dimensions From Taper.—Sometimes the dimension at one end of the piece and the taper are given and the dimension at the other end is to be calculated. This may be done by the following rules:

Rule I.—*To find the dimension at the large end, multiply the length of the piece in feet (or in inches) by the taper per foot (or per inch) and to the product add the dimension at the small end.*

Rule II.—*To find the dimension at the small end, multiply the length of the piece in feet (or in inches) by the taper per foot (or per inch) and subtract the product from the dimension at the large end.*

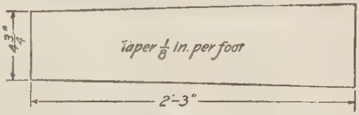


FIG. 72

EXAMPLE 1.—The piece shown in Fig. 72 has a taper of $\frac{1}{8}$ inch per foot, is $4\frac{3}{4}$ inches wide at the small end, and is 2 feet 3 inches long. What is the width at the large end?

SOLUTION.—By rule I, and remembering that 2 ft. 3 in.=$2\frac{1}{4}$ ft.,

$$2\frac{1}{4}\times\frac{1}{8}=\frac{9}{4}\times\frac{1}{8}=\frac{9}{32}$$

Then, the width at the large end is

$$\frac{9}{32}+4\frac{3}{4}=5\frac{1}{32}\text{ in. Ans.}$$

EXAMPLE 2.—A piece 16 inches long has a taper of $\frac{1}{4}$ inch per inch and is $7\frac{3}{4}$ inches in diameter at the large end. Find the diameter at the small end.

SOLUTION.—By rule II,

$$16\times\frac{1}{4}=4$$

Then, the diameter at the small end is

$$7\frac{3}{4}-4=3\frac{3}{4}\text{ in. Ans.}$$

COMMERCIAL CALCULATIONS
(PART 1)

SPECIAL METHODS IN APPLICATION OF FUNDAMENTAL RULES

1. At this point the student should be able to apply with ease the fundamental rules of arithmetic and to perform all the operations with fractions, both common and decimal. The first essential of an accountant is accuracy. A result obtained in any calculation is worthless if it is not correct. In business life a clerk or accountant whose work shows incompetency in this respect will not be tolerated. Speed ranks in importance next to accuracy. A man's services are valued according to what he can accomplish. A rapid and accurate accountant is, of course, preferred to the man who makes calculations slowly.

We shall now take up some principles which are supplemental to those treated in the preceding Sections, and which, if followed, will be a great aid to one wishing to attain accuracy and rapidity.

RAPID ADDITION

2. There is scarcely anything more useful to the bookkeeper and business man than the ability to add rapidly and correctly; but this can be acquired only by persevering practice. Any time employed in practicing addition will be well spent. If the student will practice addition ten or fifteen minutes daily for a month or so, he will be greatly benefited.

In order to become expert in adding, it is absolutely essential that when two figures are seen or heard pronounced, the

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student can instantly give their sum. Thus, 15 should suggest itself as soon as 6 and 9, or 8 and 7, are seen or heard pronounced. It should not be necessary to say mentally, 6 and 9 is 15, but 6, 15, the 9 not being pronounced either mentally or orally. (In no case should the student contract the habit of adding aloud; it is not necessary, and it is a very difficult habit to break.)

In adding 5, 6, 1, 9, 7, 5, 2, 4, 8, 9, do not say 5 and 6 is 11, and 1 is 12, and 9 is 21, etc., but *think* 5, 11, 12, 21, 28, 33, 35, 39, 47, 56, repeating the sums about as fast as they can be pronounced.

3. To add rapidly, it is necessary for the student to become accustomed to group the figures of a column and add the sums of the groups. It is most convenient to choose the groups, as far as possible, so that the sum of each group shall be either 10 or 20. An example will show how this is accomplished:

$$\begin{array}{r} \left.\begin{array}{r}3\\9\\2\\5\end{array}\right\} \\ \left.\begin{array}{r}8\\2\end{array}\right\} \\ \left\{\begin{array}{r}6\\4\end{array}\right. \\ \left.\begin{array}{r}3\\5\\0\\ \left\{\begin{array}{r}9\\1\end{array}\right. \\ 2\end{array}\right\} \\ \hline 59 \end{array}$$

Commencing at the bottom of the column, we see at once that 1 and 9 form a group whose sum is 10; hence we mentally say 2, 12 instead of 2, 3, 12. Now the next two digits, 5 and 3, we group together, and instead of adding 5 and 3 separately, we add the sum, 8. The next two figures, 4 and 6, form a group whose sum is 10, and so do the next two figures, 2 and 8. Looking now at the four figures at the top of the column, we readily see that 5, 2, and 3 form a last group whose sum is 10, leaving only the 9 outside of a group. In adding the column, we would repeat mentally 2, 12, 20, 30, 40, 50, 59. Another grouping would readily appear to the skilful accountant; the 2 at the bottom and the 5 and 3 above the 0 form a group whose sum is 10. Recognizing this group, the mental addition would be: 10, 20, 30, 40, 50, 59.

This process of forming groups may be extended to include those whose sums are 15 or 20. By a judicious selection of the figures composing a group, its sum may usually be made either 10, 15, or 20, and these sums should always be sought in preference to others, since two 15's make 30, and the numbers

10, 20, and 30 are added with little mental labor. Thus, in the following example, the grouping is shown by the braces. The mental addition is 15, 30, 37, 47, 67, 72. After some practice, the student will be able to recognize a group whose sum is 10 or 20 almost instantly; he should persevere in the solution of examples until he is able to form the groups rapidly and with ease. It is not always possible to get consecutive numbers which will form groups whose sums are 10, 15, or 20. Such groups, however, can often be found by skipping one or more figures in the column. For instance, in the following example, the first two figures, 7 and 3, make a 10, and skipping the fourth figure, 9, the third and fifth figures, 4 and 6, make another 10. Skipping the 8, the 3, 4, and 3 at the top of the column make another 10, thus making three groups of 10 and the figures 9 and 8. In adding, however, we should not leave the figures skipped to be added at the end of the operation, but should add them as they occur. In this example, we would say 10, 20, 29, 39, 47. The order of adding may be shown by the following arrangement:

$$\begin{array}{l}5\\ \left.\begin{array}{r}9\\3\\8\end{array}\right\}20\\ \left.\begin{array}{r}6\\4\end{array}\right\}10\\ 7\\ \left.\begin{array}{r}2\\9\\4\end{array}\right\}15\\ \left.\begin{array}{r}8\\7\end{array}\right\}15\\ \hline 72\end{array}$$

$$\begin{array}{r}3\\4\\8\\3\\0\\6\\9\\4\\3\\7\\ \hline 47\end{array}$$

$$\overbrace{7+3}^{10}+\overbrace{6+4}^{10}+9+\overbrace{3+4+3}^{10}+8.$$

The columns can be added from the top downwards if this order is preferred.

The following examples may be used for practice:

(1)	(2)	(3)	(4)	(5)
3	13	317	1127	13103
5	27	226	2632	61706
2	92	232	1946	43285
6	35	518	3217	39134
3	64	396	1184	96286
4	22	435	1936	45173
0	30	707	2003	78135
9	97	992	5114	26232
3	18	311	976	19375
1	33	417	5634	8428

The answers are: (1) 36; (2) 431; (3) 4,551; (4) 25,769; (5) 430,857.

4. It is frequently advantageous to be able to add *horizontally*, as it is termed; by this is meant the adding of several numbers as they stand in a horizontal row, without arranging the numbers in vertical columns. Thus,

123+567+792+221+546=2,249. Ans.

When adding in this manner, straight addition must be employed; i. e., the method of Art. 3 cannot be used. The process would be 6, 7, 9, 16, 19; 5, 7, 16, 22, 24; 7, 9, 16, 21, 22. As an example of this method, consider the accompanying table, which is supposed to give, by days, the grain export, in bushels, of a certain city for one week. It is required to find the amount of grain exported each day, the total amount of each kind of grain exported during the week, and, finally, the total amount of grain exported during the week.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Totals
Corn..........	28,325	15,230	35,715	29,128	75,183	46,217	*****
Wheat.........	35,719	41,719	50,108	32,546	59,275	81,126	*****
Oats..........	12,136	9,237	18,265	7,268	6,950	17,230	*****
Barley........	18,230	15,738	21,375	15,928	19,263	13,637	*****
Rye...........	5,275	6,829	7,201	11,325	7,825	13,261	*****
Totals........	*****	*****	*****	*****	*****	*****	*****

The student should find the totals, and prove that the results are correct by adding the totals in the right-hand column and then adding the totals in the bottom row; the two results should be the same, namely, 757,270 bushels. The other results are: corn, 229,804; wheat, 300,493; oats, 71,086; barley, 104,171; rye, 51,716; Mon., 99,685; Tues., 88,759; Wed., 132,664; Thurs., 96,195; Fri., 168,496; Sat., 171,471.

SPECIAL METHOD OF SUBTRACTION

5. A method of subtraction which is called the *making change* method is frequently used. Suppose a customer hands the clerk a five-dollar bill in payment of a purchase amounting to $3.76. The clerk does not subtract $3.76 from $5 by the ordinary rules of subtraction but he adds enough to $3.76 to make \$5. He would count out 4 cents, two 10-cent pieces, and a dollar, since \$3.76 and 4 cents is \$3.80, \$3.80 and 20 cents is \$4, and an additional dollar makes \$5. To verify his reckoning he counts as he hands the customer the change—three seventy-six, three eighty, four, five—omitting to say dollars and cents, as these words are, of course, understood. A similar method may be used in written work.

Thus, if the minuend is 8,453 and the subtrahend 844, the remainder is found as follows:

minuend	8453	
subtrahend	844	
remainder	7609	Ans.

Instead of subtracting 4 from 13, we add to 4 a number that will make 13; this number is 9, since 4 and 9 is 13. Write the 9 and carry the 1, as in addition; then $4+1=5$. Since $5+0=5$, for the next figure in the subtrahend we write 0, as shown. There is nothing to carry; hence, since 8 is greater than 4, we consider 4 to be 14, and find what number added to 8 will make 14; this is 6, which we write below the line, as shown. We have 1 to carry, but, as we have no figure in the subtrahend to add it to, we say 1 and 7 is 8, and write the 7 below the line, as shown.

Again, consider the following example:

minuend	10000	
subtrahend	8763	
remainder	1237	Ans.

Here we say 3 and 7 is 10, and write the 7; then, 7 and 3 is 10 (carrying the 1 and adding it to the 6), and write the 3; next, 8 and 2 is 10, and write the 2; finally, 9 and 1 is 10, and write the 1. The student may use whatever method of subtraction he prefers, but it may be stated that the making-change method is becoming more and more popular.

6. The method just described is particularly useful when it is desired to subtract from some number the sum of several numbers, the entire process being performed at one operation. For example, suppose it were desired to subtract the sum of

1,538, 512, 987, 645, 884, and 763 from 7,061. The work might be arranged as follows:

1538	7061	*minuend*
512	1732	Ans.
987		
645		
884		
763		

We add the right-hand column, 3, 7, 12, 19, 29. We cannot subtract 29 from 1 in the minuend, but if 2 is added to 29 the sum will be 31, a number ending in 1. Hence, write 2 under 1 and carry 3. Adding the second column, 3, 9, 17, 21, 30, 33; 3 added to 33 makes 36; hence, write 3 under 6 and carry 3. Adding the third column, 3, 10, 18, 24, 33, 43; 7 added to 43 makes 50; hence, write 7 under 0 and carry 5. Adding 5 to the 1 in the fourth column makes 6, and 6 and 1 is 7; hence, write 1 under 7. The difference between the sum of the numbers and 7,061 is 1,732, the answer.

EXAMPLES FOR PRACTICE

In the following examples use the method just described. The work may be checked by adding the remainder in each case to the numbers added; the sum should be equal to the minuend.

(1)		(2)		(3)	
33604	178884	81220	350915	21824	261132
9775		21413		71737	
11628		37526		24445	
84721		41284		94336	
5109		71733		8972	

Ans.—(1) 34,047; (2) 97,739; (3) 39,818.

SPECIAL METHODS OF MULTIPLICATION

7. A quantity may be multiplied by finding the continued product of the quantity and the factors of the multiplier. For example, a number can be multiplied by 12 by multiplying the number by 3 and the product thus obtained by 4. Multiplying a number and its products in succession by 4, 6, and 8 will produce the same product as multiplying the number by 192, the continued product of 4, 6, and 8.

8. To multiply by factors.

Example.—Multiply 2,972 by 192, taking the factors 4, 6, and 8, and test the result by multiplying directly.

Solution.—

$$\begin{array}{r} 2972\\ 4\\ \hline 11888\\ 6\\ \hline 71328\\ 8\\ \hline 570624 \end{array}\ \text{Ans.} \qquad\qquad \begin{array}{r} 2972\\ 192\\ \hline 5944\\ 26748\\ 2972\\ \hline 570624 \end{array}\ \text{Ans.}$$

In this particular problem no time would be saved by using the factors in multiplication. The principle can often be used, however, to shorten the operation and may be employed to test the accuracy of the product obtained by another method.

EXAMPLES FOR PRACTICE

(*a*) Multiply 536 by 56.
(*b*) Multiply 763 by 24.
(*c*) What will 54 horses cost at $185 each?

Ans.—(*a*) 30,016; (*b*) 18,312; (*c*) $9,990.

---

9. To multiply when one part of the multiplier is a factor of another part.

Example. 1.—Multiply 1,728 by 93.

Solution.—

$$\begin{array}{rr} & 1728\\ & 93\\ \hline & 5184\\ 3\times5{,}184= & 15552\\ \hline & 160704 \end{array}\ \text{Ans.}$$

Instead of multiplying 1,728 by 9 in order to obtain the second partial product, multiply 5,184 by 3, as $3\times3\times1{,}728=9\times1{,}728$. We do so since it is easier for most people to multiply by 3 than by 9.

Example 2.—Find the cost of 324 acres of land at $28.75 per acre.

Solution.—

$$\begin{array}{rr} & \$28.75 \\ & 324 \\ \hline & 11500 \\ 8\times 11{,}500= & 92000 \\ \hline & \$9315.00 \end{array} \quad \text{Ans.}$$

Observe that 32 equals 8 times 4. The work is shortened by multiplying 11,500 by 8 instead of multiplying 2,875 by 2 and 3 according to the usual way. The example may be solved also by considering 324 as made up of the parts 3 and 24 or 3 and 3×8.

$$\begin{array}{rr} & \$28.75 \\ & 324 \\ \hline & 8625 \\ 8\times 8{,}625= & 69000 \\ \hline & \$9315.00 \end{array} \quad \text{Ans.}$$

In using this method, it is necessary to place the first figure in any partial product directly under the right-hand figure of the corresponding multiplier.

Example 3.—Multiply 5,682 by 2,408.

Solution.—

$$\begin{array}{rr} & 5682 \\ & 2408 \\ \hline & 45456 \\ \left.\begin{array}{r} 3\times 45{,}456 \\ \text{or}\quad 24\times 5{,}682 \end{array}\right\}= & 136368 \\ \hline & 13682256 \end{array} \quad \text{Ans.}$$

In multiplying, notice that 24 = 3×8, and that the right-hand figure of the second partial product is written directly under the 4 of the multiplier.

EXAMPLES FOR PRACTICE

Multiply:

(*a*) 3,859 by 567.
(*b*) 6,492 by 364.
(*c*) 3,741 by 824.
(*d*) 9,753 by 864.

Ans.—(*a*) 2,188,053; (*b*) 2,363,088; (*c*) 3,082,584; (*d*) 8,426,592.

1. An agent sells 273 vacuum cleaners for $23.75 apiece. What was the amount of the sales? Ans. $6,483.75

2. What is the cost of 123 textbooks at $1.18 apiece? Ans. $145.14

10. To multiply by a number ending with the figure 1.

EXAMPLE.—Multiply 2,386 by 81.

SOLUTION.—

$$\begin{array}{r} 2386 \\ \underline{19088} \\ 193266 \end{array} \text{ Ans.}$$

The first partial product equals 1×2,386=2,386; hence, it can be set down directly. The next partial product, which is 8×2,386=19,088, can be set down under the first according to the rules of multiplication. It is not even necessary to draw the line below the partial products, but such abbreviated methods should not be used when one expects to have his work reviewed by some one else. In such cases the operations employed should be clearly indicated.

11. To multiply by a number that is nearly a hundred or a thousand.

EXAMPLE 1.—Multiply 2,872 by 499.

SOLUTION.—

$$\begin{array}{rr} 500\times 2{,}872= & 1436000 \\ 1\times 2{,}872= & \underline{2872} \\ & 1433128 \end{array} \text{ Ans.}$$

We notice that 499 is 1 less than 500, which is a number easy to use as a multiplier. 500 times a number less 1 times that number is evidently 499 times that number.

EXAMPLE 2.—Find the cost of $797\frac{1}{2}$ yards of cloth at $\$2.62\frac{1}{2}$ a yard.

SOLUTION.—We observe that $797\frac{1}{2}$ is $2\frac{1}{2}$ less than 800.

$$\begin{array}{lr} 800\times \$2.625= & \$2100.00 \\ 2\frac{1}{2}\times 2.625=\frac{1}{4}\times 10\times 2.625= & \underline{6.56} \\ & \$2093.44 \end{array} \text{ Ans.}$$

To multiply by $2\frac{1}{2}$, multiply by 10 and divide by 4, since $2\frac{1}{2}$ equals $\frac{10}{4}$.

EXAMPLES FOR PRACTICE

Multiply:

(*a*) 1,234 by 999.

(*b*) 5,937 by 798.

(*c*) 5,246 by 91.

(*d*) 1,234 by 9,995.

Ans.—(*a*) 1,232,766; (*b*) 4,737,726; (*c*) 477,386; (*d*) 12,333,830.

1. A man bought 97 yards of cloth at $1.25 a yard. How much did the cloth cost him? Ans. $121.25

2. How much does a farmer pay for 195 bags of corn at $1.35 a bag? Ans. $263.25

SPECIAL METHODS IN DIVISION

12. In the study of fractions it has been shown that both terms of a fraction may be multiplied by or divided by any number and the value of the fraction will remain the same. Similarly in division we may multiply or divide the dividend and divisor by the same number without changing the value of the quotient. This principle can sometimes be applied with profit, as when the divisor is a mixed number or when the divisor may be factored.

Example 1.—A real-estate agent bought $18\frac{1}{3}$ acres of land for $4,000. What was the price paid per acre?

Solution.—

```
18⅓ )4000
 3      3
---    ---
55  )12000.000($218.181 or $218.18  Ans.
     110
     ---
      100
       55
       ---
       450
       440
       ---
        100
         55
         ---
         450
         440
         ---
          100
           55
          ---
```

We multiply the divisor and dividend each by 3 in order to clear the divisor of the fraction; $3 \times 18\frac{1}{3} = 55$, and $3 \times 4{,}000 = 12{,}000$; $12{,}000 \div 55 = 218.18$.

Example 2.—Divide 1,000 by 32.

Solution.—

```
4)32   4)1000
  --     ----
   8    ) 250
       ------
         31¼  Ans.
```

32 is divisible by 4; hence, we divide the divisor and dividend each by 4, and obtain 8 and 250. $250 \div 8 = 31\frac{1}{4}$. We could as well divide by 8 and then by 4; thus,

$$\begin{array}{rr} 8\underline{)32} & 8\underline{)1000} \\ 4 & \underline{)\ \ 125} \\ & 31\frac{1}{4} \end{array}$$

The advantage of this method is that we can reduce the divisor to a smaller number and then possibly use short division. In some cases the division can be performed mentally.

Example 3.—Divide 32,784 by 4,000.

Solution.—

$$\begin{array}{rl} 4.000\underline{)32.784} & \\ 8.196 & \text{Ans.} \end{array}$$

We divide the divisor and dividend each by 1,000 by pointing off three decimal places from the right. The divisor becomes 4.000, or 4, and the dividend 32.784. By short division we find that $32.784 \div 4 = 8.196$.

We observe that when dividing by a number ending in ciphers, the ciphers may be dropped and as many decimal places pointed off in the dividend as there are ciphers dropped.

13. We will give another example to illustrate the use of this principle.

Example.—In a city of 30,000 inhabitants the assessed value of the property is $125,685,000. What is the average value of the property assessed to each person?

Solution.—The divisor ends in four ciphers, which may be dropped if four decimal places are pointed off in the dividend.

$$\begin{array}{rl} 3\underline{)12568.5000} & \\ \$4189.50 & \text{Ans.} \end{array}$$

EXAMPLES FOR PRACTICE

Divide:

(*a*) 2,256 by $5\frac{1}{2}$.
(*b*) 34,678 by 2,000.
(*c*) 3,300 by $3\frac{1}{7}$.

Ans.—(*a*) $410\frac{2}{11}$; (*b*) 17.339; (*c*) 1,050.

14. Short Method of Subtracting a Product.—It is frequently necessary to multiply a number by a single digit and then to subtract the product from another number. This is most conveniently done as in the following example, in which 314,159 is multiplied by 7 and the product is subtracted from 7,208,451.

$$\begin{array}{rrl}
314159 & \underline{7208451} & \textit{minuend} \\
\underline{7} & 5009338 & \text{Ans.}
\end{array}$$

EXPLANATION.— 7×9=63; 63 cannot be subtracted from 1 in the minuend, so we add the smallest number to 63 that will make the sum end with 1. This number is evidently 8, since 63+8=71; hence, write 8 under 1 and carry 7. Then, 7×5 =35, and 7 is 42; 42 and 3 is 45; write 3 under 5 in the minuend, and carry 4. Then, 7×1=7, and 4 is 11; 11 and 3 is 14. Write 3 under 4 and carry 1. 7×4=28, and 1 is 29; 29 and 9 is 38. Write 9 under 8 and carry 3. 7×1=7 and 3 is 10; 10 and 0 is 10. Write 0 under 0 and carry 1. 7×3=21, and 1 is 22; 22 and 0 is 22. Write 0 under 2 and carry 2. 2 and 5 is 7. Write 5 under 7. The result, 5,009,338, is the answer.

SHORT METHOD OF DIVISION

15. The following method saves about half the figures over the usual method, and we think that there will be fewer mistakes made when using it.

EXAMPLE.—Divide 39,913,910 by 5,494.

SOLUTION.—

$$\begin{array}{rll}
\textit{dividend} & \textit{divisor} & \\
\underline{39913910}) & \underline{5494} & \\
\underline{14559} & 7265 & \textit{quotient} \\
\underline{35711} & & \\
\underline{27470} & & \\
0000 & &
\end{array}$$

EXPLANATION.—The method of multiplication and subtraction described in the last article is used in this case; the divisor is written on the right of the dividend, and the quotient underneath the divisor. The different figures of the quotient are obtained in exactly the same manner as by the preceding method. Thus, the divisor is contained in the first five figures of the dividend 7 times, and 7 is written for the first figure of the quotient. Now, instead of multiplying the divisor by 7, writing the product under the first five figures of the dividend, and then subtracting, we multiply each figure of the divisor by 7 and subtract from the dividend, writing only the remainder. Thus, 7 times 4 is 28, and 5 is 33; write the 5 under the 3 in the dividend, and carry 3. Then, 7 times 9 is 63 and 3 is 66, and 66 and 5 is 71; write the 5 and carry 7. 7 times 4 is 28 and 7 is 35, 35 and 4 is 39; write the 4 and carry 3. 7 times 5 is 35 and 3 is 38, and 38 and 1 is 39; write the 1. Now bring down the next figure of the dividend, 9, and annex it to the remainder. 14,559 ÷ 5,494 = 2; write 2 as the second figure of the quotient. Then, as above, $2\times4=8$, and $8+1=9$; write the 1 under the 9, as shown $2\times9=18$, and $18+7=25$; write the 7 and carry the 2. $2\times4=8$, $8+2=10$, and $10+5=15$; write the 5 and carry the 1. $2\times5=10$, $10+1=11$, and $11+3=14$; write the 3. Bringing down 1, the next figure of the dividend, 35,711 ÷ 5,494 = 6, the third figure of the quotient. Proceed in the same way with the remaining figures.

A fast computer would work as follows: In multiplying by 6, he would repeat to himself 6, 24, and 7 is 31 (writing the 7 and carrying the 3). 6, 54, 57, and 4 is 61 (writing the 4 and carrying the 6). 6, 24, 30, and 7 is 37 (writing the 7 and carrying the 3). 6, 30, 33, and 2 is 35 (writing the 2).

The object of writing the divisor on the right is to make it easier to multiply by the figures of the quotient; it also saves space, as may readily be seen. The student is strongly advised to learn this method thoroughly, and always to use it. The process may seem slow and confusing at first, but rapidity can soon be acquired. The best way to attain facility in division is first to practice dividing by small numbers, from 2 to 12, and using the method of short division. After he has become proficient in this, he should practice long division by the method just described.

ALIQUOT PARTS

16. Numbers that form simple fractions of another number are called **aliquot parts** of the latter number. Thus, $12\frac{1}{2}$ cents is an aliquot part of a dollar, since it equals $\frac{1}{8}$ of a dollar.

17. To multiply by a fraction of 100.

EXAMPLE 1.—Find the cost of 2,876 cattle at $25 a head.

SOLUTION.— $100 \times 2{,}876 = 287{,}600$; $25 \times 2{,}876 = \frac{287{,}600}{4} = \$71{,}900$. Ans.

Since 25 is $\frac{1}{4}$ of 100, we multiply by 100 and divide the product by 4.

EXAMPLE 2.—What is the cost of 250 dozen pencils at 25 cents a dozen?

SOLUTION.— 4) 2 5 0.0 0
$ 6 2.5 0 Ans.

Since the price of each dozen is 25 cents or $\frac{1}{4}$ dollar, we multiply by $\frac{1}{4}$, or what amounts to the same thing, we divide by 4.

EXAMPLE 3.—What is the cost of 84 penknives at $33\frac{1}{3}$ cents each?

SOLUTION.— 3) 8 4
$ 2 8 Ans.

$33\frac{1}{3}$ cents is $\frac{1}{3}$ of a dollar, hence we multiply 84 by $\frac{1}{3}$ by dividing 84 by 3.

18. The fractions of 100 given in the following table are often met with in business transactions and should be memorized.

$2\frac{1}{2} = \frac{1}{40}$ of 100
$3\frac{1}{3} = \frac{1}{30}$ of 100
$4 = \frac{1}{25}$ of 100
$5 = \frac{1}{20}$ of 100
$6\frac{1}{4} = \frac{1}{16}$ of 100
$8\frac{1}{3} = \frac{1}{12}$ of 100
$10 = \frac{1}{10}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100
$16\frac{2}{3} = \frac{1}{6}$ of 100
$20 = \frac{1}{5}$ of 100
$25 = \frac{1}{4}$ of 100
$33\frac{1}{3} = \frac{1}{3}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100
$50 = \frac{1}{2}$ of 100
$62\frac{1}{2} = \frac{5}{8}$ of 100
$66\frac{2}{3} = \frac{2}{3}$ of 100
$75 = \frac{3}{4}$ of 100
$87\frac{1}{2} = \frac{7}{8}$ of 100

EXAMPLE 1.—Find the cost of 3 dozen textbooks at $87\frac{1}{2}$ cents each.

SOLUTION.— 3 dozen = 36; $87\frac{1}{2}$ cents = $\frac{7}{8}$ dollar.

$$\overset{9}{\cancel{36}}\times\frac{7}{\underset{2}{\cancel{8}}}=\frac{63}{2}=\$31.50.\quad\text{Ans}$$

EXAMPLE 2.—Find the cost of 2,875 pounds of sugar at $6\frac{1}{4}$ cents a pound.

SOLUTION.— $6\frac{1}{4}$ cents = $\frac{1}{16}$ dollar, and since 16 is 4×4, we may divide by 4 twice.

$$\begin{array}{r@{\,}l} 4) & 2875 \\ \hline 4) & \;\;718.75 \\ \hline & \;\;\;179.6875 \text{ or } \$179.69. \quad \text{Ans.} \end{array}$$

EXAMPLE 3.—What is the cost of 360 baskets of peaches at $\$1.12\frac{1}{2}$ a basket?

SOLUTION.— $\$1.12\frac{1}{2}=\$1\frac{1}{8}$.

$$\begin{array}{r} 8)\,360 \\ 45 \\ \hline \$\,405 \end{array}\quad\text{Ans.}$$

At \$1 a basket, the cost would be \$360; but, since the cost is $\$1+\$\frac{1}{8}$, the entire cost is $\$360+\frac{1}{8}$ of \$360.

EXAMPLE 4.—A merchant bought 225 yards of cloth at $66\frac{2}{3}$ cents a yard. What was the amount of the bill?

SOLUTION.— $66\frac{2}{3}$ cents is $\frac{2}{3}$ of a dollar.

$$\begin{array}{r} 3)\,225 \\ 75 \\ \hline \$\,150 \end{array}\quad\text{Ans.}$$

At \$1 a yard the cloth would cost \$225; but, since the cost is $\frac{2}{3}$ of a dollar $=\$1-\$\frac{1}{3}$, the entire cost is $\$225-\frac{1}{3}$ of \$225.

EXAMPLES FOR PRACTICE

1. What is the cost of 455 yards of cloth at 20 cents a yard? Ans. \$91

2. Find the amount received for 32 dozen eggs at $37\frac{1}{2}$ cents a dozen. Ans. \$12

3. A man earning $\$1.87\frac{1}{2}$ a day works 24 days in one month. How much did he receive for the month? Ans. \$45

4. What is the cost of 12 dozen pairs of gloves at $1.25 a pair?
Ans. $180

5. If a boy earns $87\frac{1}{2}$ cents a day, how much will he earn in 96 days?
Ans. $84

6. At $\$1.33\frac{1}{3}$ each, what will 60 chairs cost? Ans. $80

19. Use of Aliquot Parts in Division.—The aliquot parts of 100 can often be used to shorten the work of division.

EXAMPLE 1.—A boy earned $4.75 in a week. His wages are $12\frac{1}{2}$ cents an hour; how many hours did he work?

SOLUTION.— $12\frac{1}{2}$ cents $=\frac{1}{8}$ dollar; $4.75\div\frac{1}{8}=4.75\times 8=38$ hours. Ans.

EXAMPLE 2.—Divide 487,634 by 25.

SOLUTION.—Since $487{,}634\div 25=487{,}634\div\frac{100}{4}$, we may divide by 25 by multiplying by 4 and then dividing by 100, or by dividing by 100 and multiplying the quotient by 4. The work would actually be performed as follows:

$$\begin{array}{r} 4876.34 \\ 4 \\ \hline 19505.36 \end{array} \quad \text{Ans.}$$

487,634 is divided by 100 by pointing off two decimal places.

EXAMPLES FOR PRACTICE

1. How many penknives at $33\frac{1}{3}$ cents each may be purchased for $60?
Ans. 180 penknives

2. The bill for a lot of cotton goods amounted to $6.75. If the goods cost $12\frac{1}{2}$ cents a yard, how many yards were there in the lot?
Ans. 54 yards

3. At a cost of $\$1.12\frac{1}{2}$ a dozen, how many dozen handkerchiefs can be bought with $90? Ans. 80 dozen

4. How many books costing $\$1.62\frac{1}{2}$ a piece can be purchased for $71.50? Ans. 44 books

USEFUL APPLICATIONS

20. Methods of Multiplying by a Mixed Number.—In business transactions the multiplication of a mixed number by an integer, an integer by a mixed number, or a mixed number by a mixed number, is of very frequent occurrence. Unless the numbers are very small, which is not usually the case, it is very inconvenient to reduce the mixed numbers to improper fractions, multiply, and then reduce the product to a mixed number. A better way is to use one of the methods given below:

EXAMPLE 1.—Multiply 825 by $29\frac{3}{4}$.

SOLUTION.—*First Method*

$$\begin{array}{r} 825\phantom{\frac{3}{4}} \\ 29\frac{3}{4} \\ \hline 4\,)\,2475\phantom{\frac{3}{4}} \\ \hline 618\frac{3}{4} \\ 7425\phantom{\frac{3}{4}} \\ 1650\phantom{\frac{3}{4}} \\ \hline 24543\frac{3}{4} \end{array} \quad \text{Ans.}$$

Second Method

$$\begin{array}{r} 825\phantom{\frac{3}{4}} \\ 29\frac{3}{4} \\ \hline 206\frac{1}{4} \\ 412\frac{2}{4} \\ 7425\phantom{\frac{3}{4}} \\ 1650\phantom{\frac{3}{4}} \\ \hline 24543\frac{3}{4} \end{array} \quad \text{Ans.}$$

EXPLANATION.—*First Method:* Since $\frac{3}{4}$ of 825 is the same as $\frac{1}{4}$ of 3 times 825, we first find 3 times 825 and take $\frac{1}{4}$ of the product. We then multiply by 29 in the usual way, taking care to put the units place of 7,425 under the units place of 618.

In the *second method* we divide 825 by 4, obtaining $206\frac{1}{4}$. We then multiply $206\frac{1}{4}$ by 2 and write the product, $412\frac{2}{4}$, directly below. The sum of $206\frac{1}{4}$ and $412\frac{2}{4}$ is evidently equal to $\frac{3}{4}$ of 825.

EXAMPLE 2.—Multiply $86\frac{5}{8}$ by $17\frac{3}{4}$.

SOLUTION.—

$$\begin{array}{rr} & 86\frac{5}{8} \\ & 17\frac{3}{4} \\ \hline \frac{3}{4}\times 86 = & 64\frac{1}{2} \\ \frac{5}{8}\times 17 = & 10\frac{5}{8} \\ \frac{5}{8}\times\frac{3}{4} = & \frac{15}{32} \\ 17\times 86 = \left\{\begin{array}{l} 602 \\ 86 \end{array}\right. & \\ \hline & 1537\frac{19}{32} \end{array} \quad \text{Ans.}$$

In this case there are four multiplications made; 86 is multiplied by $\frac{3}{4}$; 17 by $\frac{5}{8}$; $\frac{5}{8}$ by $\frac{3}{4}$; and 86 by 17.

21. In actual practice a result is usually given to the nearest cent; hence, an accountant would make it a point to neglect fractions that would not affect the result. Again, calculations must be made rapidly and often mentally without recourse to paper and pencil. The following illustrates a method that would be used in mental reckoning:

EXAMPLE 1.—Find, to the nearest cent, the cost of $17\frac{3}{4}$ pounds of meat at $13\frac{1}{2}$ cents a pound.

SOLUTION.—

$$\begin{array}{rl}
17 \text{ lb. @ } 13¢ = \$\,2.21 & \\
\frac{3}{4} \text{ lb. @ } 13¢ = \quad .10 & \text{nearly} \\
17\frac{3}{4} \text{ lb. @ } \frac{1}{2}¢ = \quad \underline{.09} & \text{nearly} \\
\$\,2.40 & \text{Ans.}
\end{array}$$

NOTE.—The characters lb., @, and ¢ are, respectively, the symbols for *pounds*, *at*, and *cents*.

We first find the cost of 17 pounds at 13 cents a pound, then of $\frac{3}{4}$ of a pound at the same price. The sum of the two products will equal the cost of $17\frac{3}{4}$ pounds at 13 cents. We next find the cost of $17\frac{3}{4}$ pounds at $\frac{1}{2}$ cent. The entire sum is the cost of $17\frac{3}{4}$ pounds at $13\frac{1}{2}$ cents a pound. A clerk in making the calculations would merely set down the numbers 221, 10, and 9 as he computed them.

EXAMPLE 2.—Find, to the nearest cent, the cost of 12 pounds 7 ounces of chicken at 23 cents a pound.

SOLUTION.—

$$\begin{array}{rl}
12 \times .23 = \quad 2.76 & \\
\frac{7}{16} \times .23 = \quad \underline{.10} & \\
\$\,2.86 & \text{Ans.}
\end{array}$$

The cost of 12 pounds has been found first. An ounce is $\frac{1}{16}$ of a pound. The cost of 7 ounces at 23 cents could be found easily as follows: 8 ounces, or $\frac{1}{2}$ pound, will cost $11\frac{1}{2}$ cents, and 1 ounce will cost about $1\frac{1}{2}$ cents; hence, the cost of 7 ounces will be $1\frac{1}{2}$ cents less than $11\frac{1}{2}$ cents, or 10 cents.

22. A bookkeeper will find that in actual service he will need to use certain prices and quantities more than others. By the familiarity obtained from practice, he will soon learn "by heart" the products of the principal combinations used. He will, also, by a little ingenuity devise means of handling the less frequent cases presenting more than the average difficulty.

EXAMPLES FOR PRACTICE

1. Find the cost of $89\frac{2}{3}$ yards of silk velvet at $\$4\frac{3}{4}$ per yard.
Ans. $\$425\frac{11}{12}$

2. How much must be paid for $83\frac{1}{2}$ tons of hay at $\$16\frac{5}{8}$ a ton?
Ans. $\$1,388\frac{3}{16}$

3. How far can a man ride on a bicycle in $14\frac{5}{8}$ hours at the rate of $9\frac{3}{4}$ miles per hour?
Ans. $144\frac{5}{8}$ miles

4. Find the products of:

(*a*) $28\frac{7}{8}\times17\frac{2}{3}$.
(*b*) $44\frac{7}{9}\times16\frac{5}{8}$.
(*c*) $127\frac{4}{5}\times69\frac{2}{7}$.
(*d*) $86\frac{5}{12}\times78\frac{4}{5}$.
(*e*) $53\frac{3}{4}\times27\frac{1}{2}$.

Ans.—(*a*) $510\frac{1}{8}$; (*b*) $744\frac{31}{72}$; (*c*) $8,854\frac{5}{7}$, (*d*) $6,809\frac{19}{30}$; (*e*) $1,478\frac{1}{8}$.

---

23. To find the cost of articles sold by the hundred or the thousand.

EXAMPLE.—Find the cost of 1,635 pounds of sugar at $5.35 a hundred.

SOLUTION.— 1,635 equals $16\frac{35}{100}$ hundreds or 16.35 hundreds. Since each hundred costs $5.35, the entire cost is equal to the product of 16.35 and $5.35. The operation is performed as follows:

$$\begin{array}{r}
16.35 \\
5.35 \\
\hline
8175 \\
4905 \\
8175 \\
\hline
87.4725
\end{array}\text{ or \$87.47. Ans.}$$

24. Rule.—I. *To find the cost of articles sold by the hundred, point off two decimal places in the quantity and multiply by the price per hundred.*

II. *To find the cost of articles sold by the thousand, point off three decimal places in the quantity and multiply by the price per thousand.*

EXAMPLE 1.—A builder bought 28,960 feet of cypress lumber at $65.50 a thousand feet. What was the amount of the bill?

SOLUTION.—Point off three decimal places in 28,960 and multiply by the price.

$$\begin{array}{r} 28.960 \\ 65.50 \\ \hline 1448000 \\ 14480 \\ 17376 \\ \hline 1896.88000 \end{array} \text{ or } \$1,896.88. \quad \text{Ans.}$$

To check the result mentally we may find the cost of 30 thousand feet at $65. 30×65=$1,950. This result would indicate that there was no great error in the original solution such as that due to a misplaced decimal point.

The work may be checked, also, as follows:

$$\begin{array}{lr} \text{29 thousand feet at \$65.50} = & \$1899.50 \\ \text{40 feet at \$6.55 per hundred} = & 2.62 \\ \cline{2-2} & \$1896.88 \end{array}$$

EXAMPLE 2.—The tax in a certain city is $18.95 on each $1,000 of the valuation of property. What is the tax on a property valued at $11,200?

SOLUTION.— 11.2×$18.95=$212.24. Ans.

25. A bookkeeper is, of course, not expected to use several methods in the same class of problems, yet he should be able to select the best method for his particular class of work.

26. To find the cost of material sold by the ton of 2,000 pounds.

EXAMPLE 1.—What is the amount of the bill for 48,760 pounds of coal sold at $6.75 a ton?

SOLUTION.— $$48{,}760 \text{ pounds} = \frac{48.760}{2} = 24.38 \text{ tons.}$$

$$24.38 \times 6.75 = 164.565, \text{ or } \$164.57. \quad \text{Ans.}$$

The student will observe that pounds can be changed to tons of 2,000 pounds by pointing off three decimal places in the pounds and then dividing by 2.

Example 2.—Potatoes are sold in some localities by the ton. What should a farmer receive for 1,760 pounds of potatoes when the price is $14.75 a ton?

Solution.—The work may be set down as follows:

$$
\begin{array}{r}
2\,\underline{)\,1.760} \\
.88
\end{array}
\qquad\qquad
\begin{array}{r}
14.75 \\
\underline{.88} \\
11800 \\
\underline{11800} \\
12.9800
\end{array}
$$

The required amount is $12.98. Ans.

27. When prices are quoted by the ton of 2,000 pounds, it is sometimes most convenient to reckon the cost on the hundred pounds.

Example.—What is the cost of 740 pounds of poultry feed at $34.40 per ton?

Solution.—The price per hundred is $34.40 ÷ 20 = $1.72.

$$
\begin{array}{rrl}
7 \times \$1.72 = & \$12.04, & \text{the cost of 700 lb.} \\
.40 \times \$1.72 = & \underline{.69}, & \text{the cost of 40 lb.} \\
 & \$12.73 & \text{Ans.}
\end{array}
$$

The result may also be obtained as follows by the method given in Art. **26**.

$$
\begin{array}{r}
2\,\underline{)\,.740} \\
.37
\end{array}
\qquad\qquad
\begin{array}{r}
34.40 \\
\underline{.37} \\
24080 \\
\underline{10320} \\
12.7280
\end{array}
$$

Therefore, the cost of the feed will be $12.73. Ans.

EXAMPLES FOR PRACTICE

1. At $3.25 per hundred, what must be paid for 3,485 feet of Georgia pine? Ans. $113.26

2. What will be the cost of 1,400,345 bricks at $7.75 a thousand? Ans. $10,852.67

3. Find the retail price of 7,384 pounds of coal at $6.75 a ton. Ans. $24.92

CONSTRUCTION OF TABLES

28. A bookkeeper who is required to multiply by the same number frequently should make a table, giving the products of this number and each figure to 10 or more. For example, a coal dealer selling coal at $6, $6.50, and $6.75 per ton would arrange a table as shown below and thus save himself much labor.

Pounds	Price per Ton			Pounds	Price per Ton		
	$6.00	$6.50	$6.75		$6.00	$6.50	$6.75
100	.30	.33	.34	1,100	3.30	3.58	3.71
200	.60	.65	.68	1,200	3.60	3.90	4.05
300	.90	.98	1.01	1,300	3.90	4.23	4.39
400	1.20	1.30	1.35	1,400	4.20	4.55	4.73
500	1.50	1.63	1.69	1,500	4.50	4.88	5.06
600	1.80	1.95	2.03	1,600	4.80	5.20	5.40
700	2.10	2.28	2.36	1,700	5.10	5.53	5.74
800	2.40	2.60	2.70	1,800	5.40	5.85	6.08
900	2.70	2.93	3.04	1,900	5.70	6.18	6.41
1,000	3.00	3.25	3.38	2,000	6.00	6.50	6.75

Example.—Find by the table the cost of 1,565 pounds of coal at $6.75 a ton.

Solution.— 1500 pounds cost $5.06
60 pounds ($\frac{1}{10}$ of 600) cost .20
5 pounds ($\frac{1}{100}$ of 500) cost .02
1565 pounds cost $5.28 Ans.

29. By use of the table, a bill may be in excess of the exact amount by a cent, as one-half cent or more is called a whole cent. In case greater precision is required, the tables can be computed to tenths of a cent.

COMPLEX EXPRESSIONS

SYMBOLS OF AGGREGATION

30. The **vinculum** ——, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation**, and are used to include numbers that are to be considered together; thus, $13\times\overline{8-3}$, or $13\times(8-3)$ shows that 13 is to be multiplied by the difference between 8 and 3.

$$13\times(8-3)=13\times5=65$$
$$13\times\overline{8-3}=13\times5=65$$

When the vinculum or parenthesis is not used, we have

$$13\times8-3=104-3=101$$

31. When several numbers are connected by the signs +, −, ×, and ÷, the operations indicated by the multiplication and the division signs are performed first, and the operations indicated by the addition and the subtraction signs are performed in order from left to right.

EXAMPLE 1.—What is the value of $3+8-2+6$?

SOLUTION.— $3+8=11$; $11-2=9$; $9+6=15$. Ans.

EXAMPLE 2.—Simplify $26-5+8\times2$.

SOLUTION.—Here 8 is to be multiplied by 2, and the expression then becomes $26-5+16$.

$26-5=21$; $21+16=37$. Ans.

EXAMPLE 3.—What is the value of $8\times3\frac{1}{2}-24\div1\frac{1}{2}$?

SOLUTION.—We observe that 8 is to be multiplied by $3\frac{1}{2}$ and that 24 is to be divided by $1\frac{1}{2}$. $8\times3\frac{1}{2}=28$; $24\div1\frac{1}{2}=24\times\frac{2}{3}=16$. The expression may then be written $28-16$, and $28-16=12$. Ans.

The student in explaining a problem may need to use expressions such as are given here and he should follow the rules.

EXAMPLES FOR PRACTICE

Find the values of the following expressions:

(*a*) $(8+5-1)\div 4$.

(*b*) $5\times 24-32$.

(*c*) $5\times 24\div 15$.

(*d*) $144-5\times 24$.

(*e*) $(1{,}691-540+559)\div 3\times 57$.

(*f*) $2{,}080+120-80\times 4-1{,}670$.

Ans.—(*a*) 3; (*b*) 88; (*c*) 8; (*d*) 24; (*e*) 10; (*f*) 210.

REDUCTION OF COMPLEX FRACTIONS

32. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ denotes that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\dfrac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{3\times 7}{\dfrac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\dfrac{8+4}{16}$.

$\dfrac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4}\div\frac{3}{8}$.

33. It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show whether 9 is to be divided by $\frac{3}{8}$, or $\frac{9}{3}$ is to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line* is to be divided by *all below it.*

34. A fraction whose numerator or denominator is not a simple number is called a **complex fraction.** A complex fraction is reduced to its simplest form by reducing the numerator and the denominator to their simplest forms and then dividing the former by the latter.

EXAMPLE 1.—What is the value of $\dfrac{26\times 75\times 12}{1-.10}$?

SOLUTION.—The numerator is $26\times 75\times 12=23{,}400$; the denominator is $1-.10=.90$.

$$\frac{23{,}400}{.90}=26{,}000. \quad \text{Ans.}$$

EXAMPLE 2.—Simplify

$$\frac{13\tfrac{1}{3}\times 5\tfrac{1}{4}\times 12\tfrac{3}{8}}{3\tfrac{3}{8}\times 4\tfrac{1}{2}}$$

SOLUTION.—The numerator becomes

$$\frac{\overset{5}{\cancel{40}}}{\cancel{3}}\times\frac{\overset{7}{\cancel{21}}}{4}\times\frac{99}{\cancel{8}}=\frac{3,465}{4}$$

The denominator becomes $\frac{27}{8}\times\frac{9}{2}=\frac{243}{16}$.

The original expression now reduces to

$$\frac{3,465}{4}\div\frac{243}{16}=\frac{\overset{385}{\cancel{3,465}}}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{\underset{27}{\cancel{243}}}=\frac{1,540}{27}=57\tfrac{1}{27}.\quad\text{Ans.}$$

35. In solutions like the preceding, the work may be shortened by inverting each factor of the denominator instead of the product of the factors. Thus, inverting $\frac{27}{8}$ and $\frac{9}{2}$ and arranging the factors for continued multiplication

$$\frac{\overset{10}{\cancel{40}}}{\cancel{3}}\times\frac{\overset{7}{\cancel{21}}}{\cancel{4}}\times\frac{\overset{11}{\cancel{99}}}{\cancel{8}}\times\frac{\cancel{8}}{27}\times\frac{2}{\cancel{9}}=\frac{1,540}{27}, \text{ as before}$$

In solving problems of this kind a person must use his own judgment as to the best method. In some cases it is advisable to reduce common fractions to decimals, and in other cases common fractions can be profitably used throughout the operation.

EXAMPLES FOR PRACTICE

Simplify the following:

(*a*) $\frac{4\frac{2}{3}\times 10\frac{2}{5}\times 26\frac{1}{4}}{8\frac{2}{5}\times 4\frac{7}{8}\times 12\frac{3}{5}}$.

(*c*) $\frac{20\frac{2}{5}\times 34\frac{2}{3}\times 8\frac{5}{8}}{14\frac{1}{6}\times 3\frac{9}{10}\times 18\frac{2}{5}}$.

(*b*) $\frac{10\frac{2}{3}\times 12\frac{4}{7}\times 15\frac{3}{4}}{4\frac{7}{12}\times \frac{24}{35}\times 8\frac{2}{5}}$.

(*d*) $\frac{15\frac{3}{11}\times 32\frac{2}{3}\times 25\frac{3}{5}}{28\frac{4}{5}\times 38\frac{1}{4}\times 17\frac{7}{9}}$.

Ans.—(*a*) $2\frac{38}{81}$; (*b*) 80; (*c*) 6; (*d*) $\frac{5488}{8415}$.

CHECKING RESULTS

FINDING A RESULT APPROXIMATELY

36. The student in making calculations should be able to determine in some simple way whether or not his work is approximately correct. If he should find the amount of a bill of goods to be $500 when he knew from practical experience that such a bill would not be over $50, he would certainly realize that he had made a gross error. A bookkeeper, for example, who figures that 650 pounds of beef amounts to $600 will see that he is reckoning at a price of nearly $1 a pound, and that he has probably misplaced a decimal point. Many problems that involve considerable calculation to obtain the correct result can be checked up more or less roughly by the use of simple figures that are approximately correct. Thus, $26\frac{1}{2}$ pounds of metal at $18\frac{1}{4}$ cents a pound would amount to about $5, for at 20 cents the amount would be $5.30, and at 2 cents less per pound, that is at 18 cents, the amount would be $2\times 26\frac{1}{2} = 53$ cents less than $5.30, or $4.77. With a little further mental calculation the true result could be found. At $\frac{1}{4}$ cent, $26\frac{1}{2}$ pounds will cost 7 cents, to the nearest cent. The entire amount should be $4.77+.07=\$4.84$. Many an error on the part of an accountant can be discovered by a little reasoning, which will show that the result is impossible.

Example 1.—Find approximately by mental calculation the value of

$$\frac{12\frac{1}{4}\times 3\frac{3}{4}}{2\frac{7}{8}}.$$

Solution.— $12\frac{1}{4}\times 3\frac{3}{4}$ is nearly equal to $12\times 4=48$; $48\div 2\frac{7}{8}$ equals approximately $48\div 3=16$. Ans.

If the result obtained by pencil and paper should differ widely from 16, the fact would indicate a serious error.

EXAMPLE 2.—What will be the approximate cost to lay a sidewalk 239 feet long at 73 cents a foot?

SOLUTION.—Say, for easy figuring, that the length is 240 feet and that the price is 75 cents or $\$\frac{3}{4}$ a foot. Mentally, one would find that $\frac{3}{4}$ of 240 is 180; hence, the approximate cost is $180. Ans.

PROOFS BY CASTING OUT NINES

37. The remainder obtained by dividing a number by 9 may be found by adding the digits; if the sum contains more than one figure, add the figures of the sum; so continue until a sum is obtained that has but one figure. If this figure is 9, the remainder is 0; otherwise, it is the figure itself. Thus, take the number 6,877; this divided by 9 gives 764 for the quotient and 1 for the remainder. Applying the principle just stated, $6+8+7+7=28$; $2+8=10$; $1+0=1$, the remainder when 6,877 is divided by 9. For 26,478, we have $2+6+4+7+8=27$; $2+7=9$. This result means that the number 26,478 is a multiple of 9; hence, when it is divided by 9, the remainder is 0. This process is called **casting out nines**; it is so called because all the nines are rejected from the number, only the remainder being considered. Thus, in the first case above, 764 nines were rejected (cast out) from 6,877, and the remainder was 1.

If in adding the digits a 9 occurs, it need not be considered; or if two or more figures added together make 9, they may be neglected. For example, consider 364,982; here, since 3 and 6 is 9, we need consider only the figures 4, 8, and 2. For, $3+6+4+9+8+2=32$; $3+2=5$; also, $4+8+2=14$, and $1+4=5$, the same result.

38. The student will find the following test useful in determining whether the answer in multiplication is correct:

Find the sum of the digits in the multiplicand. If the sum consists of more than one figure, add the digits of the sum, and so continue until the sum is one figure. Do the same with the multiplier. Multiply together the final sums thus obtained, and if the result consists of more than one figure, add its digits *until one figure is obtained. If this result is the same as is obtained by adding the digits of the product until one figure is obtained, the work is probably correct.*

To illustrate, multiply 837,295 by 4,631.

SOLUTION.— *multiplicand* 837295
multiplier 4631
product 3877513145

PROOF.— $8+3+7+2+9+5=34;\ 3+4=7$
$4+6+3+1=14;\ 1+4=5$
$35;\ 3+5=8$
$3+8+7+7+5+1+3+1+4+5=44;\ 4+4=8$

The proof given is not absolute, because two or more errors might cause the product to fulfil the conditions of the test. But if, on trial, the final sum of the digits of the product does not agree with that of the product of the final sums of the multiplicand and the multiplier, it is certain that the work is incorrect.

39. The principle of casting out nines may be applied to addition. The number remaining after all the nines are cast out of all of the figures in the terms to be added should be the same as the remainder in the sum. Take the following for an illustration:

$$\begin{array}{r} 5368 \\ 7397 \\ 9243 \\ 2878 \\ \hline 24886 \end{array}$$

The remainder found by casting out the nines in the figures of the four terms to be added is 1. The same remainder is found in the sum; hence, the result is probably correct.

To prove subtraction, cast out the nines in the remainder and subtrahend and see whether the sum of the remainders in these terms equals the remainder in the minuend.

To prove division, cast out the nines in the divisor and the quotient; multiply together the remainders so obtained and add to the remainder in this product the figure left in the remainder after casting out the nines. The result should equal the remainder in the dividend. Observe the following proof.

```
2867)834000(290
     5734
     -----
     26060
     25803
     -----
       2570  remainder
```

The remainder in 2,867 is 5; the remainder in 290 is 2; 2×5=10; the remainder is 1; the remainder in 2,570 is 5; 1+5=6. The remainder in 834,000 is also 6. Hence, the division is probably correct.

PERCENTAGE

40. One of the most commonly used arithmetical terms is *per cent.* For illustration, the profits of a business, the changes in prices, the proportionate parts of a number, are usually expressed as so many per cent. One per cent. of a number is one hundredth of that number; two per cent. is two hundredths; etc. One per cent. of 100 is 1, one per cent. of 200 is 2. Two per cent. of 100 is 2, of 200 is 4. The symbol for per cent. is %; 5% is read 5 per cent.

41. That part of arithmetic which treats of the computation by hundredths is called **percentage.**

EXAMPLE.—Wages in a mill are to be increased 10%. How much increase will a person receive who is earning $2 a day?

SOLUTION.— 1% or one hundredth part of $2 is 2 ct., and 10% is 10×2=20 ct. The increase is, therefore, 20 ct. a day. Ans.

42. When the symbol % is written after a number and it is desired to drop the symbol, the number should be changed to a fraction whose denominator is 100. For example, 5% may be written either as a common or a decimal fraction, $\frac{5}{100}$ or .05.

As just illustrated, a number of per cent. may be written in any of these ways, and the student is cautioned against making the error of using two methods at the same time; thus, 5% should not be written .05%, as the latter means $\frac{5}{100}$ of 1%.

EXAMPLES FOR PRACTICE

The following examples are to be solved mentally. Write down your answers and when the work is completed compare your answers with those given at the end of the exercise.

Find: (*a*) 1% of 300.
(*b*) 5% of 300.
(*c*) 10% of 300.
(*d*) 25% of 300.
(*e*) 10% of 700.
(*f*) 15% of 400.
(*g*) 1% of 250.
(*h*) 4% of 250.
(*i*) 50% of 1,000.
(*j*) 100% of 600.
(*k*) $3\frac{1}{2}$% of 800.
(*l*) $3\frac{3}{4}$% of 1,200.
(*m*) 10% of 350.
(*n*) 150% of 400.
(*o*) 89% of 100.
(*p*) 1% of 1,800.
(*q*) 1% of 1,850.
(*r*) 1% of 70,000.

Ans.—(*a*) 3; (*b*) 15; (*c*) 30; (*d*) 75; (*e*) 70; (*f*) 60; (*g*) 2.5; (*h*) 10; (*i*) 500; (*j*) 600; (*k*) 28; (*l*) 45; (*m*) 35; (*n*) 600; (*o*) 89; (*p*) 18; (*q*) 18.5; (*r*) 700.

---

43. The number of which the hundredth is taken is called the **base.** The number denoting the hundredths considered is called the **rate.** The result obtained by taking the portion of the number denoted by the rate is called the **percentage.** Thus, in finding 5% of 200, we get 10 as the result. In this particular example, 200 is the base, 5% is the rate, and 10 is the percentage. The base is always considered to be 100%. One hundredth part of the base is 1% of it, two hundredths of the base is 2% of it, etc.

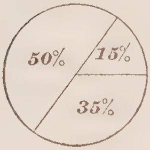

Fig. 1

44. The circle shown in Fig. 1 is divided into three parts. In the large portion there is $\frac{50}{100}$ of the circle, in the next smaller portion $\frac{35}{100}$, and in the smallest portion $\frac{15}{100}$ of the circle. The figure shows to the eye what relation 50%, 35%, and 15% bear, respectively, to the whole, or 100%.

Example.—A manufacturer bought 3,830 pounds of wool, in which there was a loss of 28% due to waste. How many pounds of waste were there in the lot?

Solution.— 1% of 3,830$=\frac{3830}{100}=$38.3 lb.
28% of 3,830$=28\times38.3=$1,072.4 lb. Ans.

Instead of finding 1% first, then 28%, the result can be obtained directly by multiplying 3,830 by .28. Thus:

$$\begin{array}{r} 3830 \\ .28 \\ \hline 30640 \\ 7660 \\ \hline 1072.40 \text{ lb.} \quad \text{Ans.} \end{array}$$

45. Rule.—*To find the percentage when the base and rate are given, write the rate as a decimal and then multiply the base by the rate.*

EXAMPLES FOR PRACTICE

What is:

(*a*) 7% of 1,860?

(*b*) 15% of 1,326?

(*c*) $3\frac{3}{4}$% of 1,850?

Ans.—(*a*) 130.2; (*b*) 198.9; (*c*) 69.375.

1. Ten years ago a city had a population of 125,000. The population has increased 12% since that time. How many inhabitants has it now? Ans. 140,000

Note.—Find the increase and add it to the given population.

2. A railroad's receipts in a month were $567,800, of which 72% was for freight. What were the receipts for freight? Ans. $408,816

ALIQUOT PARTS IN PERCENTAGE

46. When the rate per cent. is an aliquot part of 100 per cent., it is usually more convenient to use the equivalent fraction in computations. The principal aliquot parts are shown in the following table:

Per Cent.	Decimal	Fraction	Per Cent.	Decimal	Fraction
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	$8\frac{1}{3}$%	.08$\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$12\frac{1}{2}$%	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$16\frac{2}{3}$%	.16$\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$33\frac{1}{3}$%	.33$\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$ or $\frac{1}{3}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$37\frac{1}{2}$%	.37$\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$ or $\frac{3}{8}$
75 %	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$62\frac{1}{2}$%	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$
$6\frac{1}{4}$%	.06$\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$ or $\frac{1}{16}$	$87\frac{1}{2}$%	.875	$\frac{87\frac{1}{2}}{100}$ or $\frac{7}{8}$

EXAMPLE 1.—What is $16\frac{2}{3}$% of 684?

SOLUTION.— $16\frac{2}{3}\% = \frac{1}{6}$. $\frac{1}{6}$ of $684 = 114$. Ans.

EXAMPLE 2.—A furniture dealer paid $48 for a dozen chairs which he sold at a profit of 25% on the purchase price. What was his profit and what price did he receive?

SOLUTION.— $25\% = \frac{1}{4}$.
$\frac{1}{4}$ of \$48=\$12, the profit. Ans.
Selling price was \$48+\$12=\$60. Ans.

The student should become familiar with the most common aliquot parts of 100, and use the fractions wherever possible in percentage computations.

EXAMPLES FOR PRACTICE

Solve the following by aliquot parts:

(*a*) What is 50% of 1,964?
(*b*) What is $33\frac{1}{3}$% of $630?
(*c*) What is $6\frac{1}{4}$% of 1,760?
Ans.—(*a*) 982; (*b*) $210; (*c*) 110.

AMOUNT AND DIFFERENCE

47. The **amount** is the sum of the base and the percentage.

48. The **difference** is the remainder obtained when the percentage is subtracted from the base.

49. The terms amount and difference are ordinarily used when there is an increase or a decrease in the base. For example, suppose the population of a village is 1,500 and it increases 25 per cent. This means that for every 100 of the original 1,500 there is an increase of 25, or a total increase of 15×25 =375. This increase added to the original population gives the *amount,* or the population after the increase. If, on the other hand, the population decreases 375, the final population is 1,500−375=1,125, and this is the *difference.* The original population, 1,500, is the base on which the percentage is computed; the 25% is the rate, and the increase or decrease, 375, is the percentage. If the base increases, the final value is the amount, and if it decreases, the final value is the difference.

50. To find the relations existing between the amount or the difference and the base and the rate, let us consider an example.

Example.—In a factory where 2,100 men are employed, the force is increased 8%. How many new men are employed, and how many men are at work after the increase?

Solution.— 8% of 2,100=2,100×.08=168 men=number of new men employed. Ans. The total number of men after the force is increased is 2,100+168=2,268 men. Ans.

In this example, the original number, 2,100, is the base, and the final number, 2,268, is the amount. For every 100 men originally in the shop, there are 8 more men, or 108 men, after the force is increased.

We may consider the original force as 100%, and, as the increase is 8% of the original, the total force is 100%+8% =108%. 108%=1.08.

Total force is 1.08×2,100=2,268. Ans.

51. If, in the above example, the force had been decreased 8%, the number of men thrown out of work would have been 168, and those left at work, $2{,}100-168=1{,}932$. As before, the base is 2,100, but the final number, 1,932, is the difference, since the base has decreased. Out of every 100 men formerly at work, 8 have been discharged, leaving $100-8=92$ still at work. Since 8% of the force has been thrown out of work, $100\%-8\%=92\%$ remains. The force that remains equals $.92\times2{,}100=1{,}932$, the same result as before.

52. In the case of small fractions of a per cent. care should be observed to read the number correctly. For example, .01% is read $\frac{1}{100}$ of 1%, and equals $\frac{1}{100}$ of $\frac{1}{100}=\frac{1}{10000}$, or .0001. Similarly, .5% or 0.5% is read $\frac{5}{10}$ of 1%. If the number of per cent. is expressed as a common fraction, it is read in a similar manner. Thus, $\frac{1}{8}\%$ is read $\frac{1}{8}$ of 1%, and equals $\frac{1}{8}$ of $\frac{1}{100}=\frac{1}{800}$, or .00125.

EXAMPLES FOR PRACTICE

Solve the following:
(*a*) What is $12\frac{1}{2}\%$ of $900?
(*b*) What is $\frac{4}{5}\%$ of 627?
(*c*) What is $33\frac{1}{3}\%$ of 54?
Ans.—(*a*) $112.50; (*b*) 5.016; (*c*) 18.

1. If gunpowder contains 75% of saltpeter, 10% of sulphur, 15% of charcoal, how much of each is there in 2,000 pounds of powder?

Ans. { Saltpeter, 1,500 lb.
Sulphur, 200 lb.
Charcoal, 300 lb.

2. A man owning a ship worth $225,000, sells $\frac{1}{4}$ of it to A, 20% of the remainder to B, and 35% of what then remains to C. How much do A, B, and C each pay for their shares?

Ans. { A, $56,250
B, $33,750
C, $47,250

3. A house which cost $4,000 was sold for 7% less than it cost. What was the selling price? Ans. $3,720

4. A school having 200 pupils lost during the year $12\frac{1}{2}\%$ of the number. How many pupils were left? Ans. 175

5. The receipts of a business in 1 year were $25,346.90. During the next year the receipts were increased 16%. What were the receipts for that year? Ans. $29,402.40

COMMISSION AND BROKERAGE

53. **Commission,** or **brokerage,** is the sum paid an agent for transacting business for another person; as, for buying or selling merchandise or property, for collecting or investing money, etc.

54. The agent or party who transacts the business is called a **commission merchant,** or **broker;** the party for whom the business is transacted is called the **principal.** The term broker is applied to one who sells and buys stocks, bonds, bills of exchange, and money securities.

55. A **consignment** is a shipment of goods from one party to another; the party that ships the goods is called the **consignor,** or **shipper,** and the party to whom they are shipped is called the **consignee.**

56. When goods are sold on credit, the agent charges an additional amount for guaranteeing the payment of the sale. This extra charge is called **guaranty.**

57. The **gross proceeds** of a sale or collection is the total amount realized by the agent before deducting his commission and other expenses connected with the transaction. The **net proceeds** is the amount due the principal after the commission and all other charges have been deducted.

58. An **account sales** is a detailed statement made by the agent to his principal, showing the goods sold and the prices obtained, giving a list of the charges and expenses, and the net proceeds due the principal. The charges include freight, cartage, storage, insurance, inspection, advertising, commission, and guaranty.

59. The **prime cost** of a purchase is the sum paid by the agent for the goods or property. The **gross cost** is the prime cost plus the commission and expenses incident to the purchase.

60. An **account purchase** is a detailed statement made by the agent to his principal, showing the cost of goods or property bought, the expenses attending the purchase, and the gross cost.

61. The commission, or brokerage, is usually computed at a certain per cent. of the gross proceeds of a sale or the prime cost of a purchase. In some cases, however, it is computed at a certain price per unit of weight or measure; as, so much per ton, per bushel, or per barrel. Examples in commission are solved by the rules of percentage. Either the gross proceeds or the prime cost is the *base;* the net proceeds is the *difference;* the gross cost is the *amount;* the commission is the *percentage;* and the rate of commission is the *rate.* The remittance from the principal to the purchasing agent, including both the investment and the commission, is an *amount.* The following rule is derived directly from the principles of percentage:

62. Rule.—*To find the commission, multiply the prime cost or the selling price by the rate of commission.*

Example.—A real estate agent sells a house and lot for $4,375 and receives 2% commission. What is the commission and what is the net proceeds?

Solution.—Commission=selling price×rate=$4,375×.02=$87.50. Ans.

Net proceeds=selling price−commission=$4,375−$87.50=$4,287.50. Ans.

EXAMPLES FOR PRACTICE

What is the commission:

(*a*) If the gross proceeds is $300 and the rate of commission is $3\frac{1}{2}$%?

(*b*) If the gross proceeds is $9,375 and the rate of commission is 2%?

(*c*) If the prime cost is $831.75 and the rate of commission is $1\frac{1}{4}$%?

(*d*) If the prime cost is $960 and the rate of commission is $\frac{5}{8}$%?

Ans.—(*a*) $10.50; (*b*) $187.50; (*c*) $10.40; (*d*) $6.

1. A commission merchant sold a quantity of wool for $4,650. He charged $2\frac{1}{2}$% commission, 2% guaranty, and the transportation, storage, and other expenses amounted to $184. How much should he send his principal? Ans. $4,256.75

INSURANCE

63. **Insurance** is a contract by which one party, the **underwriter,** or **insurer,** agrees, for a consideration, to make good a loss sustained by another party.

64. Insurance is of two kinds, **property insurance** and **personal insurance.** Property insurance includes *fire insurance* (indemnity for loss or damage by fire); *marine insurance* (indemnity for losses at sea); *transit insurance* (indemnity for loss of, or damage to, merchandise during transportation); *stock insurance* (indemnity for loss of live stock); and *accident insurance* (indemnity for breakage of fragile materials, as plate glass, etc.).

Personal insurance includes *life insurance,* which secures the payment of a certain amount to a specified person at the death of the party insured, or after the lapse of a specified time; *accident insurance,* which secures the payment of a certain sum in case of accident to the insured; *health insurance,* which secures the payment of a weekly sum during sickness; and insurance against the dishonesty of employes.

65. A **policy** is a written contract between the insurance company and the party insured; it contains a description of the property insured, the conditions on which the insurance is taken, and the amount to be paid in case of loss.

66. A **premium** is the amount paid to the insurer for assuming the risk of loss or damage. The premium is a certain per cent. of the amount of insurance, as $\frac{5}{8}\%$, $\frac{3}{4}\%$. The rate of premium depends on the nature of the risk and on the length of time the insurance has to run. It is customary to speak of the rate of premium as the cost per $100 of insurance; as 60 cents per $100, $1.20 per $100, etc.

67. In property insurance, all computations are based on the rules of percentage. The amount of insurance is the *base,* the premium is the *percentage,* and the rate of premium is the *rate.*

68. Rule.—*To find the premium, multiply the amount of insurance by the rate of premium.*

EXAMPLE.—A house and furniture are insured against fire for $3,250, the rate of premium being $\frac{3}{4}\%$, or 75 cents per $100 per year. What is the yearly premium?

SOLUTION.— $\frac{3}{4}\% = .0075$. Using the rule,
Premium $= \$3,250 \times .0075 = \$24.37\frac{1}{2}$. Ans.

EXAMPLES FOR PRACTICE

1. A store and contents valued at $16,400 are insured for $\frac{3}{5}$ of their value at $1\frac{1}{8}\%$ premium. What is the cost per year of insurance?
Ans. $110.70

2. A boat load of 8,600 bushels of corn, worth 32 cents per bushel, is insured for $\frac{3}{4}$ of its value at $1\frac{5}{8}\%$ premium. If the corn is totally destroyed, what will be the owner's loss? Ans. $721.54

3. A building worth $9,600 is insured for $\frac{3}{4}$ of its value in three companies. The first company takes $\frac{1}{3}$ of the risk at $\frac{1}{2}\%$ premium; the second $\frac{2}{5}$ of it at $\frac{3}{4}\%$; and the third the remainder at $\frac{7}{8}\%$. What is the total premium? Ans. $50.40

4. A fire insurance company took a risk of $42,000 at $\frac{3}{4}\%$ premium, and reinsured $\frac{1}{3}$ of it in another company at $\frac{1}{2}\%$, and $\frac{1}{2}$ of it in a third company at $\frac{5}{8}\%$. What did the company gain by reinsuring?
Ans. $61.25

INTEREST

SIMPLE INTEREST

69. Interest is money paid for the use of money belonging to another.

70. The **principal** is the sum for which interest is paid.

71. The **rate of interest** is the per cent. of the principal that is paid for its use for a given time, usually a year.

72. The **amount** is the sum of the principal and interest. If, for example, $100 is loaned for a year at 6 per cent. interest, the principal is $100, the rate is 6 per cent., the interest is 6 per cent. of $100 or $6, and the amount is $\$100 + \$6 = \$106$. If the principal is loaned for 2 years, the interest is $2 \times \$6 = \$12$, and the amount $\$100 + \$12 = \$112$.

73. The **legal rate** is the rate established by law.

74. **Usury** is a rate that exceeds the legal rate. The penalty for usury is, in some states, the forfeiture of all interest; in others, the forfeiture of both principal and interest. In a number of states, no legal notice is taken of usury.

75. The finding of interest is the direct application of the principle of percentage when the base and rate are given.

EXAMPLE 1.—Find the interest for 1 year on $1,000 at 5%.

SOLUTION.—

$$\begin{array}{r} \$1000 \\ .05 \\ \hline \$50.00 \end{array} \text{ Ans.}$$

EXPLANATION.— 5% expressed as a decimal is .05. The interest is found by multiplying the principal, which is $1,000, by .05, the rate.

EXAMPLE 2.—Find the interest on $1,275.50 for 3 years at 6%.

SOLUTION.—

$$\begin{array}{rl} \$1275.50 & \\ .06 & \\ \hline 76.5300 & = \text{Interest for 1 yr.} \\ 3 & \\ \hline \$229.59 & = \text{Interest for 3 yr. Ans.} \end{array}$$

The example may be solved also by finding the interest on $1 for 1 year, then for 3 years, and multiplying this by the principal. Thus,

$$.06 \times 3 \times 1{,}275.50 = \$229.59. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Find mentally the interest on:

(*a*) $300 for 1 yr. at 5%.
(*b*) $3,000 for 1 yr. at 5%.
(*c*) $700 for 1 yr. at 4%.
(*d*) $700 for 2 yr. at 4%.
(*e*) $2,000 for 4 yr. at 6%.

Solve the following on paper:

Find the interest on:

(*f*) $725 for 1 yr. at 5%.
(*g*) $4,200 for 3 yr. at $4\frac{1}{2}$%.
(*h*) $3,762.87 for 5 yr. at 5%.
(*i*) $2,617.75 for $2\frac{1}{2}$ yr. at 3%.

Ans.—(*a*) $15; (*b*) $150; (*c*) $28; (*d*) $56; (*e*) $480; (*f*) $36.25; (*g*) $567; (*h*) $940.72; (*i*) $196.33.

76. To find the interest when the time is in months.

EXAMPLE.—What is the interest on $900 at 5% for 7 months?

SOLUTION.—
$900
.05
12)45.00=Interest for 1 yr.
3.75=Interest for 1 mo.
7
$26.25=Interest for 7 mo.

EXPLANATION.—The interest for 7 months equals 7 times that for 1 month. The interest for 1 month is found by dividing the interest for 1 year by 12.

EXAMPLES FOR PRACTICE

Find the interest on:
(*a*) $825 for 6 mo. at 6%.
(*b*) $930 for 2 mo. at 4½%.
(*c*) $876.27 for 11 mo. at 10%.
(*d*) $572.84 for 1 yr. 5 mo. at 5%.
Ans.—(*a*) $24.75; (*b*) $6.98; (*c*) $80.32; (*d*) $40.58.

77. To find the interest for a number of days.

In computing interest, a year is usually regarded as consisting of 12 months of 30 days each.

EXAMPLE.—What is the interest on $800 for 21 days at 6%?

SOLUTION.—
$800
.06
12)48.00 =Interest for 1 yr.
30)4.00 =Interest for 1 mo.
.1333=Interest for 1 da.
21
1333
2666
$2.7993

To the nearest cent, the interest is $2.80. Ans.

78. The interest for a number of days can also be found by considering the days as that many 360ths of a year and finding that fraction of the yearly interest. In the preceding example, the interest for 21 days equals $\frac{21}{360}$ of $48 = $2.80. Most persons who do not have occasion to compute interest frequently can check their calculations more readily and feel rather more sure of their results by obtaining the interest for a month and then for the fraction of a month.

Example.—Find the interest on $248.80 for 5 years 9 months 29 days at 5%.

```
Solution.—           $248.80
                         .05
                12)12.4400=Interest for 1 yr.
                 30)1.037 =Interest for 1 mo.
                     .034 =Interest for 1 da.
         5×12.44=$62.20   =Interest for 5 yr.
         9×1.037=   9.333=Interest for 9 mo.
        29× .034=    .986=Interest for 29 da.
                 $72.519   or $72.52. Ans.
```

The interest is found for 1 year, 1 month, and 1 day, respectively, and the total for 5 years 9 months and 29 days is computed as shown.

EXAMPLES FOR PRACTICE

Find the interest on:
(*a*) $600 for 1 yr. 4 mo. 15 da. at 6%.
(*b*) $2,160 for 2 yr. 2 mo. at 5%.
(*c*) $1,800 for 7 mo. 20 da. at 4½%.
(*d*) $725.50 for 7 yr. 5 mo. 23 da. at 7½%.
Ans.—(*a*) $49.50; (*b*) $234; (*c*) $51.75; (*d*) $407.03.

SIXTY-DAY METHOD

79. Among accountants one of the most popular methods of computing interest is known as the sixty-day method. In business dealings generally, loans are made for 30 days, 60 days, or some multiple of these. The sixty-day method meets with favor, as by its use the interest for such periods can be quickly reckoned.

80. Since 60 days is considered as $\frac{1}{6}$ of a year, the interest on $1 for 60 days at 6% is $\frac{1}{6}$ of 6 cents, or 1 cent. The interest on any sum for 60 days at 6% is, therefore, equal to 1 hundredth of the principal. Hence, *if the decimal point of any sum be moved two places to the left, it will give the interest on that sum for 60 days at 6%.*

Thus, the interest on $3,472.75 for 60 days at 6% is $34.73 −, and on $692 it is $6.92.

Example.—Find the interest on $4,268 for 30 days at 6%.

Solution.—The interest on $4,268 for 60 da. at 6% is $42.68. For 30 da. it is one-half of that for 60 da.; ½ of $42.68=$21.34. Ans.

81. In finding the interest by the sixty-day method for any number of days, the work can be shortened and arranged before the eye in an orderly manner by finding the sum or the difference of the aliquot parts of 60 days that will make the given number. For example, to find the interest for 35 days, we would observe that 35 days equals the sum of 30 days and 5 days, each of which is an aliquot part of 60 days, and compute the interest for these separately.

Example 1.—Find the interest on $980 for 35 days.

Solution.— 2) $9.80=Interest for 60 da.
6) 4.90=Interest for 30 da.
.82=Interest for 5 da.
$5.72 Ans.

Explanation.—The interest for 30 days is ½ of that for 60 days, and the interest for 5 days is ⅙ of that for 30 days.

Example 2.—Find the interest on $8,368 for 99 days at 6%.

Solution.—
$83.68 =Interest for 60 da.
41.84 =Interest for 30 da. = ½ of that for 60 da.
8.368=Interest for 6 da. = $\frac{1}{10}$ of that for 60 da.
4.184=Interest for 3 da. = ½ of that for 6 da.
$138.072=Interest for 99 da. Ans.

82. In case the interest rate is other than 6%, the interest is first calculated at that rate, and then a proportionate part is added or subtracted corresponding to the difference between the given rate and 6%. The following partial table will illustrate:

Interest at
7 % = Interest at 6% plus $\frac{1}{6}$ of itself.
8 % = Interest at 6% plus $\frac{1}{3}$ of itself.
9 % = Interest at 6% plus $\frac{1}{2}$ of itself.
5 % = Interest at 6% less $\frac{1}{6}$ of itself.
4 % = Interest at 6% less $\frac{1}{3}$ of itself.
4$\frac{1}{2}$% = Interest at 6% less $\frac{1}{4}$ of itself.

Example 1.—What is the interest at 9% on $1,264.76 for 49 days?

Solution.—

$12.6476 = Interest for 60 da. at 6%.
6.324 = Interest for 30 da. = $\frac{1}{2}$ of that for 60 da.
3.162 = Interest for 15 da. = $\frac{1}{2}$ of that for 30 da.
.632 = Interest for 3 da. = $\frac{1}{10}$ of that for 30 da.
.211 = Interest for 1 da. = $\frac{1}{3}$ of that for 3 da.
$10.329 = Interest for 49 da. at 6%.
5.165 = Interest for 49 da. at 3%.
$15.494 = Interest for 49 da. at 9%. Ans.

Example 2.—What is the interest at 4$\frac{1}{2}$% on $3,000 for 98 days?

Solution.—

$30.00 = Interest for 60 da. at 6%.
15.00 = Interest for 30 da. at 6% = $\frac{1}{2}$ of that for 60 da.
3.00 = Interest for 6 da. at 6% = $\frac{1}{5}$ of that for 30 da.
1.00 = Interest for 2 da. at 6% = $\frac{1}{3}$ of that for 6 da.
$49.00 = Interest for 98 da. at 6%.
12.25 = Interest for 98 da. at 1$\frac{1}{2}$% = $\frac{1}{4}$ of that at 6%.
$36.75 = Interest for 98 da. at 4$\frac{1}{2}$%. Ans.

Example 3.—Find the interest on $258.50 for 29 days at 6%.

Solution.—

$2.585 = Interest for 60 da.
1.293 = Interest for 30 da. = $\frac{1}{2}$ of that for 60 da.
.043 = Interest for 1 da. = $\frac{1}{30}$ of that for 30 da.
$1.25 = Interest for 29 da. Ans.

EXAMPLES FOR PRACTICE

By the sixty-day method, find the interest on:

1. $8,000 for 87 days at 6%.
2. $6,050 for 96 days at 3%.
3. $875.28 for 77 days at 3$\frac{1}{2}$%.

4. $1,468.80 for 123 days at 4%.
5. $23,750 for 108 days at 4½%.
6. $42,690 for 176 days at 3⅔%.
7. $7,200 for 225 days at 5%.
8. $468.24 for 101 days at 5½%.
9. $6,880 for 186 days at 7%.
10. $7,600 for 143 days at 7½%.

Ans.—(1) $116; (2) $48.40; (3) $6.55+; (4) $20.07+; (5) $320.625; (6) $765.26−; (7) $225; (8) $7.23−; (9) $248.83−; (10) $226.42−.

COMMERCIAL CALCULATIONS

(PART 2)

APPLICATIONS OF PERCENTAGE

1. To find the base when the rate and percentage are given.

Example 1.—If 3% of a number is 12, what is the number?

Solution.— 3%=12; 1%=$\frac{1}{3}$ of 12=12÷3=4.
100%=100×4=400. Ans.

Explanation.— 1% is evidently $\frac{1}{3}$ as great as 3%. Having found 1% of the number, we find 100%, or the entire number, by multiplying 1% of it by 100.

Example 2.—A mechanic received an increase of 30 cents a day in his salary, which was 15% of his former pay. What pay had he been receiving?

Solution.— 15%=30 cents; 1%=$\frac{1}{15}$ of 30 cents=2 cents.
100%=100×2 cents=$2.00. Ans.

In this example 15% is the rate, 30 cents is the percentage, and the base or 100% is to be found.

EXAMPLES FOR PRACTICE

Solve the following mentally:
(*a*) 25 is 5% of what number?
(*b*) 40 is 20% of what number?
(*c*) 4 is $\frac{1}{2}$% of what number?
(*d*) 7 is 2% of what number?
(*e*) 81 is 90% of what number?
(*f*) 50 is 100% of what number?
(*g*) 500 is 25% of what number?

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(*h*) 800 is 200% of what number?

(*i*) The rate is 12%, the percentage is 96; what is the base?

Ans.—(*a*) 500; (*b*) 200; (*c*) 800; (*d*) 350; (*e*) 90; (*f*) 50; (*g*) 2,000; (*h*) 400; (*i*) 800.

2. The illustrations just given show that when the percentage and rate are known the base can be found by dividing the percentage by the number of hundredths which equals the rate and then multiplying the result thus obtained by 100. The following rule gives a more direct method.

3. Rule.—*To find the base, divide the percentage by the rate expressed decimally.*

Example 1.— 45 is 15% of what number?

Solution.— 15%=.15.

.15) 45.00 (300 Ans.
45
00

The division is performed according to the rule for division of decimals.

Example 2.—In a certain village there are 308 children, who form 28% of the total population. How many inhabitants are there in the village?

Solution.— 28%=.28.

.28) 308.00 (1100. Ans.
28
28
28
00

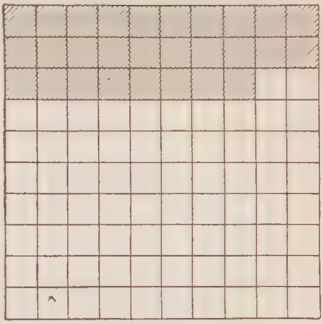


Fig. 1

4. The conditions stated in the last example may be illustrated by the aid of Fig. 1, which is divided into 100 parts. Each small division is 1% of the square. The shaded part covers 28 small divisions, or 28% of the entire square. The entire figure represents the total population and the shaded portion represents the number of children. Since the value of 28 parts is known, the value of 100 parts can be obtained by dividing the known number by .28.

5. Aliquot parts of 100 can often be used to advantage in finding the base when the rate and percentage are known.

Example 1.— 40 is $16\frac{2}{3}\%$ of what number?

Solution.— $16\frac{2}{3}\%=\frac{1}{6}$.
$\frac{1}{6}$ of a number=40
$\frac{6}{6}$ of the number=6×40=240. Ans.

Example 2.—In a fire a merchant lost 40% of his stock of goods. If his loss was estimated to be $8,700, what was the estimated value of his entire stock?

Solution.— $40\%=\frac{2}{5}$.
$\frac{2}{5}$ of the stock=$8,700
$\frac{1}{5}$ of the stock=$\frac{1}{2}$ of $8,700=$4,350
$\frac{5}{5}$ of the stock=5×$4,350=$21,750. Ans.

EXAMPLES FOR PRACTICE

1. The percentage is 725 and the rate is 29%. What is the base?
Ans. 2,500

2. The percentage is 1,326 and the rate is 32%. What is the base?
Ans. $4,143.75

3. In the manufacture of certain goods, 43% of the cost is for material. What is the entire cost of the goods if the cost for material is $2,800? Ans. $6,511.63

4. The receipts of a business place this year were $128,276, which is 92% of the receipts of last year. What were last year's receipts?
Ans. $139,430.43

---

6. To find the rate when the base and percentage are known.

Example 1.—A farmer owning 200 fowls sold 20 of them. What per cent. did he sell?

Solution.— 1% of 200=2.
20÷2=10%. Ans.

Explanation.—We first find 1% of the entire number to be 2. Since 2=1%, he sold as many per cent. as the number of times that 2 is contained in 20.

Example 2.—What per cent. of 500 is 75?

Solution.— 1% of 500=5.
$75=\frac{75}{5}$, or 15% of 500. Ans.

EXAMPLES FOR PRACTICE

Solve mentally the following examples:

What per cent. of

(*a*) 500 is 200?
(*b*) 500 is 400?
(*c*) 700 is 28?
(*d*) 400 is 30?
(*e*) 100 is 6?
(*f*) 100 is $\frac{1}{4}$?
(*g*) 300 is 600?
(*h*) 200 is 300?

Ans.—(*a*) 40%; (*b*) 80%; (*c*) 4%; (*d*) $7\frac{1}{2}$%; (*e*) 6%; (*f*) $\frac{1}{4}$%; (*g*) 200%; (*h*) 150%.

---

7. When the base and percentage are given, instead of finding the rate step by step as shown, in actual practice it would be found by the following rule:

8. Rule.—*To find the rate, divide the percentage by the base.*

EXAMPLE 1.—What per cent. of 240 is 60?

SOLUTION.—

```
240 ) 60.00 ( .25=25%  Ans.
      480
      1200
      1200
```

EXPLANATION.—It is evident that 60 is $\frac{60}{240}$ of 240, since 1 is one 240th part of 240. We find that $\frac{60}{240}=.25$, which equals 25%. In this example, 240 is 100%, or the base; 60 is the percentage; and the rate is to be found.

It must be observed that when the rate is to be expressed with the words per cent. or with the symbol % the decimal point is moved two places to the right.

EXAMPLE 2.—In a city of 24,367 inhabitants, 2,876 are unable to read. What percentage of the population cannot read?

SOLUTION.—

```
24367 ) 2876.000 ( .118+=11.8%.  Ans.
        24367
         43930
         24367
        .195630
         194936
```

EXPLANATION.— 24,367 is the base. This is the number on which the percentage is to be figured; 2,876 is the percentage. The division is made according to the rule for division of decimals.

9. In actual practice the student may be at a loss to know how far to carry the division in problems similar to that given in Art. **8.** This is a question to be determined by the circumstances of the case. In the preceding example, if the information is for a table to show the relative illiteracy in various cities, two decimal places would probably be sufficient and the result would be called 12%. If, however, the percentage is to be used again in some exact calculation, the division should be carried to a greater number of figures.

Example.—In shipping freight from one city to another, the freight passes over two different railroads. The distance on the first road is 136 miles, and on the second is 280 miles. What percentage of the freight charges is due to each of the roads?

Solution.— Total mileage is 136+280=416 mi.
136÷416=.327=32.7% for first road. Ans.
280÷416=.673=67.3% for second road. Ans.

Explanation.—The total mileage, or 416 miles, is considered to be 100%. The percentage that each distance is of 416 is then found.

10. The only difficulty that problems in percentage give is in determining what the given quantities stand for or represent.

Example 1.—What per cent. of 40 is 30?

Solution.— 30 is $\frac{30}{40}$ of 40.
$\frac{30}{40}=\frac{3}{4}=.75$, or 75%. Ans.

Explanation.—In this case, 30 is to be measured in hundredths of 40. Therefore, 40 is the base. The rate is to be found.

Example 2.—What per cent. of 30 is 40?

Solution.— 40 is $\frac{40}{30}$ of 30.
$\frac{40}{30}=\frac{4}{3}=1.33\frac{1}{3}$, or $133\frac{1}{3}$%. Ans.

The unit of measure is one hundredth part of 30; evidently, 30 is the base.

Example 3.—The price of a line of goods has advanced 25%. The present price is $6.25 a dozen. What was the former price?

Solution.—The increase of 25% was made on the former price. The former price is 100%; the present price, $6.25, equals the former price plus the advance, or 100%+25%=125%. $6.25 is, therefore, the percentage and 125% is the rate.

125%=1.25; base=$6.25÷1.25=$5. Ans.

To prove the result, we may add to $5, 25% of itself.

25% of $5=$1.25; $5+$1.25=$6.25. Ans.

Example 4.—A fruit dealer bought a lot of oranges. After losing 18% of the lot on account of cold weather, he had 656 boxes left. How many boxes did he buy?

Solution.—The loss is based on the amount purchased, which we shall call 100%. He loses 18%, and he has left 100%−18%=82%. It is now seen that 656 is the percentage and 82% is the rate.

82%=656; 100%=656÷.82=800. Ans.

11. The solutions of the foregoing examples should be studied very carefully, as they illustrate typical cases and as similar problems will be met frequently later in this Section.

In solving any indirect problem in percentage, the student should carefully analyze the statement of the problem to determine what quantity represents 100% and whether this quantity is known or is to be computed.

EXAMPLES FOR PRACTICE

1. A man's salary is $1,800 per year and he saves $225. (*a*) What per cent. of his salary does he save? (*b*) What per cent. of it does he spend?

Ans. $\begin{cases} (a)\ 12\frac{1}{2}\% \\ (b)\ 87\frac{1}{2}\% \end{cases}$

2. A man has 32% of his money invested in stocks, 18% in grain, and the remainder, which is $7,620, in real estate. What is the total value of his property? Ans. $15,240

3. If wool loses 32% of its weight in washing, how many pounds of unwashed wool are required to produce 35,360 pounds of washed wool? Ans. 52,000 lb.

4. In 1910 the population of a city was 85,000, which was 36% more than the population in 1900. What was the population in 1900? Ans. 62,500

5. A man bequeathed to a charity 32% of his estate. To another charity he gave $23,100, which was 23% less than the amount given to the first charity. (*a*) What was the value of the estate? (*b*) What per cent. of the estate was given to the second charity?

Ans. $\begin{cases} (a)\ \$93{,}750 \\ (b)\ 24{,}64\% \end{cases}$

PROFIT AND LOSS

12. **Profit and loss** treats of the gains or losses arising in business transactions.

If the price for which merchandise is sold is greater than the cost of the merchandise, the difference is *profit*, or *gain*. If the selling price is less than the cost, the difference is *loss*.

13. The **gross cost** of merchandise is its first cost plus the expenses of purchase, transportation, and storage. Such expenses are commission, freight, insurance, drayage, etc.

14. The **net selling price** is the gross selling price, less all discounts and expenses of sale.

15. Computation in profit and loss are made according to the rules of percentage. In this Section, unless statement is made to the contrary, the gross cost of the merchandise is considered to be the *base*, upon which the *rate* of profit or loss is computed. The profit or loss is the *percentage*. If the merchandise is sold at a profit, the net selling price is the *amount*; if at a loss, the net selling price is the *difference*.

16. **Rule.**—*To find the profit or loss, multiply the gross cost by the rate of gain or loss.*

EXAMPLE.—A house costing \$3,000 is sold for 22% above cost. What is the profit?

SOLUTION.—Profit=cost×rate=\$3,000×.22=\$660. Ans.

17. **Rule.**—*To find the rate of profit or loss, divide the difference between the selling price and gross cost by the gross cost; or divide the profit or loss by the gross cost.*

EXAMPLE.—A merchant sold for \$768 a lot of dry goods for which he paid \$900. What was the per cent. loss?

SOLUTION.— Loss=\$900−\$768=\$132.

Rate of loss=loss÷cost=\$132÷\$900=.14⅔, or 14⅔%. Ans.

18. **Rule.**—*To find the selling price, the cost and rate of gain or loss being given, multiply the cost by 100 per cent. plus the rate of gain, or by 100 per cent. minus the rate of loss.*

Example.—If hay is bought for $8 per ton, and if baling and shipping cost $5.50 per ton additional, at what price must it be sold to yield a profit of 16%?

Solution.—Gross cost=$8+$5.50=$13.50. The selling price=100% +16%, or 116% of cost. 116% expressed as a decimal becomes 1.16.

Selling price=cost×(100 per cent.+rate)=$13.50×1.16=$15.66. Ans.

19. In business calculations, it is more usual to calculate the profit separately and add it to the cost. The selling price can be found directly, however, by the rule. A process which is the reverse of the application of this rule is given in the next article.

20. **Rule.**—*To find the cost, the selling price and rate of gain or loss being given, divide the selling price by 100 per cent. plus the rate of gain, or by 100 per cent. minus the rate of loss.*

Example.—A dealer sold drugs for $112 and gained 75%. What was the cost of the drugs, and what was the profit?

Solution.—Selling price=100%+75%=175% of cost. 175%=1.75.

Cost=selling price÷(100%+rate)=$112÷1.75=$64. Ans

Profit=$112−$64=$48. Ans.

EXAMPLES FOR PRACTICE

What is the profit or loss

(*a*) If the gross cost is $85 and the rate of gain is 32%?

(*b*) If the gross cost is $837.50 and the rate of loss is 12%?

(*c*) If the gross cost is $240 and the rate of gain is $16\frac{2}{3}$%?

What is the rate of gain or loss

(*d*) If the gross cost is $6.50 and selling price is $9.10?

(*e*) If the gross cost is $14.00 and selling price is $12.50?

(*f*) If the gross cost is $3,500 and profit is $500?

What is the selling price

(*g*) If the cost is $945 and the rate of gain is $33\frac{1}{3}$%?

(*h*) If the cost is $3.50 and the rate of gain is $12\frac{1}{2}$%?

(*i*) If the cost is $125 and the rate of loss is 18%?

What is the cost

(*j*) If the selling price is $575 and the rate of gain is 15%?

(*k*) If the selling price is $28 and the rate of loss is $12\frac{1}{2}$%?

(*l*) If the selling price is $3.50 and the rate of gain is 26%?

Ans.—(*a*) $27.20; (*b*) $100.50; (*c*) $40; (*d*) 40%; (*e*) $10\frac{5}{7}$%; (*f*) $14\frac{2}{7}$%; (*g*) $1,260; (*h*) $3.94; (*i*) $102.50: (*j*) $500; (*k*) $32; (*l*) 2.77\frac{7}{9}$.

1. A house and lot which cost $3,250 is sold at a profit of 12%. What is (*a*) the profit and (*b*) the selling price? Ans. $\begin{cases}(a)\ \$390\\(b)\ \$3{,}640\end{cases}$

2. What must be the selling price of a suit of clothes which cost $18 in order that the profit may be 33⅓%? Ans. $24

3. A harvesting machine costs a hardware merchant $90 net, and $6 for freight and cartage. If sold for $108, what is the gain per cent.? Ans. 12½%

4. A carload of cattle is sold for $875, which is at a loss of 16%. What was the cost of the cattle? Ans. $1,041.67

5. A sells a steam tug to B, gaining 14%, and B sells it to C for $4,104 and gains 20%. How much did the tug cost A? Ans. $3,000

6. How much must hay sell for per ton, to gain 25%. if when sold for $8.40 per ton, there is a gain of 16⅔%? Ans. $9

7. Six horses were sold at $125 each; three of them at a profit of 25%, and the others at a loss of 25%. What was the net gain or loss? Ans. $50 loss

PROFITS BASED ON SELLING PRICE

21. In marking goods, some merchants reckon the profits and the selling expenses as a percentage of the selling price. This method is claimed to be the logical one, inasmuch as the cost of running a business is always reckoned as a percentage of the amount of sales, not of the cost of goods. Discounts, salesmen's commissions, taxes, etc. are also based on the selling price. Moreover, there is less chance of overestimating the profits when they are based on the selling price. To show that the percentage of profit on the selling price is not the same as that of the cost price, suppose an article costing $1 is sold at an advance of 25% on the cost price. The profit equals 25% of $1=$.25. The selling price equals $1+$.25=$1.25. The profit based on the selling price equals $.25÷$1.25=.20, or 20%. It is thus seen that 25% of the cost price equals only 20% of the selling price.

Example.—An article costing $1 is sold for $2. (*a*) What per cent. of the cost is the gain? (*b*) What per cent. of the selling price is the gain?

Solution.—(*a*) $2−$1=$1, the gain.
$1 is 100% of $1, or 100% of the cost. Ans.
(*b*) $1 is 50% of $2, or 50% of the selling price. Ans.

From the example just solved it has been shown that a profit of 100% on the cost price is equal to a profit of 50% on the selling price. Thus, it is seen that it may make considerable difference what the percentage is based on.

22. Rule.—*To find the selling price, add together the percentage estimated for selling expenses and the percentage of net profit desired and subtract from 100%. Divide the cost price by the remainder, and the result will be the selling price.*

Example.—A merchant estimates that his expenses are 20% of his sales. He desires to make a profit of 10% on his receipts. What should he ask for an article that costs him $12.25?

Solution.—The profit and cost of selling=10%+20%=30% of selling price. The purchase price+30% of selling price=entire selling price. The purchase price is, therefore, 100%−30%, or 70%, of selling price.

Selling price is $12.25÷.70=$17.50. Ans.

23. The relation of the purchase price, profit, and cost of selling can be shown by Fig. 2.

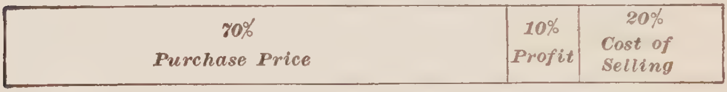


Fig. 2

Example 1.—A manufacturer allows his salesmen 40% of their sales and proposes to make a profit of 15% on the sales. What will be the selling price of an article that costs $1.95?

Solution.—Cost price=100%−40%−15%=45% of the selling price.

$1.95÷.45=$4.33$\frac{1}{3}$, selling price. Ans.

Example 2.—A merchant estimates that in order to do business at a reasonable profit, the difference between the cost price and the selling price should equal 20% of the latter. In marking his goods, what per cent. should he add to the cost?

Solution.—For goods he sells for $1 he would pay 20% less than $1, or 80 cents. An advance of 20 cents on 80 cents equals $\frac{20}{80}$=25%. The merchant would, therefore, make the marked price 25% more than the cost price. Ans.

24. The student may at first be confused to some extent owing to the different methods of computing the selling price. If, however, he will consider whether profits and selling expenses are to be based on the cost or the selling price he will have no difficulty.

TRADE DISCOUNTS

25. **Trade discounts** are reductions made by manufacturers, jobbers, or merchants from their list or catalog prices.

In many branches of business, manufacturers and dealers list their goods at a fixed price for each article, and allow a rate of discount according to the conditions of the market, the size of the order, and the plan of payment. In fact, two or even three discounts are sometimes allowed on an order. If it becomes necessary to raise or lower the price of the goods, the rate of discount is increased or decreased, the list price remaining the same. The system of discounts thus saves the expense of publishing a new price list every time prices change.

26. Merchandise is frequently sold at *time prices;* that is, payment is to be made in, say, 30, 60, or 90 days after date of sale, and a certain rate of discount is allowed if payment is made at an earlier date. Business houses usually make announcements such as the following upon their bill heads: "Terms: 4 mo., or 5% 60 days"; "Terms: 60 days net; 30 days, 3% off; 10 days, 5% off." Even when no discount is stated in the terms, sellers will usually deduct the legal interest for the time remaining, if the payment is made before it becomes due. Thus, if a payment due in 3 months is made 1 month after the sale, the seller should deduct the interest for the remaining 2 months.

27. Trade discounts are computed by the rules of percentage, the list price of the goods being the base. When several discounts are allowed, the first discount is computed on the list price, the second is computed on the remainder after deducting the first discount, and so on, each remainder being regarded as a base for the computation of the next discount. The several discounts, if there are more than one, form a **discount series.**

28. Rule.—*To find the selling price, multiply the list price by the rate, and subtract the discount thus obtained from the list price. If there is a discount series, compute the second discount, using the first remainder as a base, and subtract the discount from the remainder. Repeat the process, using each successive remainder as a base for computing the next discount. The last remainder is the selling price.*

Example 1.—The list price of an article is \$62.50 and a discount of 40% is allowed. What is the selling price?

Solution.—

Discount=\$62.50×.40=\$25.
Selling price=\$62.50−\$25=\$37.50. Ans.

Example 2.—On a bill of goods amounting to \$720, discounts of 30%, 10%, and 5% are allowed. What is the selling price?

Solution.—

First discount=\$720×.30=\$216.
Remainder=\$720−\$216=\$504.
Second discount=\$504×.10=\$50.40.
Remainder=\$504−\$50.40=\$453.60.
Third discount=\$453.60×.05=\$22.68.
Selling price=\$453.60−\$22.68=\$430.92. Ans.

29. The trade discounts usually allowed are aliquot parts of 100%, and the labor of computation may be shortened by using the fractions corresponding to the rates of discount.

Example.—The gross amount of a bill of hardware is \$640, and discounts of 25%, 10%, and 5% are allowed. What is the net amount of the bill?

Solution.—

$25\% = \frac{1}{4}$, $10\% = \frac{1}{10}$, $5\% = \frac{1}{20}$.

The solution is arranged as shown. To multiply \$640 by 25%, or $\frac{1}{4}$, we divide by 4; then the discount, \$160, is subtracted and the remainder, \$480, is divided by 10. The second discount, \$48, is subtracted, and the remainder is divided by 20. The final remainder, \$410.40, is the net amount, or selling price.

4)	\$640	
	\$160	1st discount.
10)	\$480	1st remainder.
	\$ 48	2d discount.
20)	\$432	2d remainder.
	\$ 21.60	3d discount.
	\$410.40	net amount. Ans.

30. When a discount series is allowed, business men usually reduce the series to an equivalent single discount; if there is a large number of sales, much labor of computation is saved by using the equivalent discount rather than the series.

Example.—What single discount on the gross price is equivalent to a discount series of 25%, 20%, and 10%?

Solution.—We will call the list price 100%. A discount of 25% makes the price 100% − 25% = 75% of the list price. A further discount of 20% makes the price 100% − 20%, or 80%, of the 75%; 80% of 75% = .8 × 75% = 60%. A third discount, which is 10%, reduces the price to 100% − 10%, or 90% of the 60%. .9 × 60% = 54%. This result can be obtained by continued multiplication as follows:

100% − 25% = 75% = .75; 100% − 20% = 80% = .8;
100% − 10% = 90% = .9
.75 × .8 × .9 = .54 = 54%
100% − 54% = 46%, the discount. Ans.

31. Rule.—*To reduce a discount series to an equivalent single discount, subtract each rate of discount from 100 per cent., and multiply the remainders together. Subtract the product from 100 per cent., and the remainder will be the equivalent discount.*

Example 1.—The cost of a line of goods is $350. What must they be marked to give a profit of 20% on the cost, and allow a discount of 30% on the marked price?

Solution.—The profit is $350 × .20 = $70; therefore, the actual selling price is $350 + $70 = $420. This is what remains after deducting 30% from the marked price. Since the 30% discount is computed on the marked price, that price must be the base, and the selling price, $420, obtained by subtracting the discount, is 100% − 30% = 70% of the base. Hence, the base or marked price = $420 ÷ (100% − 30%) = $420 ÷ .7 = $600. Ans.

Example 2.—The cost of manufacturing hats is $36 per dozen. At what price per dozen must they be marked that the manufacturer may realize 16⅔% profit on the cost after allowing the trade discounts of 20% and 12½%?

Solution.—16⅔% = ⅙; profit = $36 × ⅙ = $6; selling price = $36 + $6 = $42. 100% − 20% = .8; 100% − 12½% = .875; .8 × .875 = .7 = 70%. The equivalent single discount is 100% − 70% = 30%.

Marked price = $42 ÷ .7 = $60, per doz. Ans.

32. Rule.—*To find the price at which goods must be marked to insure a given profit after allowing a discount or a* *discount series, add to the cost, including the selling expenses, the profit required, and divide the sum by 100% minus the discount series or equivalent single discount.*

33. A bill in which discounts of 10% and 5% have been allowed is shown in Fig. 3.

Scranton, Pa., June 10, 19___

TECHNICAL SUPPLY CO.
Sold to
BROWN & BROS.
Denver, Colo.

10 rolls Blueprint Paper @ $1.50 per roll	$15	00
162 lb. Drawing Paper @ $.15 per lb.	24	30
1 gross Drawing Ink	21	60
	$60	90
Less 10% and 5%	8	83
	$52	07

FIG. 3

EXAMPLES FOR PRACTICE

Reduce the following discount series to equivalent single discounts:

(*a*) 25% and 16%.
(*b*) 30%, 20%, and 5%.
(*c*) 60%, 10%, and 5%.
(*d*) 40%, 20%, 12½%, and 4%.

Ans.—(*a*) 37%; (*b*) 46.8%; (*c*) 65.8%; (*d*) 59.68%.

TAXES

34. **Taxes** are sums of money levied on persons, properties, or incomes, for public purposes. Thus, taxes are levied to support the state, county, and city governments; to support schools and charities; and to make improvements, such as paved streets and sewers.

35. A **capitation,** or **poll,** tax is a tax levied on persons; a **property** tax is a tax levied on real estate or personal property; an **income** tax is levied on incomes or salaries. The term *real estate* applies to such property as lands and buildings. Other property, such as bonds, notes, goods, and money, is called *personal property*. The poll tax is usually a fixed amount for each citizen over 21 years of age. The property tax is reckoned as a certain per cent. of the assessed valuation of the property subject to taxation.

36. Rule.—*To find a property tax, multiply the assessed value of the property by the rate of taxation.*

EXAMPLE.—What property tax must a person pay who owns real estate assessed at $34,000 and personal property assessed at $12,500, the rate of taxation being 8 mills per $1, that is, rate=.008?

SOLUTION.—Total assessed value=$34,000+$12,500=$46,500.
Tax=$46,500×.008=$372. Ans.

EXAMPLES FOR PRACTICE

What is the tax if the assessed value of the property is
(*a*) $6,300 and the rate of taxation is $1\frac{1}{8}$%?
(*b*) $34,300 and the rate of taxation is $6\frac{1}{2}$ mills per $1?
(*c*) $9,430 and the rate of taxation is 85 cents per $100?
Ans. (*a*) 70.87\frac{1}{2}$; (*b*) $222.95; (*c*) 80.15\frac{1}{2}$.

1. I own real estate worth $8,500 and personal property worth $3,750; both are assessed at $\frac{3}{5}$ of their value. The rate of taxation is 1.2%, but I receive a discount of $2\frac{1}{2}$ per cent. of my taxes for prompt payment. How much do I pay the tax receiver? Ans. $86

2. A has 5 lots worth $1,300 each; B has 4 lots worth $1,000 each; C has 2 lots worth $1,500 each; D has 7 lots worth $800 each, and E has 1 lot worth $1,150. A tax of $1,417.50 for a street improvement is to be divided among them. What should each pay?

NOTE.—To find the rate, divide the amount levied for improvement by the total value of the property.

Ans.—A, $455; B, $280; C, $210; D, $392; E, $80.50.

3. The rate of taxation for a certain state is $3\frac{1}{2}$ mills per $1. How much state tax must be raised by a county whose valuation is fixed by the State Board of Equalization at $13,876,394? Ans. $48,567.38

4. If the income tax is 1% on all income in excess of $4,000 a year, what tax will a person pay whose annual income is $18,230?
Ans. $142.30

PROMISSORY NOTES

37. A **promissory note** is a written promise to pay a certain sum at a certain time.

38. The **maker,** or **drawer,** of a note is the person that promises to pay; the **payee** is the person to whom the note is payable; and the **holder** is the person that owns it.

39. The **face** of a note is the sum promised to be paid. This sum should be written both in figures and in words.

40. Notes are of two kinds—notes *bearing interest* and notes *not bearing interest.* When no rate of interest is specified, the legal rate in the state or county where the note is made is to be understood. If a note not bearing interest is not paid when due, it bears interest at the legal rate after that time until paid.

41. The legal rate of interest for money varies in the different states of the Union and depends on two facts:

1. The rate per cent. prescribed by statute in each state must be used when no other rate is agreed upon; this prescribed rate is as low as 5% in some states and as high as 8% in others.

2. If the rate is agreed upon by contract it can be no higher than 6% in some states, and is without any limit in others. Call loans by banks in New York and Pennsylvania—which are loans that may be called in at the pleasure of the bank—may command any rate agreed upon, provided the loan is $5,000 or more.

42. The punishment for usury—which is an interest charge higher than the admissible legal rate—involves various penalties, from the forfeiture of the excess of interest to the forfeiture of both principal and interest.

43. If a debt falls due on a Sunday or on a legal holiday or half holiday, some states require that it shall be paid on the day before, and some on the day following the legal holiday or half holiday.

44. Every state has a statute of limitation according to which, after the expiration of a certain time, a claim for money does not hold in law. This time is different for notes, closed and open accounts, judgment notes, and sealed documents involving claims for money.

45. Rates of interest, penalties for usury, payments with respect to holidays, and statutes of limitation are changed so frequently that any table showing them at a given time is speedily out of date. The student who wishes information relative to any of these matters can get it at any bank, so it is deemed best to omit it here.

46. A note should be so written as to show where it was made and when, the sum promised to be paid, whether it does or does not bear interest, and should bear the words "value received." The law assumes that no one is to be compelled to pay unless he has received what he deems an equivalent. If "value received" is omitted, the holder may have to prove that the maker of the note did actually receive value for the money promised in it.

47. A note usually specifies where it is to be paid—usually at a bank. If no place is designated, the holder's place of business is understood, or his residence.

48. If a note contains the words "or order," it is a **negotiable note,** and may pass like a bank note from one person to another. If the holder of a negotiable note wishes to dispose of it, he is required to guarantee its payment by *indorsing* it—that is, by writing his name across the back of the note. There are several kinds of indorsements. Thus, if the holder is John Smith, he may, on the back of the note, write *John Smith.* This is an indorsement *in blank,* and makes John Smith responsible for the payment of the note.

He may write *Pay to William Jones.* The note is then payable only to William Jones.

If he indorses it *Pay to William Jones, or order,* it is payable to William Jones, or to any one to whom William Jones may order it to be paid.

He may indorse it *Pay to William Jones, or bearer,* and it is payable to any person that presents it.

If it be indorsed *Pay to bearer,* it is payable to the person that presents it for payment.

49. A **joint-and-several** note is a note signed by two or more persons, who become collectively and individually responsible for its payment.

50. A note is **protested** when the holder, or a notary public at his direction, notifies the indorsers in legal form and within the time prescribed by law that the note is unpaid. Unless the note is protested in this manner, the indorsers are not responsible for its payment.

51. Forms of notes used in actual business are given below:

$250. *New York, N. Y., Sept. 17, 1912*

On demand I promise to pay George Camp, or order, Two Hundred and Fifty Dollars, value received.

Howard Gray

$1,000. *Toronto, Ont., July 5, 1913*

Three months after date, for value received, I promise to pay Stephen Girard, or order, One Thousand Dollars, with interest at 5%. *Charles Goldwin*

$3,000. *Philadelphia, Pa., July 5, 1913*

Six months after date, we, or either of us, will pay to George Owen, Three Thousand Dollars, value received, with interest at 6%.

George Kirwin
Henry Potter
Erastus Kirby

Payable at the First National Bank.

The first two notes above are negotiable.

Responsibility for the payment of a joint-and-several note, such as the last of those above, rests on the first person signing. If he cannot pay, the second person signing is called upon, and so on.

52. In some states, *three days of grace* are allowed before a note must be paid. If, in a state where grace is allowed, a bank discounts a note, interest is charged for days of grace.

EXAMPLES FOR PRACTICE

1. Write a negotiable demand note for $600, with interest at 6%, and make Brown, Jones & Co. the payee.

2. Write a non-negotiable note for $4,000, with interest at 5%, payable in 30 days, yourself being the maker and Howard Crosby the payee.

3. How much will pay the following note when due without grace?

575\frac{50}{100}$. *New York, N. Y., Sept. 19, 1914*

Sixty days after date, for value received, I promise to pay Ralph Newton, or order, Five Hundred Seventy-Five and $\frac{50}{100}$ Dollars, with interest at 4½%. *Henry Miles*

Ans. $579.82

BANK DISCOUNT

53. Bank discount is the charge made by a bank for paying a note or other obligation before it is due. This charge is the interest on the amount of the obligation from the time it is discounted until its **maturity**; that is, the date on which it is due. This interest is subtracted from the face of the obligation, and its holder receives for it the remainder, which is called the **proceeds.** Hence, bank discount is inequitable, since interest is charged not only upon the sum actually paid for the obligation, but also upon the discount.

54. If a note for $1,000, with 60 days to run before it is due, is discounted at a bank, the interest on $1,000 is found for 60 days, and is subtracted from $1,000. If the rate of discount is 6%, the holder will receive as proceeds $1,000 − $10 = $990.

55. The **term of discount** is the time from the discounting of the note to its maturity.

56. In the case of an interest-bearing note, the sum discounted is the amount of the note at maturity; that is, the face of the note plus the interest.

57. Banks usually require that a discounted note shall be payable at the bank that discounted it, and ordinarily they do not discount notes having more than 90 days to run.

58. To find the time when a note matures, the term of discount, the discount, and the proceeds.

Example.—Find (*a*) the discount and (*b*) the proceeds of the following note:

$\$484\frac{60}{100}$. *Newark, N. J., Oct. 4, 1913*

Sixty days after date, for value received, I promise to pay William Hall, or order, Four Hundred Eighty-Four and $\frac{60}{100}$ Dollars, at the Ninth National Bank. *Henry Parshall*

Discounted, Oct. 20, 1913, at 6%.

Solution.—

(*a*) Maturity, Dec. 3, 1913.
Term of discount, 44 days.
Discount, $3.55. Ans.
(*b*) Proceeds, $484.60−$3.55=$481.05. Ans.

Explanation.—Sixty days after Oct. 4 is Dec. 3, the date of legal maturity. From the time of discount, Oct. 20, to Dec. 3 is 44 days, for which the interest at 6% is $3.55. Subtracting the discount, $3.55, from the face of the note, $484.60, gives $481.05, the proceeds.

59. The maturity of a note, when grace is allowed, is on the last day of grace. The time of maturity is generally indicated on such notes. Thus, Mar. 7/10, written on a note means that it matures nominally on Mar. 7, and legally on Mar. 10.

Example.—Find (*a*) the discount and (*b*) the proceeds of the following note, allowing for days of grace:

$1,060. *Austin, Tex., August 6, 1913*

For value received, I promise to pay, three months after date, to Ernest Hazard, or order, One Thousand Sixty Dollars, with interest at 8%.

Emil Reeves

Discounted, Sept. 1, 1913, at 6%.

Solution.—

(*a*)	Maturity, Nov. 6/9, 1913.		
	Amount of note at maturity	$1081.91	
	Terms of discount, 69 days		
	Discount	12.44	Ans.
(*b*)	Proceeds	$1069.47	Ans.

60. Rule.—I. *If the note bears interest, find its amount at the time of legal maturity.*

II. *Find the interest on the face of the note at the given rate of discount, or, if it is an interest-bearing note, on the amount of the note at the date of legal maturity at the given rate of interest, and the result will be the bank discount.*

III. *Subtract the bank discount from the face of the note, or from its amount at maturity, and the remainder will be the proceeds.*

EXAMPLES FOR PRACTICE

1. Find (*a*) the bank discount and (*b*) the proceeds of a note for \$5,000, due in 60 days, discounted at 6%.

Ans. (*a*) \$50
(*b*) \$4,950

2. Find (*a*) the bank discount and (*b*) the proceeds of a grace note for \$4,000 due in 90 days, discounted at 5%.

Ans. (*a*) \$51.67
(*b*) \$3,948.33

3. *\$8,476 $\frac{00}{100}$.* *Scranton, Pa., Jan. 8, 1914*

Six months after date I promise to pay to Charles Brown, or order, Eight Thousand Four Hundred Seventy-Six Dollars, value received, with interest at 6%. *Howard Bristow*

Find the proceeds of the foregoing note, if discounted April 24, 1914, at 5%. Ans. \$8,639.34

PARTIAL PAYMENTS

61. A debt or obligation may be discharged at one payment; or, from time to time, payments *in part* may be made, and finally at a time of *settlement* the remainder of the debt may be paid. Now, it is obvious that interest should be allowed upon such payments as are made, since interest is charged upon the obligation itself. But, if a payment should be less than the interest upon the debt since a previous payment had been made, to subtract such payment from the debt with accrued interest would result in increasing the principal. This would be a species of compound interest which, in many states, is illegal.

62. When a partial payment of a note is made, the date of payment and its amount are written on the back of the note, and this record of it is called an **indorsement.**

The following rule for partial payments, formulated by the United States Supreme Court, is used in most states:

63. United States Rule.—*Find the amount of the principal to the time when the payment, or the sum of the payments, is greater than the interest then due. From the amount subtract the payment or the sum of the payments, and treat the remainder as a new principal. Proceed in this manner to the date of settlement, and the last amount will be the sum still due.*

Find the difference between dates by putting down the year, the number of the month and the day of the month in the case of both minuend and subtrahend. Then count 30 days to the month, and subtract regularly as in the case of denominate numbers.

EXAMPLE.—

$1,200. New York, Sept. 16, 1912

On demand I promise to pay John Crawford, or order, Twelve Hundred Dollars, with interest at 6%, value received.

Edward G. Carson

Indorsements: Jan. 1, 1913, $120; May 7, 1913, $300; Dec. 22, 1913, $16; Sept. 19, 1914, $400. What is due Jan. 1, 1915?

SOLUTION.—

Principal	$1200
Interest from Sept. 16, 1912, to Jan. 1, 1913 (3 mo. 15 da.)	21
Amount	1221
First payment	120
New principal	1101
Interest from Jan. 1, 1913, to May 7, 1913 (4 mo. 6 da.)	23.12
Amount	1124.12
Second payment	300
New principal	824.12
Interest from May 7, 1913, to Sept. 19, 1914 (1 yr. 4 mo. 12 da.)	67.58
Amount	891.70
Sum of third and fourth payments	416.00
New principal	475.70
Interest from Sept. 19, 1914, to Jan. 1, 1915 (3 mo. 12 da.)	8.09
Amount due at time of settlement	$ 483.79
	Ans

The third payment of $16 is less than the interest due at the time it was made; hence, according to the rule, it is added to the next payment of $400 and the interest is computed to the time of the fourth payment.

EXAMPLES FOR PRACTICE

Find the amount due at the time of settlement on each of the following:

1. *$2,000.* *Philadelphia, July 1, 1911*

One year after date, for value received, I promise to pay Wm. Gray, or order, Two Thousand Dollars, with interest at 6%.

Henry G. Brown

Indorsements: Dec. 16, 1911, $350; Mar. 1, 1912, $25; Oct. 25, 1912, $400; June 14, 1913, $275. Time of settlement, July 1, 1914.

2. A note for $3,000 at 6% dated Mar. 5, 1913, bore the following indorsements: Dec. 20, 1913, $400; Mar. 14, 1914, $60; Nov. 30, 1914, $360; July 15, 1915, $600. Time of settlement, Jan. 1, 1916.

Ans.—(1) $1,216.80; (2) $2,024.38.

EXACT INTEREST

64. It is the common practice to consider 360 days as a year in computing interest. This gives a result too great, since 1 day is not as much as $\frac{1}{360}$ of a year but is $\frac{1}{365}$ of a year. The United States government and some banks reckon interest by counting each day as $\frac{1}{365}$ of a year or $\frac{1}{366}$ of a year in case of a leap year. Interest thus computed is called exact interest.

Example.—Find the exact interest on $1,000 for 89 days at 6%.

Solution.—Interest for 1 yr.=6% of $1,000=$60.
Interest for 89 da.=$\frac{89}{365}$ of $60=$14.63. Ans.

65. Should one be obliged to calculate exact interest frequently, he should make a table giving the interest on various sums for various periods; as, for example, on each thousand dollars from one to nine for 1 day to 100 days.

EXAMPLES FOR PRACTICE

Find the exact interest on:

1. $10,000 for 123 days at 6%.
2. $12,800 for 168 days at 6%.
3. $6,400 for 213 days at 5%.
4. $22,800 for 2 yr. 73 da. at 4%.

5. $13,000 from Jan. 17, 1913, to Nov. 29, 1913, at 4½%.

Ans.—(1) $202.19+; (2) $353.49; (3) $186.74−; (4) $2,006.40; (5) $506.47—.

MERCHANTS' RULE

66. When the time from the date of a note or other obligation is less than a year, settlement is usually made by a method called the *merchants' rule.*

67. Rule.—I. *By the method of exact interest, find the amount of each of the several payments from the time each is made to the date of settlement.*

II. *Subtract the sum of these amounts from the amount of the obligation from its date to the time of settlement. The remainder will be the amount still due.*

Example.—Face of note, $2,000; rate, 6%; date of note, Dec. 31, 1914; time of settlement, Nov. 15, 1915. Indorsements: Mar. 10, 1915, $200; June 1, 1915, $300; Aug. 20, 1915, $400; Oct. 1, 1915, $500. What was due at time of settlement?

Solution.—

Principal		$2000
Interest on $2,000 for 319 da.		104.88
Amount		$2104.88
Amount of $200 for 250 da.	$208.22	
Amount of $300 for 167 da.	$308.24	
Amount of $400 for 87 da.	$405.72	
Amount of $500 for 45 da.	$503.70	
Sum of payments, with interest		$1425.88
Amount due at time of settlement		$679.00

The table on pages 31 and 32 can be used to find the difference of time between dates.

COMPOUND INTEREST

68. If the interest on a principal is added to the principal at regular intervals to form by each addition a new principal for the next interval, the resulting interest is called **compound interest.** Thus, if $100 be placed at compound interest at 6%, with the understanding that the interest is to be compounded annually, the principal will be $100 for the first year, $106 for the second year, $112.36 for the third year, etc.

69. Most savings banks allow compound interest, although in most states its payment cannot be legally enforced, even though it be specified in a contract.

The method of compounding interest is used principally in determining the amount a given principal will yield in a given time on the assumption that the interest is invested at the same rate as soon as it is due. Thus, in computing the probable returns on loans made for long periods, this method is used.

70. If the interest is to be compounded annually it is added to the principal at the end of each year. If it be compounded semiannually, one-half of the annual rate is taken as the rate, and the interest is added at the end of each period of 6 months; if quarterly, one-fourth the annual rate is taken; etc.

EXAMPLE.—Find the amount of $800, at compound interest for 2 years at 6%, interest compounded semiannually.

SOLUTION.—

$800 =prin. 1st 6 mo.
24 =int. 1st 6 mo.=$800×.03
$824 =prin. 2d 6 mo.
24.72=int. 2d 6 mo.=$824×.03
$848.72=prin. 3d 6 mo.
25.46=int. 3d 6 mo.=$848.72×.03
$874.18=prin. 4th 6 mo.
26.23=int. 4th 6 mo.
$900.41=amount for 2 yr. Ans.

71. When the time is given in years, months, and days, the interest is compounded for the greatest number of entire periods included in the time, and the simple interest on the last amount as the principal is found for the remaining time.

Thus, if the time in the preceding example were 1 year 8 months 20 days, $874.18 would be taken as the principal for the last 2 months 20 days, and the simple interest on this principal for 2 months 20 days would be computed. This interest, which is $11.66, added to $874.18 gives $885.84, the amount for 1 year 8 months 20 days.

72. Compound interest is calculated in actual business by means of a table. The accompanying compound-interest table shows the amount of $1 at several different rates, and for

COMPOUND-INTEREST TABLE

Showing the amount of $1, at various rates, compound interest, from 1 to 20 years, interest compounded annually

Yr.	1½ Per Cent.	2 Per Cent.	2½ Per Cent.	3 Per Cent.	3½ Per Cent.	4 Per Cent.
1	1.015000	1.020000	1.025000	1.030000	1.035000	1.040000
2	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600
3	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864
4	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859
5	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653
6	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319
7	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932
8	1.126493	1.171659	1.218403	1.266770	1.316809	1.368569
9	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312
10	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244
11	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454
12	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032
13	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074
14	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676
15	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944
16	1.268986	1.372786	1.484506	1.604706	1.733986	1.872981
17	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901
18	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817
19	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849
20	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123

Yr.	5 Per Cent.	6 Per Cent.	7 Per Cent.	8 Per Cent.	9 Per Cent.	10 Per Cent.
1	1.050000	1.060000	1.070000	1.080000	1.090000	1.100000
2	1.102500	1.123600	1.144900	1.166400	1.188100	1.210000
3	1.157625	1.191016	1.225043	1.259712	1.295029	1.331000
4	1.215506	1.262477	1.310796	1.360489	1.411582	1.464100
5	1.276282	1.338226	1.402552	1.469328	1.538624	1.610510
6	1.340096	1.418519	1.500730	1.586874	1.677100	1.771561
7	1.407100	1.503630	1.605781	1.713824	1.828039	1.948717
8	1.477455	1.593848	1.718186	1.850930	1.992563	2.143589
9	1.551328	1.689479	1.838459	1.999005	2.171893	2.357948
10	1.628895	1.790848	1.967151	2.158925	2.367364	2.593742
11	1.710339	1.898299	2.104852	2.331639	2.580426	2.853117
12	1.795856	2.012197	2.252192	2.518170	2.812665	3.138428
13	1.885649	2.132928	2.409845	2.719624	3.065805	3.452271
14	1.979932	2.260904	2.578534	2.937194	3.341727	3.797498
15	2.078928	2.396558	2.759031	3.172169	3.642482	4.177248
16	2.182875	2.540352	2.952164	3.425943	3.970306	4.594973
17	2.292018	2.692773	3.158815	3.700018	4.327633	5.054470
18	2.406619	2.854339	3.379932	3.996019	4.717120	5.559917
19	2.526950	3.025600	3.616527	4.315701	5.141661	6.115909
20	2.653298	3.207136	3.869684	4.660957	5.604411	6.727500

periods from 1 to 20 years. More extensive tables can be purchased through bookdealers. Having the amount of $1 at any given rate and for any number of periods, we multiply it by the number of dollars in any given principal. The result will be the amount of that sum for the given time. If the original principal be subtracted from this amount, the remainder is the compound interest required.

EXAMPLE.—What is the amount and the compound interest of $600 for 4 years at 6%, interest compounded annually?

SOLUTION.—The amount of $1 for 4 years at 6% is, according to the table, page 26, $1.262477. The corresponding amount of $600 is

$600×1.262477=$757.49. Ans.
The interest is $757.49−$600=$157.49. Ans.

73. If the interest is computed semiannually, find by the table the amount for double the number of years at half the rate.

EXAMPLE.—A young man deposited $1,000 in a savings bank. What sum would he have at the end of 10 years if the bank pays 4% interest per annum, and credits the interest semiannually?

SOLUTION.—The rate for each half year is 2%; the number of interest periods is 20. Hence, the amount is equal to that for 20 years at 2%. According to the table, the amount of $1 for 20 years at 2% is $1.485947. Hence, the amount of $1,000 for the same time is $1,000×1.485947 =$1,485.95. Ans.

EXAMPLES FOR PRACTICE

Find the compound interest on the following amounts for the times mentioned, and prove the correctness of your result by using the table. Unless otherwise stated, the interest is to be compounded annually.

1. On $1,600 for 2 years at 8%.
2. On $1,280 for 5 years at 4%.
3. On $480 for 3 years at 8%, interest compounded semiannually.
4. On $3,000 for 2 years 11 months 12 days at 6%, interest compounded semiannually.

Ans.—(1) $266.24; (2) $277.32; (3) $127.35; (4) $571.72.

ADDITIONAL RULES OF INTEREST

74. In the problems in interest so far considered, the principal, the rate, and the time have been given, and the interest or the amount has been found. One may have occasion to compute the principal, the rate, or the time when the other terms necessary for the solution of the problem are known. No new principles are required, as the applications involve those of percentage.

75. Given the principal, the interest, and the time, to find the rate.

The interest for one year is obtained by multiplying the principal by the rate. Then, conversely, the rate is obtained by dividing the interest for 1 year by the principal. The interest for one year is equal to the given interest divided by the time in years.

Example.—A pawnbroker charged \$8 for a loan of \$200 for $2\frac{1}{2}$ months. What rate of interest did he receive?

Solution.— \$8 for $2\frac{1}{2}$ months is equivalent to $\$8 \div 2.5 = \$3.20$ a mo., or to $\$3.20 \times 12 = \$38.40$ a year.

$$\text{Rate} = 38.40 \div 200 = .19\tfrac{2}{10}, \text{ or } 19\tfrac{2}{10}\%. \quad \text{Ans.}$$

76. Given the principal, interest, and rate, to find the time.

It is evident that if the given interest is divided by the interest for 1 year, the time required in years will be obtained. Or if the given interest is divided by the interest for 1 day, the time in days will be obtained.

Example 1.—In what time will the interest on \$1,000 at 5% be \$80?

Solution.—The interest on \$1,000 for 1 yr. at 5% is \$50. The required time is, therefore, $80 \div 50 = 1\frac{3}{5}$ yr. Ans.

Example 2.—If the principal is \$978.26 and the interest at 6% is \$4.08, what is the time in days?

Solution.—Interest on \$978.26 for 60 da. at 6% is \$9.78. Interest for 1 day is $\$9.78 \div 60 = \$.163$.

$$\text{Time} = 4.08 \div .163 = 25 \text{ days.} \quad \text{Ans.}$$

77. Given the amount, time, and rate, to find the principal.

The amount for any time and rate can be obtained by multiplying the principal by the amount of $1 for the same time and rate. Consequently, the principal can be found by dividing the amount by the amount of $1 for the same time and rate.

Example.—What sum placed at interest for 1 year at 6% will amoun to $1,000?

Solution.—The amount of $1 for 1 yr. at 6% is $1.06.

The principal=$1,000÷1.06=$943.396, or $943.40. Ans.

It is seen that the principal is, therefore, $\frac{100}{106}$ of the amount for this rate and time.

PRESENT WORTH

78. The **present worth** of an obligation is a sum such that, if it be put at interest at a specified rate for a given time, it will amount to the obligation.

Thus, if the specified rate is 5%, a debt of $105 due in 1 year is worth $100 *now,* since $100 placed at interest at 5% will in one year amount to $105.

The present worth may be found as shown in the solution of the example in the preceding article. In business practice it is customary when a debt is paid before it is due to deduct the interest; in equity, that is, in justice, the amount paid should be the present worth of the obligation. Usually, however, loans of this character are of such short duration that the discount is reckoned in the ordinary way.

Example.—A debt of $773, which has 9 mo. 20 da. yet to run, is discounted at 5%. What is: (*a*) the present worth? (*b*) the true discount? (*c*) the bank discount?

Solution.—(*a*) 9 mo. 20 da.=290 da., counting 30 days to the month. The interest on $1 for 9 mo. 20 da. at 5% equals $\frac{290}{360}\times.05=\$.04028$. The amount of $1 for the given time and rate=$1.04028.

Present worth=$773÷1.04028=$743.07. Ans.

(*b*) The true discount is the difference between the present worth and the amount of the obligation.

True discount=$773−$743.07=$29.93. Ans.

(*c*) The bank discount is $773×.04028=$31.14. Ans.

Proof.—Calculate the interest on $743.07 for 9 mo. 20 da. at 5%. It is $29.93; hence, the solution is correct.

79. To find the face of a note when the proceeds, time, and rate are given.

Sometimes one may be required to find what the face of a note should be in order that the proceeds may be equal to a given sum. The face multiplied by the proceeds of $1 equals the proceeds, hence the given proceeds divided by the proceeds of $1 will give the face of a note.

Example.—The proceeds of a note discounted at a bank for 45 days at 6% amounted to $1,488. What was the face of the note?

Solution.—Interest on $1 for 60 da. at 6%=$.01. Interest on $1 for 45 da. at 6%=$.0075.

Proceeds of $1 due in 45 days=$1−$.0075=$.9925. Face of note =$1,488÷$.9925=$1,499.24. Ans.

EXAMPLES FOR PRACTICE

Find the principal, when:

(*a*) Interest=$96, time=2 years, rate=4%.

(*b*) Interest=$131.25, time=2 years 6 months, rate=6%.

Find the time, when:

(*c*) Principal=$4,800, interest=$652, rate=6%.

(*d*) Principal=$680, interest=$163.20, rate=5%.

Find the rate, when:

(*e*) Principal=$2,875, time=4 years 7 months 6 days, interest=$529.

(*f*) Principal=$760, time=3 years 9 months 18 days, interest =$144.40.

Ans.—(*a*) $1,200; (*b*) $875; (*c*) 2 yr. 3 mo. 5 da.; (*d*) 4 yr. 9 mo. 18 da.; (*e*) 4%; (*f*) 5%.

1. A note for 60 days discounted at 4½% yields $81,815.33. What is its face? Ans. $82,433.58

2. Write in proper form a 60-day note payable at the Chemical Bank of New York, which, when discounted when the note is made, will yield at 5%, $7,850 proceeds. Ans. Face, $7,915.97

TABLE FOR FINDING THE NUMBER OF DAYS BETWEEN TWO DATES

80. The table on pages 31 and 32 may be used for finding the time between two dates, also for finding a date that occurs a certain number of days later or earlier than a given date. It will be observed that in the table there are twelve columns, one for each month of the year, and in each column there are as many lines as there are days in the month. We shall now illustrate the use of the table.

In the column headed Feb., we find, opposite 24 in the left-hand column, the numbers 55 and 310. The first number, 55, indicates that Feb. 24 is the 55th day of the year, and the

TABLE FOR FINDING THE NUMBER OF DAYS BETWEEN TWO DATES

Day of Month	Jan.		Feb.		March		April		May		June		Day of Month
1	1	364	32	333	60	305	91	274	121	244	152	213	1
2	2	363	33	332	61	304	92	273	122	243	153	212	2
3	3	362	34	331	62	303	93	272	123	242	154	211	3
4	4	361	35	330	63	302	94	271	124	241	155	210	4
5	5	360	36	329	64	301	95	270	125	240	156	209	5
6	6	359	37	328	65	300	96	269	126	239	157	208	6
7	7	358	38	327	66	299	97	268	127	238	158	207	7
8	8	357	39	326	67	298	98	267	128	237	159	206	8
9	9	356	40	325	68	297	99	266	129	236	160	205	9
10	10	355	41	324	69	296	100	265	130	235	161	204	10
11	11	354	42	323	70	295	101	264	131	234	162	203	11
12	12	353	43	322	71	294	102	263	132	233	163	202	12
13	13	352	44	321	72	293	103	262	133	232	164	201	13
14	14	351	45	320	73	292	104	261	134	231	165	200	14
15	15	350	46	319	74	291	105	260	135	230	166	199	15
16	16	349	47	318	75	290	106	259	136	229	167	198	16
17	17	348	48	317	76	289	107	258	137	228	168	197	17
18	18	347	49	316	77	288	108	257	138	227	169	196	18
19	19	346	50	315	78	287	109	256	139	226	170	195	19
20	20	345	51	314	79	286	110	255	140	225	171	194	20
21	21	344	52	313	80	285	111	254	141	224	172	193	21
22	22	343	53	312	81	284	112	253	142	223	173	192	22
23	23	342	54	311	82	283	113	252	143	222	174	191	23
24	24	341	55	310	83	282	114	251	144	221	175	190	24
25	25	340	56	309	84	281	115	250	145	220	176	189	25
26	26	339	57	308	85	280	116	249	146	219	177	188	26
27	27	338	58	307	86	279	117	248	147	218	178	187	27
28	28	337	59	306	87	278	118	247	148	217	179	186	28
29	29	336			88	277	119	246	149	216	180	185	29
30	30	335			89	276	120	245	150	215	181	184	30
31	31	334			90	275			151	214			31

number 310 indicates that there are 310 more days in the year after Feb. 24. Again, in the tenth line of the Aug. column, we

TABLE FOR FINDING THE NUMBER OF DAYS BETWEEN TWO DATES—(*Continued*)

Day of Month	July		Aug.		Sept.		Oct.		Nov.		Dec.		Day of Month
1	182	183	213	152	244	121	274	91	305	60	335	30	1
2	183	182	214	151	245	120	275	90	306	59	336	29	2
3	184	181	215	150	246	119	276	89	307	58	337	28	3
4	185	180	216	149	247	118	277	88	308	57	338	27	4
5	186	179	217	148	248	117	278	87	309	56	339	26	5
6	187	178	218	147	249	116	279	86	310	55	340	25	6
7	188	177	219	146	250	115	280	85	311	54	341	24	7
8	189	176	220	145	251	114	281	84	312	53	342	23	8
9	190	175	221	144	252	113	282	83	313	52	343	22	9
10	191	174	222	143	253	112	283	82	314	51	344	21	10
11	192	173	223	142	254	111	284	81	315	50	345	20	11
12	193	172	224	141	255	110	285	80	316	49	346	19	12
13	194	171	225	140	256	109	286	79	317	48	347	18	13
14	195	170	226	139	257	108	287	78	318	47	348	17	14
15	196	169	227	138	258	107	288	77	319	46	349	16	15
16	197	168	228	137	259	106	289	76	320	45	350	15	16
17	198	167	229	136	260	105	290	75	321	44	351	14	17
18	199	166	230	135	261	104	291	74	322	43	352	13	18
19	200	165	231	134	262	103	292	73	323	42	353	12	19
20	201	164	232	133	263	102	293	72	324	41	354	11	20
21	202	163	233	132	264	101	294	71	325	40	355	10	21
22	203	162	234	131	265	100	295	70	326	39	356	9	22
23	204	161	235	130	266	99	296	69	327	38	357	8	23
24	205	160	236	129	267	98	297	68	328	37	358	7	24
25	206	159	237	128	268	97	298	67	329	36	359	6	25
26	207	158	238	127	269	96	299	66	330	35	360	5	26
27	208	157	239	126	270	95	300	65	331	34	361	4	27
28	209	156	240	125	271	94	301	64	332	33	362	3	28
29	210	155	241	124	272	93	302	63	333	32	363	2	29
30	211	154	242	123	273	92	303	62	334	31	364	1	30
31	212	153	243	122			304	61			365	0	31

find 222 and 143. These numbers show that Aug. 10 is the 222d day of the year and that there are 143 more days in the year after that date.

EXAMPLE 1.—A note bearing interest was made on Nov. 15 and paid Feb. 1 of the following year. How many days did the note draw interest?

SOLUTION.—In this case we find the number of days in the year after Nov. 15 and add this to the number of days to be counted in the next year. In the fifteenth line in the column headed Nov. the second number is 46. Feb. 1 is the 32d day of the year. Hence, from Nov. 15 to Feb. 1 is $46+32=78$ days. Ans.

In case the period considered includes the last day of February of a leap year—that is, February 29—the additional day must be considered.

EXAMPLE 2.—In a leap year, find the days from Jan. 28 to May 4.

SOLUTION.—Jan. 28 is the 28th day and May 4 is the 124th day of the common year. The difference in time is $124-28=96$ days. But as Feb. 29 has not been counted, we add one day more and obtain $96+1=97$ days. Ans.

STOCKS AND BONDS

81. If work involving a large expenditure of money is to be undertaken, it is usual to organize a company and procure a charter under the laws of some state. The chartered company then issues shares, which are sold to persons who have money to invest and are willing to incur the chances of loss. These shares are known as stock.

The advantage of having a charter is that a chartered company can do business just as an individual—that is, it may sue or be sued for debts, enter into contracts, etc. Moreover, the shareholders, in case the business is unprofitable, are, as a rule, liable for the debts of the company only to the amount of their shares. The exception may be noted that owners of stock in national banks are responsible for an amount equal to the face value of their stock in addition to their shares. The members of an unincorporated concern may be compelled to pay all of its debts.

82. The **par value** of shares is the amount—usually $25, $50, or $100—specified in the certificates issued to the subscribers of a corporation.

83. When a company gains in its business, it pays its shareholders part of its profits, called **dividends.**

Dividends are *declared* quarterly, semiannually, or annually, and are usually paid at the general office of the company.

84. If the business is conducted at a loss, the shareholders may be required to make good the loss. Such payments are called **assessments.**

85. When the dividends on stock are considered high, the stock will usually sell above par; if no dividends are paid or if they are small, the stock, as a rule, will sell below par. The price of stock is, of course, influenced by other considerations than present dividends. The stability and the risk of the business, the assets of the company, and the outlook for the future are factors that affect the price.

86. In some corporations the stock is divided into two classes, called **preferred** and **common.** Dividends on preferred stock are payable first and do not exceed a fixed rate. The dividends on common stock depend on the profits of the business after the dividends on the preferred stock are provided for.

87. A **bond** is a written obligation under seal to pay a certain sum at a specified time.

Bonds are issued by national and state governments, cities, counties, towns, and incorporated companies in order to provide money for current or extraordinary expenses, or for such improvements as may be desired. The bonds are secured by the property of those who issue them, and bear interest payable quarterly, semiannually, or annually.

88. **Registered bonds** are numbered, and the names of their purchasers are recorded. When registered bonds are sold, the transfer must be recorded on the books of the company that issued them. Sometimes bonds have small forms called **coupons** attached, stating the amount of interest due at certain times. These coupons may be cut off and exchanged for money at the general office of the company or deposited in any bank for collection. Owing to the ease with which they may be transferred from one party to another, unregistered bonds usually sell for a slightly higher price than registered ones. Corporation bonds are, as a rule, issued in denominations of $1,000 or multiples thereof. Some bonds are issued in denominations of $100 and of $500.

89. Bonds are usually designated by the interest they bear, or by the time when they are payable. Thus, "U. S. 4's, 1932" are bonds of the United States government bearing interest at 4%, and payable in 1932.

90. A **stock broker** is a person whose business consists in buying and selling bonds or stocks for others. His compensation is a certain per cent. of the *par value* of the stocks bought or sold. The compensation of a broker is called **brokerage.**

91. The price quoted in stock market reports is that for $100 par. For example, when Canadian Pacific is quoted at 125, each $100 par sells for $125. In some localities and in some classes of stock, prices are quoted as the price per share. For example, the par value of a share of Pennsylvania Railroad Company stock is $50. The New York market may quote the stock at 130, which means that $130 is the price for each $100 par, or for two shares. In the Philadelphia market, the price would be quoted at $65, which is the price per share. Information concerning issues of stock and related matters can be obtained from stock brokers.

92. In this Section the *par value* of stocks is to be understood as 100, unless some other value is given. Whatever may be the market price of stocks and bonds, brokerage is calculated on their par value.

EXAMPLE 1.—Find the cost of 480 shares of Canadian Pacific stock bought at 123$\frac{1}{2}$, if the brokerage is $\frac{1}{8}$%.

SOLUTION.—The cost of each share is $123.50 and the brokerage is $\frac{1}{8}\%$ of $\$100=\frac{1}{8}$ of $\$1=\$.12\frac{1}{2}$.

$$\begin{array}{lr} 480\times\$123.50= & \$59280.00 \\ 480\times\$.12\frac{1}{2}\ = & 60.00 \\ \hline & \$59340.00 \end{array}\quad\text{Ans.}$$

Or, cost per share including brokerage$=\$123.50+\$.12\frac{1}{2}=\$123.62\frac{1}{2}$.

$480\times\$123.62\frac{1}{2}=\$59,340.00$. Ans.

Example 2.—What will be the cost with accrued interest of a 5% $1,000 bond at 103, including brokerage, if the interest was paid last on July 1 and the bond is sold on October 1?

Solution.—

One $1,000 bond at 103=$1030.00
Interest on $1,000 for 3 mo. at 5%= 12.50
$1042.50 Ans.

The former owner would be entitled to the interest of the bond from the date of the last payment of interest until the date of sale. When the quoted price includes the interest due, bonds are said to be sold *flat*.

Example 3.—How many shares of bank stock selling at 56 can be bought for $89,700, if the brokerage is $\frac{1}{8}\%$? The par value of the shares is $50.

Solution.—The cost of 1 share of stock at the market price is $56; the brokerage per share is $.00\frac{1}{8}\times\$50=\$0.06\frac{1}{4}$. Therefore, total cost of 1 share$=\$56+\$0.06\frac{1}{4}=\$56.06\frac{1}{4}$, and the number of shares bought$=\$89{,}700\div\$56.06\frac{1}{4}=1{,}600$ shares. Ans.

93. Rule.—I. *To find the cost of any number of shares of stock, multiply the sum of the market price per share and the brokerage by the number of shares, and the product will be the cost.*

II. *To find the number of shares that can be bought for a given sum, divide the given sum by the cost of one share, including the brokerage, and the quotient will be the number of shares.*

Example 1.—How much must be invested in railroad stock that pays a quarterly dividend of $2\frac{1}{2}\%$, in order to have an income of $4,000, if it is bought at $104\frac{1}{2}$, brokerage being $\frac{1}{8}\%$?

Solution.—The expression "a dividend of $2\frac{1}{2}\%$" means $2\frac{1}{2}$ per cent. on the *par* value of the stock. Hence, since the *quarterly* dividend is $2\frac{1}{2}\%$, the *annual* income per share of $100 will be $10. Consequently, to obtain an annual income of $4,000, there must be bought $4{,}000\div10=400$ shares; then each share will cost $\$104\frac{1}{2}+\$\frac{1}{8}$, and their total cost will be 400 times as much, or $\$104\frac{5}{8}\times400=\$41{,}850$. Ans.

Example 2.—What per cent. is realized by buying 4% bonds at $89\frac{7}{8}$, brokerage being $\frac{1}{8}\%$?

Solution.—Since a bond of $100 denomination costs $\$89\frac{7}{8}+\$\frac{1}{8}=\$90$, and each $100 bond yields $4 annual income, the per cent. realized will be found by dividing $4 by the entire cost of one $100 bond. Hence,

$$\$4 \div (\$89\tfrac{7}{8} + \$\tfrac{1}{8}) = .04\tfrac{4}{9} = 4\tfrac{4}{9}\%. \quad \text{Ans.}$$

In the preceding example, consideration has not been taken of the fact that the purchaser of the bond will receive par when the bond becomes due if he holds it until that date. He pays 90 and will receive 100, hence he will realize 10% profit on the par value of the bond in addition to the interest. The 10% profit will not be realized until maturity, so the actual yield depends on the date the bond is due. The method of computing the true yield on bonds is rather complex, but tables are published from which rates may be determined.

94. Rule.—I. *To find the investment that will yield a given income, divide the income by the gain from one share, and the quotient will be the number of shares that must be bought; then multiply the cost of one share by the number of shares, and the product will be the investment.*

II. *To find the rate per cent. of income from money invested in stocks or bonds, divide the gain yielded by one share by the cost of a share, and multiply the quotient by 100.*

EXAMPLES FOR PRACTICE

1. What must be paid for 128 shares of U. S. Steel preferred stock at $128\frac{1}{4}$, brokerage $\frac{1}{8}\%$? Ans. $16,432

2. How many shares of Union Gas Co. stock at $98\frac{3}{4}$ can be bought for $39,550, the brokerage being $\frac{1}{8}\%$? Ans. 400 shares

3. How much will 68 $1,000 U. S. 4% bonds of 1925 cost at $116\frac{1}{2}$, brokerage being $\frac{1}{8}\%$? Ans. $79,305

4. The cost of some railroad stock was $18,150, for which a man paid $\$137\frac{3}{8}$ per share, and $\frac{1}{8}\%$ brokerage. How many shares did he buy? Ans. 132 shares

5. Find the cost of 240 shares of mining stock at $98\frac{3}{8}$, brokerage being $\frac{1}{4}\%$. Ans. $23,670

6. How much must be paid for 5% city bonds to yield an annual income of $1,250, if they cost $104\frac{7}{8}$, brokerage $\frac{1}{8}\%$? Ans. $26,250

7. Bank stock that pays an annual dividend of 10% is bought for $109\frac{7}{8}$, brokerage $\frac{1}{8}\%$. What per cent. is realized by investing in it? Ans. $9\frac{1}{11}\%$

8. How much more is the rate per cent. of income on 8% stock bought at $119\frac{7}{8}$ than on 6% stock bought at $107\frac{7}{8}$, brokerage in each case being $\frac{1}{8}$%? Ans. $1\frac{1}{9}$%

9. How much better is a gain of 20% on an investment at 80 than a gain of 18% on an investment at 90? Ans. 5%

EXCHANGE

95. The subject of **exchange** treats of the methods of paying debts in a distant place without transmitting money. Exchange between different parts of the *same* country is **domestic exchange,** and between *different* countries is **foreign exchange.**

DOMESTIC EXCHANGE

96. The payment of debts in a distant place may be made by means of a post-office money order, an express money order, a personal check, or a draft. At all post offices of importance and at express company offices one may obtain forms giving the rates for money orders and directions for use. If a personal check is used a delay of several days may be caused, as most business houses hold an order for goods until collection has been made on the check. Some business houses will refuse to accept a personal check. The most general method of making payments at a distance is by draft.

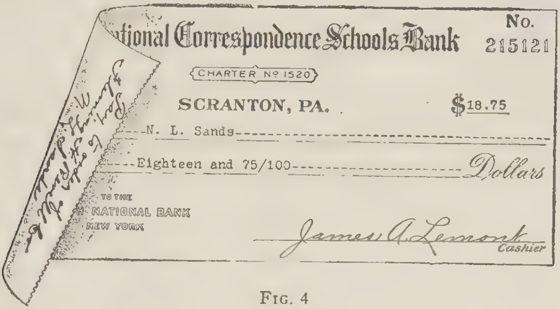
97. The Bank Draft.—A bank draft, which is an order issued on one bank by another, is one convenient means of transmission of credit. Banks in the smaller cities usually have money on deposit in the banks of large cities such as New York or Chicago, subject to draft at any time.

To illustrate the method of using a bank draft, let us suppose that N. L. Sands, of Scranton, Pa., wishes to buy some books from the publishers, Fleming H. Revell Company, New York City, the catalog price of which is $18.75, and decides to send a bank draft. Mr. Sands will apply to his bankers in Scranton for a New York draft, drawn in his own favor for $18.75. They will issue this for a few cents and charge the entire amount, say $18.85, to his bank account. Many banks do not charge their depositors for issuing bank drafts. Mr. Sands indorses the draft to the publishers by writing on the back of it,

Pay to order of
Fleming H. Revell Co.
N. L. Sands

He then incloses it in the letter with his order. Fig. 4 shows the bank draft drawn in Mr. Sands' favor with his indorsement on its back. This is a safe and economical means of paying a bill at a distance.

98. Collection by Draft.—A draft is a written order from one party to another party to pay a certain sum of money to a third party. These three parties are always connected with every draft transaction; namely, the party who makes, or signs, the draft, called the *drawer;* the party who is to pay it, or on whom it is drawn, called the *drawee;* and the party to whom the money is to be paid, called the *payee.* A draft may be drawn *at sight* or *on demand,* and it is then expected that it will be paid by the drawee when it is presented to him; or, it may be written at two, three, or any number of days' sight, which means that the drawee is allowed that many days after presentation in which to pay it. The name of the *drawee* is usually written near the lower left-hand corner of the face of the draft. The *payee* is usually a bank with which the drawer does business. The drawee accepts the draft, that is,

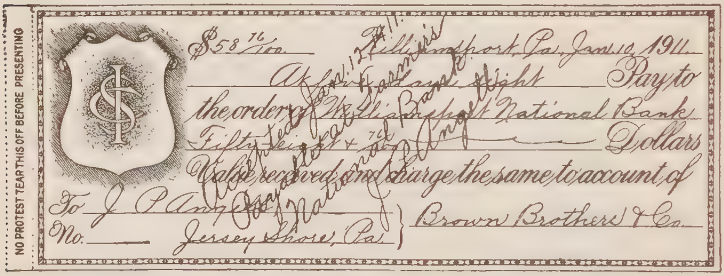


tional Correspondence Schools Bank No. 215121
CHARTER No 1520
SCRANTON, PA. $18.75
N. L. Sands
Eighteen and 75/100 Dollars
TO THE
NATIONAL BANK
NEW YORK
James A. Lemont
Cashier

Fig. 4

promises to pay it, by writing or stamping on its face the word "Accepted," together with the current date and his signature.

Suppose for example that Brown Brothers & Co., wholesale grocers, of Williamsport, Pa., have shipped $58.76 worth of groceries to J. P. Angell, retail grocer, Jersey Shore, Pa., and that they wish to collect this amount. They go to the Williamsport National Bank, where they do their banking business, and request it to forward a draft on Mr. Angell that they, Brown Brothers & Co., have already filled in. The Williamsport bank indorses the draft to a Jersey Shore bank and forwards it to that bank with instructions to collect it. The Jersey Shore bankers promptly present the draft to Mr. Angell by messenger or notify him by telephone or perhaps by mail that they have received it for collection. Mr. Angell is not compelled to pay his debt in this way; but, if he refuses to do so, he is liable to lower his credit with the banks and business men in his community. Therefore, he *accepts* the draft by writing his name across its face, usually in red ink, pays the money to the Jersey Shore bankers within the five days named, and receives the draft, marked paid, as his receipt. They remit to the Williamsport bankers, who credit to the account of Brown Brothers & Co. the $58.76 less a small commission that they charge for making the collection. In this transaction Brown Brothers & Co. are the *drawers* of the draft, the Williamsport National Bank is the *payee*, and J. P. Angell is the *drawee*. The draft, with Mr. Angell's acceptance written across its face, would appear as in Fig. 5.



NO PROTEST TEAR THIS OFF BEFORE PRESENTING

$58 76/100 Williamsport, Pa., Jan. 10, 1911.

At five days sight Pay to

the order of Williamsport National Bank

Fifty-eight & 76/100 Dollars

Value received and charge the same to account of

To J. P. Angell } Brown Brothers & Co.

No. — Jersey Shore, Pa.

Accepted Jan 12/11

Payable at Jersey Shore National Bank

J. P. Angell

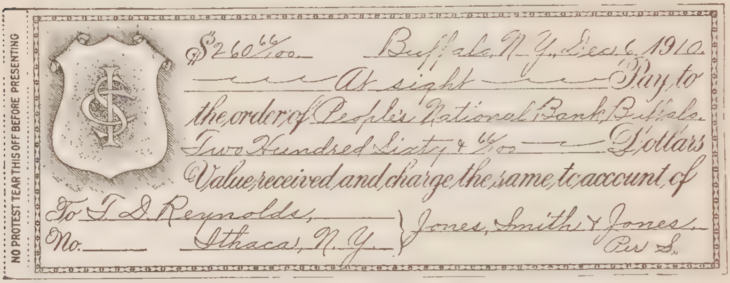
FIG. 5

When a draft is accepted it becomes in effect a promissory note.

The student should carefully observe how the form is filled in. The wording of the last line of the form may be misunderstood. The drawee, Mr. Angell, is directed to charge the amount of the draft against the drawer, Brown Brothers & Co. The name of the drawee follows the word "To" and that of the drawer is written on the right, as shown.

99. The Bill of Lading.—A bill of lading is a printed contract, furnished free by railroad and steamboat companies, on which the shipper, or *consignor,* is required to write the name and address of the person to whom the shipment is to be sent, called the *consignee,* and an itemized statement of the number of pieces, their weight, a brief description of the goods shipped, and his own name as shipper. Always two, and sometimes three, copies of the bill of lading are made at the same time and for each shipment. The first copy is called a *straight bill of lading.* It is signed by the railroad agent and returned to the shipper, who usually mails it to the consignee, so that he may take it to the railroad agent at the point of destination as proof that the goods are for him. The second, or duplicate, copy, called the *shipping order,* is signed by the shipper and retained by the railroad agent as his directions for shipping the goods. The triplicate copy is called a *memorandum* and is retained by the consignor as his receipt.

100. The Sight Draft With Bill of Lading.—Suppose a retail storekeeper, T. D. Reynolds, of Ithaca, N. Y., orders goods from Messrs. Jones, Smith & Jones, wholesalers, of Buffalo, N. Y., amounting to $260.66, and asks them to ship the goods by freight, collect on delivery. The steps in the transaction would be about as follows: Jones, Smith & Jones deliver the goods to the railroad company and receive from the freight agent a *bill of lading,* showing that the goods are consigned to themselves or to their order. The goods are not consigned to Mr. Reynolds. Jones, Smith & Jones make a sight draft on Mr. Reynolds for $260.66, attach it to the bill of lading, and hand both to their bankers in Buffalo, who forward it to a bank in Ithaca with instructions to deliver the bill of lading to Mr. Reynolds when he pays the attached draft. Fig. 6 shows the sight draft on T. D. Reynolds that went to the Ithaca bank with the bill of lading. Messrs. Jones, Smith & Jones promptly mail to Mr. Reynolds an itemized bill for the goods and notify him that their draft for $260.66 has been forwarded through their bankers. Mr. Reynolds calls at the Ithaca bank, pays the draft, receives the bill of lading, presents it to the railroad freight agent, and is allowed to take the goods.



NO PROTEST TEAR THIS OFF BEFORE PRESENTING

$260 66/100 Buffalo, N. Y., Dec. 6, 1910

At sight Pay to

the order of People's National Bank, Buffalo

Two Hundred Sixty & 66/100 Dollars

Value received and charge the same to account of

To T. D. Reynolds, Jones, Smith & Jones.

No. ____ Ithaca, N. Y. Per S.

Fig. 6

101. To find the cost of a bank draft.

The charge, called exchange, made by a bank for a draft is usually stated as a percentage or as the rate per $1,000.

Example 1.—What will a sight draft of $2,400 in San Francisco cost in New York when the charge is $\frac{1}{2}$%?

Solution.—Exchange = $\frac{1}{2}$% of $2,400 = $\frac{1}{2}$ of $24 = $12.

Total cost = $2,400 + $12 = $2,412. Ans.

Example 2.—Mr. Rodriguez, of New Orleans, wishes to pay $24,600 to a New York merchant. What will a draft cost him when exchange is $1 per $1,000?

Solution.—The exchange equals 1 thousandth part of the face value of the draft; hence, the charge may be obtained by pointing off three places in $24,600. The charge is therefore $24.60, and the entire cost of the draft equals $24,600 + $24.60 = $24,624.60. Ans.

102. To find the proceeds of sight and time drafts.

The drawer of a draft may not wish to wait for the collection to be made and may sell it. The purchaser may or may not give face value for it. If sold below its face value it is said to be sold at a discount, and if above it, at a premium. A time draft is discounted in the same manner as a promissory note. The purchaser may also deduct charges sufficient to cover cost of collection.

EXAMPLE 1.—A wholesale dealer sold a sight draft on a customer for $5,800 at $\frac{3}{4}$% discount. What were the proceeds?

SOLUTION.— Face value of draft=$5800.00
Discount=$\frac{3}{4}$% of $5,800= 43.50
Proceeds=$5756.50 Ans.

EXAMPLE 2.—A furniture manufacturer of Grand Rapids deposited in a bank a 30-day draft for $28,000 on a Memphis dealer. How much credit would he receive for the draft if no charges were made to cover collection? Assume the interest rate to be 6%.

SOLUTION.— Face value of draft=$28000
Discount on $28,000 for 30 da. at 6%= 140
Proceeds=$27860 Ans.

103. A commercial draft may be sold at a premium or a bank draft may be purchased at a discount when there is a tendency of money to move strongly in one direction. Suppose, for example, that the San Francisco banks owe considerably more to the New York banks than the latter owe in return. To adjust the difference, money would be shipped to New York. It is possible that a New York bank would, under such circumstances, sell a draft on San Francisco at a discount, and that in San Francisco a holder of a draft on New York could sell it at a premium. Such drafts would tend to lessen the shipment of currency.

In some states 3 days of grace are allowed on sight and time drafts. If one needs to know at any time whether grace is allowed in any particular state he can obtain the information from a bank.

104. To find the face of a draft when the proceeds are known.

EXAMPLE.—The proceeds of a sight draft sold at $\frac{3}{4}$% discount are $2,977.50. What is its face value?

SOLUTION.—The face can be found, when the proceeds are known, in a manner similar to that used in finding the face of a note when the proceeds are known.

Proceeds of $1=$1.00−.00$\frac{3}{4}$=$.99$\frac{1}{4}$
Face=$2,977.50÷.9925=$3,000. Ans.

105. Rule I.—*To find the cost of a sight draft, find the premium or the discount. The sum of the face of the draft and the premium, or the difference between the face of the draft and the discount, will be the cost of the draft.*

106. Rule II.—*To find the cost of a time draft, find the proceeds of the face of the draft for the time the draft has to run. Find, also, the premium or the discount, on the face of the draft. The sum of the proceeds and the premium, or the difference between the proceeds and the discount, will be the cost of the draft.*

If days of grace are allowed in the state where the draft is payable, find the proceeds for three days more than the time the draft has to run, and then proceed as before.

107. Rule III.—*To find the face of a draft when the cost or the proceeds is given, divide the cost of the draft by the cost of $1.*

EXAMPLES FOR PRACTICE

Solve the following examples:

1. Find the cost of a sight draft for $1,876 at: (*a*) $1\frac{1}{4}\%$ premium; (*b*) $\frac{1}{2}\%$ discount.
Ans. { (*a*) $1,899.45
(*b*) $1,866.62

2. The face of a sight draft is $7,875.56, and the premium is $\frac{4}{5}\%$. Find its cost. Ans. $7,938.56

3. How much will it cost to pay, by a sight draft on San Francisco, a bill of $7,528, when exchange is at $1\frac{1}{2}\%$ discount? Ans. $7,415.08

4. A man paid $484.72 for a 60-day draft, premium being $1\frac{1}{4}\%$, and money worth 6% interest. What was the face of the draft?
Ans. $483.51

5. If a draft payable 30 days after sight costs $2,800 when discount is $\frac{3}{4}\%$ and money worth 6%, what is its face? Ans. $2,835.44

6. Find the face of a sight draft costing $1,200, when exchange is at $1\frac{3}{8}\%$ discount. Ans. $1,216.73

FOREIGN EXCHANGE

108. A draft on a foreign country is usually known as a bill of exchange. Foreign bills can be used in the same manner as ordinary drafts. The face of the draft is expressed in the currency of the country on which the draft is drawn. The price of exchange fluctuates according to the balance of trade, the loans and investments in foreign countries, and other conditions that tend to cause a movement of money from one country to another. The Secretary of the Treasury of the United States issues at certain periods a statement showing the value of foreign coinage in dollars and cents.

109. The daily papers of our commercial cities give quotations showing the rates of exchange from day to day. One of these follows:

Sterling exchange was again weak and lower. Continental exchange was also lower. Rates are: Long bills, \$4.81 @ $\$4.82\frac{3}{4}$; sight drafts, $\$4.84\frac{3}{4}$ @ \$4.85, and cable transfers, $\$4.85\frac{1}{4}$ @ $\$4.85\frac{1}{2}$. Francs are quoted at $5.21\frac{7}{8}$ for long and 5.20 for short; reichsmarks, $94\frac{1}{2}$ @ $94\frac{9}{16}$ for long and $95\frac{1}{8}$ @ $95\frac{3}{16}$ for short; guilders, $39\frac{7}{8}$ @ $39\frac{15}{16}$ for long and 40 @ $40\frac{1}{16}$ for short.

NOTE.—A reichsmark (mark of the empire) is the same as a mark, about $23\frac{3}{4}$ cents. The exchange value of 4 marks, or reichsmarks, is given in commercial quotations in the daily newspapers.

Sterling exchange is Bills of Exchange payable in English money called Pounds Sterling. Long bills are those payable 30, 60, 90, or more days after being received. Short bills are those payable from sight to 30 days after being received. Sight drafts are payable at sight, that is, as soon as received. Long bills will evidently sell for less than those drawn on sight.

Edward Howe wishing to pay a debt in England may buy from Smith, Jones & Co. a draft like the following:

New York, Oct. 1, 1912

Exchange for £820-12-6 sterling.

At sight pay to Edward Howe, or order, the sum of Eight Hundred Twenty Pounds £820-12-6 sterling.

Value received, and charge to the account of

Smith, Jones & Co.

To Baring Bros. & Co.
London, England.

Example 1.—Find the cost of the foregoing draft in New York when exchange on London is $\$4.84\frac{3}{4}$.

Solution.— £820-12-6=£820 12s. 6d.=£820.625, since 6d.$=\frac{6}{12}=\frac{1}{2}$s., and 12s. 6d. $=12\frac{1}{2}$s.$=\frac{12\frac{1}{2}}{20}=$£.625. $\$4.8475\times820.625=\$3{,}977.98$. Ans.

Example 2.—What must be paid for a draft on Paris of 8,000 francs, when \$1 is quoted at $5.21\frac{1}{2}$?

Solution.— 1 franc$=\$1\div5.215$;
$(\$1\div5.215)\times8{,}000=\$1{,}534.04$. Ans.

110. **Rule.**—*To find the cost of a draft upon a foreign country, multiply the quoted value of a foreign monetary unit by the given number of such units.*

EXAMPLES FOR PRACTICE

Solve the following examples:

1. Find the cost of a draft on London, at 60 days' sight, for £987 16s., exchange being $\$4.82\frac{3}{4}$. Ans. \$4,768.60

2. When exchange on Paris is quoted at 5.23, what must be paid for a sight draft for 2,800 francs? Ans. \$535.37

3. I bought a long draft on Berlin for 8,425 reichsmarks when exchange was quoted at $94\frac{3}{4}$ per 4 reichsmarks. What did it cost? Ans. \$1,995.67

4. What must be paid for a draft on Amsterdam for 8,000 guilders, exchange being $40\frac{1}{16}$? Ans. \$3,205

DUTIES

111. **Duties,** or **customs,** are taxes levied by governments on imported goods for the purpose of producing revenue and for the protection of home industries.

112. There are two kinds of duties: *ad valorem* and *specific*. An **ad valorem** duty is estimated at a certain per cent. of the market value of the goods in the country from which they are imported; as, silks 50%, musical instruments 15%, etc. The market value of the goods is the invoice value after deducting discounts and before extra charges, such as commission, freight, boxing, etc., are added.

113. A **specific** duty is a duty levied on imported goods according to the weight, measurement, or number of the articles, without reference to their value; as, wheat 15 cents per bushel, coal 75 cents per ton, etc. Some kinds of merchandise are subject to both ad valorem and specific duties. In computing specific duties, the long ton of 2,240 pounds and the hundredweight of 112 pounds are used.

114. An **invoice** is an itemized statement of merchandise shipped. It contains the names of purchaser and seller, a description of the quality and quantity of the goods, prices, and incidental charges. Invoices are made out in the weights and measures and the currency of the country from which the goods are imported. Thus, the price and cost of goods imported from Germany would be given in *marks;* from France, in *francs;* from England in *£ s. d.*

115. The following table gives the monetary units of leading foreign nations and their equivalents in United States money. These rates are proclaimed each year by the Secretary of the Treasury, and are used in Custom House computations:

Country	Monetary Unit	Value in U. S. Gold
Canada	Dollar = 100 cents	$1.00
Great Britain	Pound = 20 shillings	4.8665
France Belgium Switzerland	Franc = 100 centimes	.193
Italy	Lira = 100 centesimi	.193
Spain	Peseta = 100 centimes	.193
Germany	Mark = 100 pfennige	.238
Denmark Norway Sweden	Crown = 100 öre	.268
Japan	Yen = 100 sen	.498

116. Before computing duties the following allowances are made: **Tare,** a deduction for the weight of boxes or crates; **leakage,** an allowance for loss of liquids imported in barrels or casks; and **breakage,** an allowance for loss of liquids imported in bottles. The **net quantity** is what remains after deducting tare, leakage, or breakage.

117. Ad valorem duties are computed by the rules of percentage; the net invoice price is regarded as the *base,* the ad valorem duty as the *percentage,* and the rate of duty as the *rate.*

Duties are not computed on fractions of a dollar; if the cents are less than 50 they are rejected; if more, they are counted as a dollar.

118. Rule.—*To find the ad valorem duty, reduce the net invoice price to United States money, if necessary, deduct allowances, and multiply the remainder (expressed in even dollars) by the rate of ad valorem duty.*

To find the specific duty, multiply the net quantity by the rate of specific duty per unit of quantity.

EXAMPLE 1.—What is the duty on an invoice of silks valued at 24,360 francs, the ad valorem rate being 60%?

SOLUTION.—According to Art. **115,** 24,360 francs=24,360×.193 =$4,701.48. Duty=$4,701×.60=$2,820.60. Ans.
The 48 cents is rejected, being less than 50 cents. (Art. **117.**)

EXAMPLE 2.—What is the duty on 820 gallons of brandy at $1.50 per gallon, leakage 3%?

SOLUTION.— Leakage=820×.03=24.6 gal.
Net quantity =820−24.6=795.4 gal.
Duty=795.4×1.50=$1,193.10. Ans.

EXAMPLES FOR PRACTICE

What is the ad valorem duty on an importation invoiced at
(*a*) £430 12s. 4d., allowing 5% breakage, rate of duty 40%?
(*b*) 36,750 lira, allowing 2% for tare, rate of duty 24%?
(*c*) 9,264 marks, rate of duty 85%?
What is the specific duty on an importation of
(*d*) 60 dozen bottles of wine at $3 per dozen, breakage 10%?
(*e*) 125 gross of empty bottles, breakage 6%, duty 10 cents per dozen?
(*f*) 3 T. 6 cwt. of iron castings at ¾ cent per pound?

Ans.—(*a*) $796.40; (*b*) $1,668.24; (*c*) $1,874.25; (*d*) $162; (*e*) $141; (*f*) $55.44.

1. An importation of musical instruments from Germany is valued at 13,670 marks. What is the duty at 17½% ad valorem? Ans. $569.28

2. An importer buys French silks at $1.80 per yard and pays a duty of 35% ad valorem, and $.60 per yard specific. At what price per yard must the silk be sold to yield a profit of 25% on the cost? Ans. $3.79

3. What is the duty at 65%, upon a consignment of 1,350 dozen kid gloves invoiced at 115 francs per dozen? Ans. $19,475.95

EQUATION OF PAYMENTS

119. Suppose that Mr. Ellison, a dealer, owes a wholesale house a bill of $100 payable June 10 and another bill of $100 payable June 30. It is evident that he may pay the entire amount on June 20, a date midway between June 10 and June 30, without paying interest or receiving discount, for he will owe on June 20, 10 days' interest on the first bill and he will be entitled to 10 days' discount on the second bill. The process of finding the equitable time when payment of several sums may be made in one payment is called **equation of payments.** The date at which several bills may be equitably paid is called the equated time of payment.

Example.—A man owes $250 due March 1, $300 due April 20, $450 due May 5, and $500 due June 25. When is the equated time of payment?

Solution.—We first find the interest that is due on some date, preferably the latest due date. This date is called the *focal date.* The latest due date is June 25. From March 1 to June 25 is 116 da., hence we compute the interest of $250 for 116 da. It does not matter what rate of interest is taken in finding the equated time, but we shall assume the rate to be 6% and use the 60-day method in calculating the interest. The interest on $250 for 60 days is $2.50, and for 116 days is $4.83.

From April 20 to June 25 is 66 da.; the interest on $300 for 66 da. at 6% is $3.30. From May 5 to June 25 is 51 da.; the interest on $450 for 51 da. is $3.83. No interest is computed on $500, since this sum is due on June 25. The total interest is $11.96; the total amount due is $1,500. We now proceed to find at what earlier date $1,500 may be paid so that no interest may be due.

The interest on $1,500 for 1 da. at $6\% = \frac{\$15.00}{60} = \$.25$

The interest on $1,500 will amount to $11.96 in $\frac{11.96}{.25} = 48$ da.

The account may be settled 48 da. earlier than June 25, that is, on May 8. May 8 is, therefore, the equated time of payment.

The arithmetical work should be arranged as follows:

Due Date	Amount	Time	Interest
Mar. 1,	$250	116 da.	$4.83
Apr. 20,	300	66 da.	3.30
May 5,	450	51 da.	3.83
June 25,	500	no da.	.00
	$1500		$.25) $11.96
			48 da., very nearly

$$\frac{\$15.00}{60}=\$.25$$

Equated time is 48 da. earlier than June 25, or May 8. Ans.

EXAMPLES FOR PRACTICE

Solve the following examples:

1. A owes B $500 due in 8 months, and $900 due in 4 months. When may he equitably pay B both debts in one payment?
Ans. 5 mo. 13 da.

2. Find the equated time for paying $400 due May 10, $500 due June 20, $900 due July 30, and $1,000 due Aug. 15. Ans. July 17

3. Tefft, Weller & Co. sold to E. King & Co. goods as follows: on June 15, $2,500 on 30 days' credit, and, on June 30, $3,600 on 20 days' credit. Find the equated date of payment. Ans. July 18

4. What is the equated time for the payment of three notes: one for $600, dated Aug. 9, 1912, for 3 months; the second for $800, dated Oct. 1, 1912, for 2 months; the third for $1,200, dated Dec. 21, 1912, for 6 months? Ans. Feb. 27, 1913

5. On Jan. 1, 1914, a merchant sold a bill of goods amounting to $3,600, payable as follows: $\frac{1}{2}$ in 30 days, $\frac{1}{3}$ in 60 days, and the remainder in 90 days. Find the equated time of payment. Ans. Feb. 20, 1914

EQUATION OF ACCOUNTS

120. The time at which an account having a debit and credit side may be equitably settled can be found in much the same manner as the equated time of payment. The process is called the **equation of accounts.** The method is employed particularly by business houses selling goods on credit, since by this method the balances of accounts may be readily determined at any time.

In finding the equated time of an account some date, called the focal date, is selected and the standing of the account with reference to that date is determined, interest being reckoned to that date on all bills due and on all payments previously made, and discount being computed on items of the account not due at that date. If the balance of interest on the credit side of the account is greater, credit should be extended for the balance of the account to equalize the excess of interest. If, on the other hand, the interest is greater on the debit side, the equated time of payment will be earlier than the focal date. The solutions of the following examples will make the method clear.

Example.—Find the equated time for the settlement of the following account:

Henry Wardell

1911				1911			
Jan.	20	Mdse., 30 da.,	800	Mar.	1	Cash,	400
Feb.	18	Mdse., 90 da.,	600		20	Cash,	600
Mar.	14	Mdse., 60 da.,	1,000	Apr.	1	Cash,	1,000
Apr.	10	Mdse., 30 da.,	1,200		20	Cash,	500

Solution.—The date of each item and the amount should be shown as in the following table. When credit is given, the date at which the bill is due is set down and not the date of sale. For example, 30 da. credit is given on the bill sold Jan. 20, making the bill due on Feb. 19. The bill of goods sold on Feb. 18 is due May 19, that sold March 14 is due May 13, and that sold April 10 is due May 10. The latest date, which is May 19, will be considered as the focal date. The interest is computed at 6%.

			INTEREST				INTEREST
Feb. 19,	$ 800	89 da.	$11.87	Mar. 1,	$ 400	79 da.	$ 5.27
May 19,	600	no da.	.00	Mar. 20,	600	60 da.	6.00
May 13,	1000	6 da.	1.00	Apr. 1,	1000	48 da.	8.00
May 10,	1200	9 da.	1.80	Apr. 20,	500	29 da.	2.42
	$3600		$14.67		$2500		$21.69
	2500						14.67
	$1100					$.183)	$7.02
							38 da.

Interest on $1,100 for 1 da. = $.183.
38 days after May 19 is June 26, the equated time of payment. Ans.

EXPLANATION.—The condition of the account on May 19 is determined. Interest is charged on each bill of goods for the time from the date the bill is due to the focal date, and credit is given for interest on each payment, reckoned from date of payment until the focal date. The total interest on the debit side is $14.67 and on the credit side is $21.69. Mr. Wardell should, therefore, receive credit for $7.02 if the bill is settled on the focal date. To find how far credit may be extended for $1,100, the balance of the account, we divide $7.02 by the interest on $1,100 for 1 day. The interest on $1,100 for 1 day is $.183. In 38 days the interest will amount to $7.02. Mr. Wardell should have credit until 38 days after May 19, or until June 26, for the balance of the account.

The earliest date can be taken as the focal date and the discount on each item computed. Again, the focal date can be taken between the earliest and latest dates. In this case interest must be computed on some items and discount on others, but with the aid of a little thought the method should not present any great difficulty in determining when the balance may be paid without discount or interest.

121. Some accountants prefer to compute the equated time by what is known as the **products method.** The preceding example is solved as follows by that method:

SOLUTION.—

Feb. 19,	$ 800×89=	$71200	Mar. 1,	$ 400×79=	$31600
May 19,	600×00=	00	Mar. 20,	600×60=	36000
May 13,	1000× 6=	6000	Apr. 1,	1000×48=	48000
May 10,	1200× 9=	10800	Apr. 20,	500×29=	14500
	$3600	$88000		$2500	$130100
	2500				88000
	$1100			$1100)	$42100
					38 da.

EXPLANATION.—In this method the interest is not actually computed, but the equivalent principal placed at interest for 1 day is found. Taking the first item, the interest on $800 for 89 days is equal to the interest on $800×89=$71,200 for 1 day. There is no interest due on $600 payable May 19. The interest on $1,000 for 6 days is equal to the interest on $1,000×6 =$6,000 for 1 day. Each item is multiplied by the number of days that the due date or date of payment precedes the focal date. When these products are added it is seen that Mr. Wardell owes on the focal date the interest on $88,000 for 1 day. He should, on the other hand, receive credit for the interest on $130,100 for 1 day. The difference in the interest accounts on the focal date is equivalent to the interest on $42,100 for 1 day. Dividing $42,100 by $1,100 gives the number of days in which the interest on $1,100 equals the interest on $42,100 for 1 day. Credit for payment of the balance may be extended to 38 days after the focal date.

It may be observed that this solution gives the same result as that in Art. **120.**

122. In the products method, it is not necessary to calculate the interest on the items. The actual calculation is easier than that by the interest method, but large numbers may be involved. In the interest method, much smaller numbers are used and an error is more likely to be detected. Moreover, by this method the balance of an account, including interest, can be found directly on any date.

EXAMPLE 1.—What will be the equated time of payment of the following account and what will be the balance due: on Jan 2? on Jan. 31? Interest at 5%.

WM. BONNER

1913				1913			
Sept.	1	Mdse., 90 da.,	800	Sept.	12	Cash,	600
	20	Mdse., 60 da.,	900	Oct.	20	Draft, 30 da.,	500
Oct.	25	Mdse., 60 da.,	1,000	Nov.	15	Cash,	800
Nov.	1	Mdse., 30 da.,	2,000	Dec.	12	Cash,	1,000

SOLUTION.—It will be noticed that one of the payments, that of $500 on Oct. 20, is a 30-da. draft. Since this draft can be cashed for its face value only after 30 da., i. e., on Nov. 19, this payment must be considered as having been made on Nov. 19. The latest date entering in the statement of the account is 60 da. after Oct. 25, or Dec. 24.

			INTEREST				INTEREST
Nov. 30,	$800	24 da.	$3.20	Sept. 12,	$600	103 da.	$10.30
Nov. 19,	900	35 da.	5.25	Nov. 19,	500	35 da.	2.92
Dec. 24,	1000	no da.	.00	Nov. 15,	800	39 da.	5.20
Dec. 1,	2000	23 da.	7.67	Dec. 12,	1000	12 da.	2.00
	$4700		$16.12		$2900		$20.42
	2900						16.12
	$1800					$.30)	$4.30
							14 da.

The interest on $1,800 for 1 da. at 6% is $.30.

The equated time is 14 da. after Dec. 24, which is Jan. 7. Ans.

If the account is settled on Jan. 2, a discount for 5 da. should be made.

Interest on $1,800 for 5 da. at 6%=$1.50; at 5% interest=$1.25.

Amount due Jan. 2 is $1,800−$1.25=$1,798.75. Ans.

Should the account be settled on Jan. 31, interest would be due from Jan. 7, or for 24 da. Interest on $1,800 for 24 da. at 5%=$6.

Amount due on Jan. 31 is $1,800+$6=$1,806. Ans.

EXAMPLE 2.—When should interest begin on the balance of the following account?

HENRY WELLINGTON

1914				1914			
June	16	Mdse.,	1,200	July	21	Cash,	800
July	21	Mdse.,	1,000	Aug.	11	Cash,	1,200
Aug.	13	Mdse.,	2,000	Sept.	2	Draft, 30 da.,	1,600
Sept.	15	Mdse.,	3,600		23	Draft, 10 da.,	1,500

SOLUTION.—

			Interest				Interest
June 16,	$1200	109 da.	$21.80	July 21,	$ 800	74 da.	$ 9.87
July 21,	1000	74 da.	12.33	Aug. 11,	1200	53 da.	10.60
Aug. 13,	2000	51 da.	17.00	Oct. 2,	1600	1 da.	.27
Sept. 15,	3600	18 da.	10.80	Oct. 3,	1500	no da.	.00
	$7800		$61.93		$5100		$20.74
	5100		20.74				
	$2700	$.45)	$41.19				
			92 da.				

Interest on $2,700 for 1 da. = $.45.

Equated time is, therefore, 92 da. preceding Oct. 3, or July 3. In this case the interest on the debit side is greater than that on the credit side. This indicates that Mr. Wellington should pay the balance of $2,700 earlier than the focal date. Had the excess of interest been on the credit side the equated time would have been later than the focal date.

VERTICAL PENMANSHIP

GENERAL INSTRUCTIONS

PRELIMINARY REMARKS

1. Advantages.—The advantages of a good handwriting are too well understood to require mention. Many competent authorities say that the position of the body assumed in vertical writing is less injurious to the health, that time and energy are saved, and that the writing looks better than slant writing.

2. Qualifications.—The only qualifications necessary are a good eye, normal hand, determination, and perseverance. Vertical writing is more easily learned, written, and read than slant writing, and it is reasonable to expect that the student who will pursue these lessons faithfully may become a good penman. The method given of presenting the subject is simple and direct, and experience has shown it to be the very best that has been devised for producing a practical handwriting. The results, however, depend as much on the student as on the plan of instruction.

3. Specimens.—Before proceeding with the lessons, write two pages of specimens, each containing one set of capitals, one set of small letters, one set of figures, and the following short letter:

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

Your address and the current date

International Correspondence Schools,
Scranton, Pa.

Gentlemen:

This is a good specimen of my writing before beginning your lessons in penmanship.

Yours truly,

Your signature

Keep one set of these specimens and send the other to us immediately.

MATERIALS

4. **Quality.**—No one can expect to do good work without good tools, and the student should be provided with the best. The difference between the price of a good and an inferior article for writing purposes is too slight to be considered, and the best materials are an incentive to do good work.

5. **Pens.**—Vertical writing requires a fairly coarse pen with a smooth point. Such a pen will not produce fine hair lines, but it will stand resistance and at the same time produce a smooth, firm line. Spencerian vertical pens graded "fine," "medium," and "coarse"; Esterbrook 556 and 570; and Gillott's vertical pens, are all good. The finer of these are recommended for the more advanced pupils.

6. **Penholder.**—There is nothing better than a plain, straight holder made of either wood or rubber, slightly tapered, and having a cork or rubber base on which to rest the thumb and fingers. A holder with a metal ferrule should be avoided. It is not only unpleasant to the touch but it is injurious to the nerves, on account of the metal being a conductor of electricity, which passes from the pen point to the hand.

7. **Ink.**—In order that one may write with confidence, the ink should flow freely, and it should be as black as

possible. Stafford's, Carter's, Arnold's, and Thomas's are all reliable.

8. **Blotter and Pen Wiper.**—A good blotter and pen wiper should always be at hand. A blotter is used not only to absorb the surplus ink but also to rest the hand on, so that the paper may not be soiled. The pen wiper should be used frequently to keep the pen clean, so that the ink will flow freely from it. A wet sponge is the best wiper. A small piece of chamois skin or a cotton cloth, however, will answer.

9. **Paper.**—Always provide a liberal quantity of good letter or foolscap paper; not necessarily the most expensive but a paper of fair weight, with a hard, smooth finish, without gloss.

POSITION

10. **Position of the Body.**—There are three good reasons why a student should assume a correct position at the desk: First, it permits free play to the circulatory and respiratory organs; second, it allows control of the muscles of the arm and hand; third, the writer feels, appears, and acts better.

Sit squarely in front of the desk, leaning slightly forwards without bending the back. The right arm should be placed on the desk at right angles to the left, both elbows off the desk and quite near the body without touching it. The left hand should be used to hold or steady the paper. The feet should rest squarely on the floor, the left foot a little in advance of the right, a position that enables one to rise without changing the position of the feet. This will give necessary firmness as well as ease and comfort. (See Fig. 1.) While writing, every muscle of the hand, arm, and body should be in a relaxed condition.

11. **Position of the Paper.**—While the ideal position of the paper for children is parallel to the front edge of the desk, as shown in Figs. 1 and 2, the person accustomed to

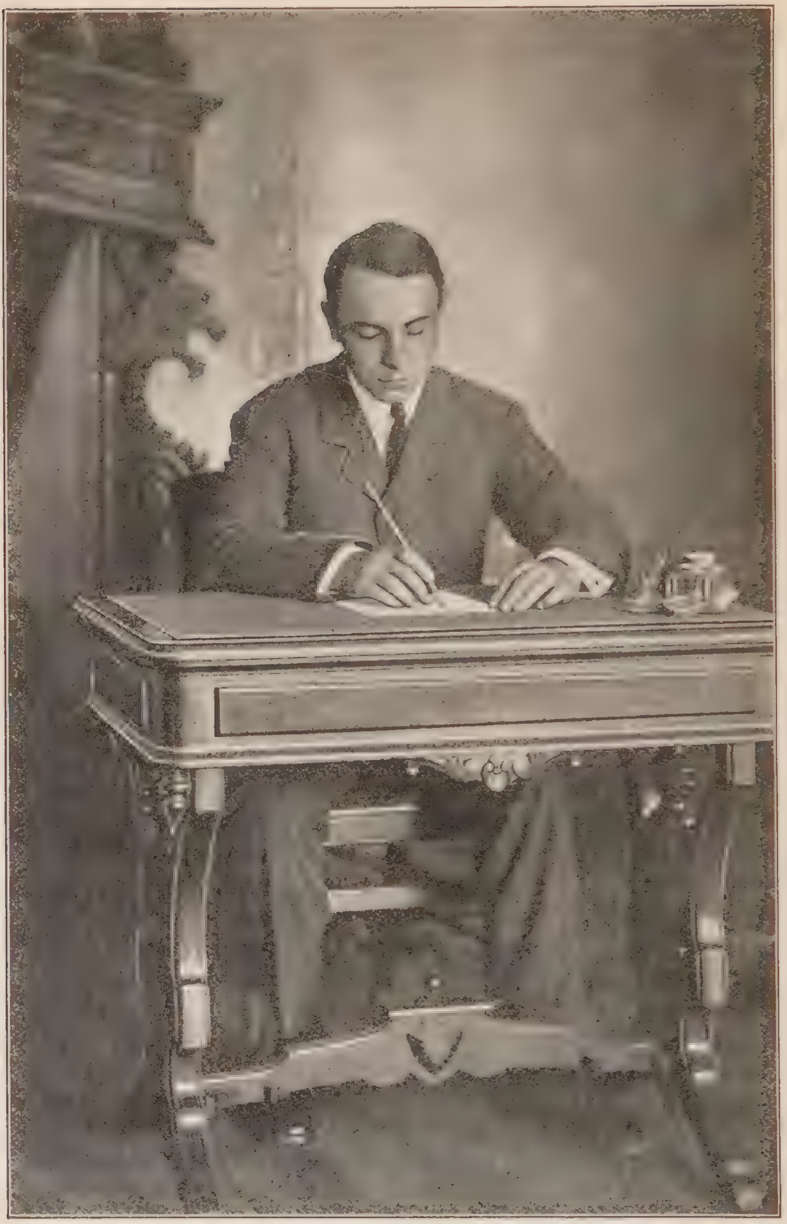


Fig. 1

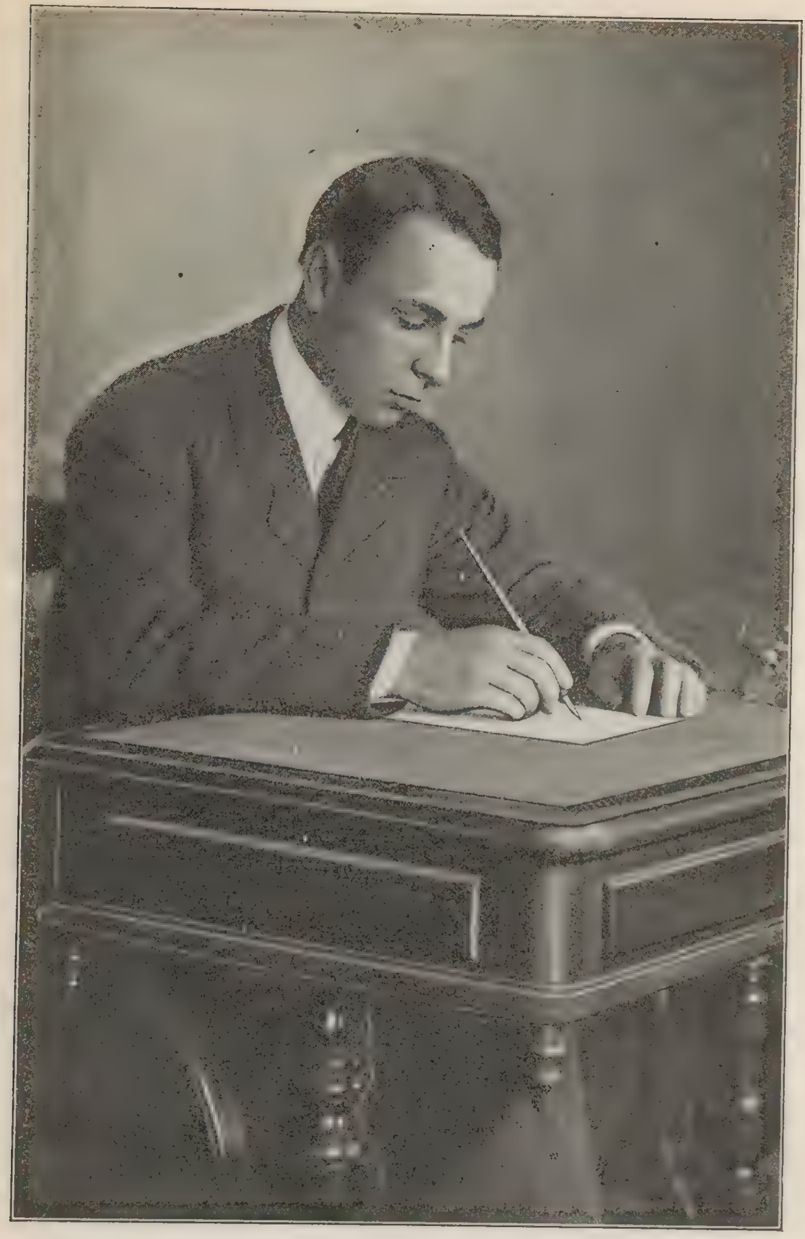


FIG. 2

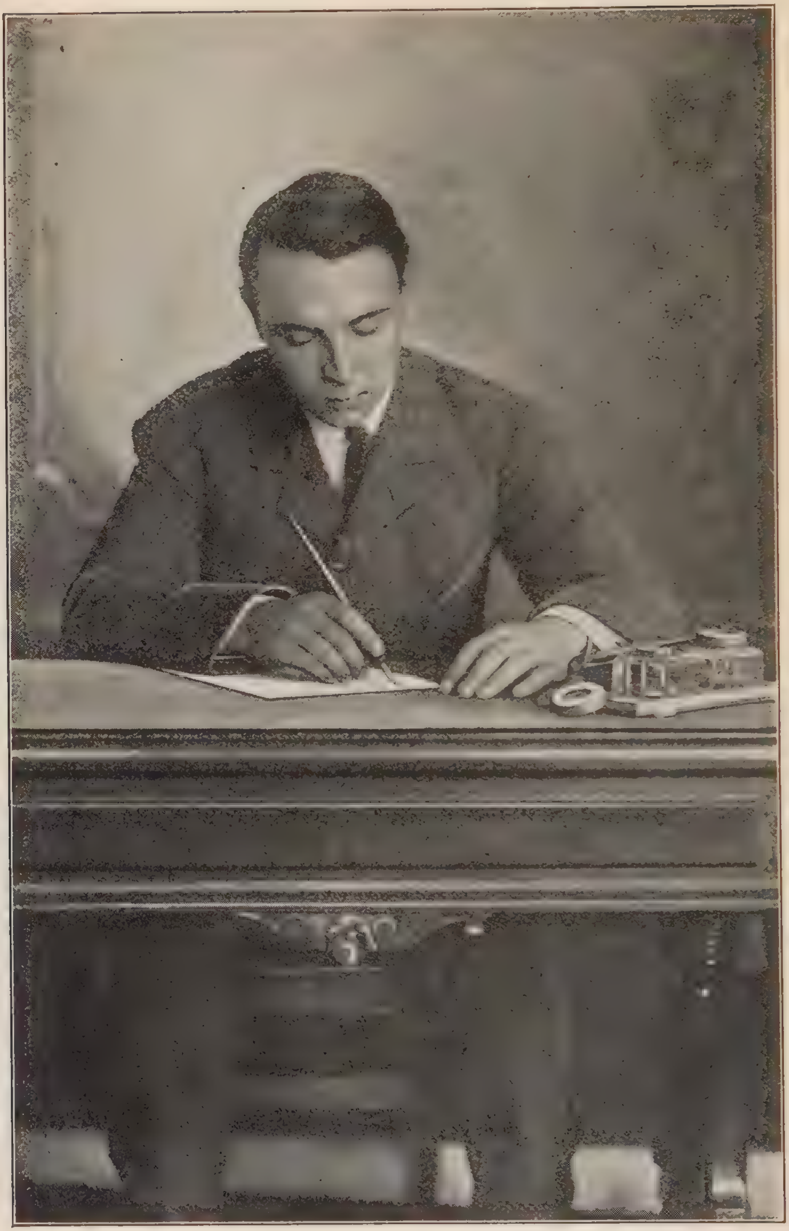


FIG. 3

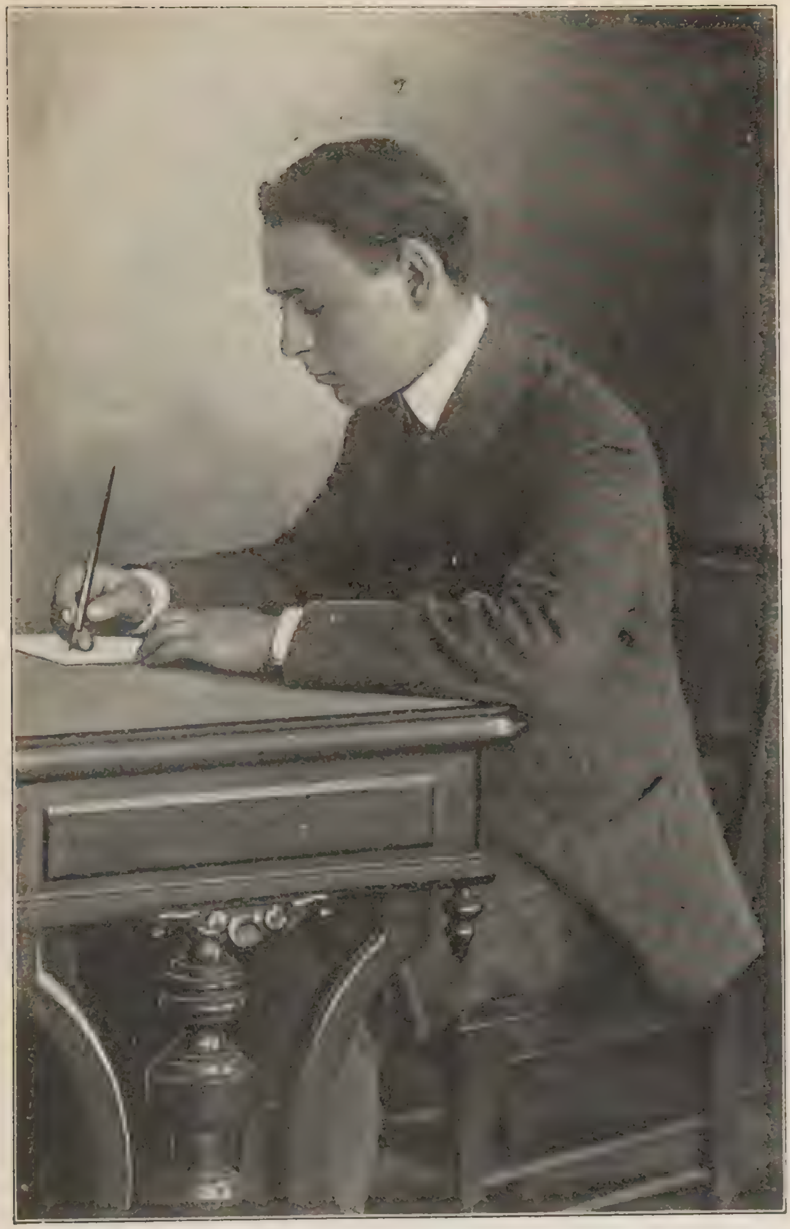


FIG. 4

slant writing will find it more natural and easy to slant the paper a little to the right, as shown in Fig. 3, so that the arm will have more room to rest on the desk, but not so much as in slant writing. The paper should be placed a little to the right of the middle of the body, as shown in Figs. 1 and 3, so that the pen will begin at a point opposite the breast bone. If the pen begins at a point to the left of the breast bone, the writing will slant to the left of the vertical, and will lead to backhand writing. It is very important that these points should be carefully observed and followed until they become a habit.

12. Position of the Hand.—One of the strongest claims made for the superiority of vertical over slant writing is the naturalness of the position of the hand in holding the pen. No turning or twisting of the hand or wrist into an abnormal position is necessary in vertical writing. Drop the hand to the side in a natural position; then raise and place it on the desk as you find it. All of the fingers will be in a slightly bent position, as shown in Figs. 2 and 3, the first finger being bent a little less than the second; the second, less than the third; the third, less than the fourth. Place the penholder between the thumb and first finger, both being in a slightly bent position, with the first finger on the top of the holder and about 1 inch from the point of the pen, as shown in Fig. 2; place the end of the thumb on the holder opposite the first joint of the second finger, as shown in Fig. 4. The second finger should drop a little to the right of the holder, so that the holder will pass opposite the root of the finger nail, as in Fig. 4. The third and fourth fingers should be drawn back so that they separate from the others at the first joint of the second finger, as in Figs. 1 and 2, resting them on the sides of the fingers, as in Fig. 4, or, better, on the nails, as in Fig. 3. This, with some shaped hands, is difficult. One of the best tests of the correctness of the position of the fingers holding the pen is that they can be moved easily and naturally and that the pen so held will move in a vertical line.

MOVEMENTS

13. There are four movements recognized in writing: the *whole-arm*, the *finger*, the *muscular* or *forearm*, and the *combined* or *mixed*.

14. The **whole-arm movement** consists of the independent motion of the whole arm from the shoulder, using all of the muscles that control the joints of the arm and hand, the only rest being on the pen and nails of the last two fingers. This movement is used in making large capitals and ornamental work, and also in writing on the blackboard, where there is no possible rest for the arm and hand.

15. The **finger movement** consists of simply moving the thumb and fingers by extending and contracting them. While this movement is not to be recommended for exclusive use, it can be used to advantage at the beginning, when learning the forms of the letters. The student will find he has a more sure control of the muscles of the joints of the hand than of the joints of the arm.

16. The **muscular,** or **forearm, movement** consists of resting the fleshy part of the forearm on the desk in a relaxed condition, the elbow just off the desk, and simply pushing and pulling the arm back and forth in the sleeve without moving the cuff of the sleeve. This brings into action the muscles of the shoulder joint, which run down from the shoulder to the spine. These are the muscles that very largely control the pull motion of the arm, and the muscles running down from the shoulder to the chest are those that very largely control the push motion. There are other muscles that control the smaller joints of the arm and hand, but they are of minor importance as compared with the shoulder muscles. To acquire a free, smooth, and rapid handwriting, these muscles must be brought under quick control, until their use has become a habit. This is possible with every one that is willing to give it thoughtful, painstaking practice.

17. The **combined, or mixed, movement** is just what its name indicates—a combination of the other movements—a simultaneous action of the arm and hand, one working in unison with the other. It is the ultimate result of the training that comes from the other movements. It becomes an ideal movement when all of the muscles controlling the joints of the shoulder, elbow, wrist, and fingers respond quickly to the mandates of the will.

DEFINITION OF TERMS

18. Before entering on the study of a subject, it is important to know the meanings of the terms used. The following are those with which the student of penmanship should be familiar:

Base Line.—The real or imaginary line on which the writing rests.

Circle.—A plane figure bounded by a curved line that is everywhere equally distant from a given center.

Connective Slant, or Line.—A curved line slanting 45° to the right of the vertical. The line may be a simple or a compound curve.

Headline.—The real or imaginary line to which the short letters extend.

Left Curve.—A curved line on the left-hand side of a circle.

Lower Turn.—The turn at the bottom of a letter.

Main Line.—A firm, straight, vertical line without shade.

Right Curve.—A curved line on the right-hand side of a circle.

Sharp Turn.—The point where two lines running in different directions meet and where the pen stops before changing direction.

Space.—The height of the small *i*, without the dot, is the unit of measurement for both height and width of letters, and is called a space.

Straight Line.—A line that does not change direction throughout its entire length.

Top Line.—The real or imaginary line to which the loop and capital letters extend.

Upper Turn.—The turn at the top of a letter.

PROPORTIONS OF LETTERS

19. The short letters are one-half the height of the loop and capital letters, the small *i* being the standard, or unit of measurement. The *t*, *d*, and *p* should not be made quite so high as the loop and capital letters; or, to be more definite, they should be one-fourth of a space shorter, as shown in Fig. 5. The *r* and *s* should be one-fourth higher than the short letters. The *p*, *f*, *y*, and *g* should extend one space below the base line, and all loop letters one and one-half spaces below, making the crossing on the base line. The crossing of the loop above the line should be a little less than one space above the base line.

The word *upright*, shown in Fig. 6, illustrates the different lengths of letters, both above and below the base line. The distances between the down strokes in letters, as well as those between letters in words, is shown in the word *nine*, Fig. 7.

It will be observed that the *o*, *a*, *c*, *d*, *g*, and *q*, shown in Fig. 8, are based very largely on the direct circle, and the *x*, *s*, and *p* on the reversed circle. All of the other small letters are based on the square.

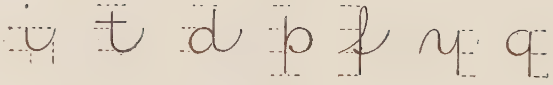


Fig. 5

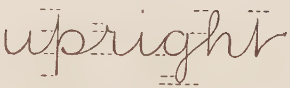
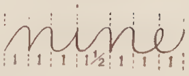


Fig. 6



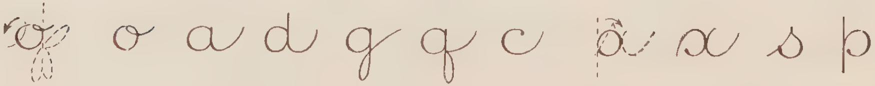


Fig. 8

THE LESSONS

20. The lessons follow in their regular order. Each consists of photoengravings of pen-written copies and the explanations and directions for practice. In preparing the work for examination, write each copy at least three times. *Never send in more than one lesson at a time.*

LESSON 1

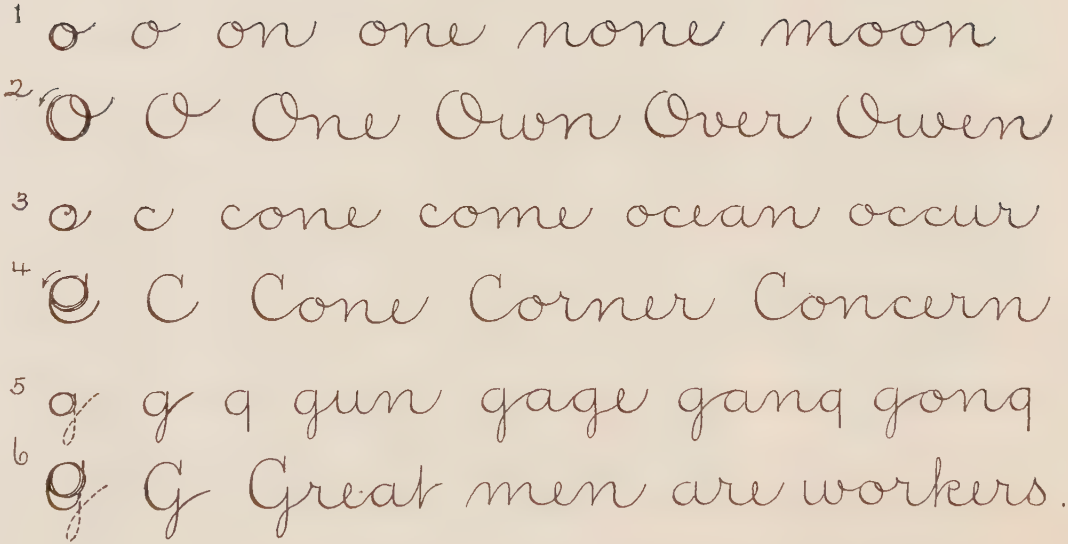
Copy 1.—*Small o:* This letter begins and ends at the headline and is made with one motion of the pen; that is, the pen continues to move from the beginning to the finish of a letter, there being no sharp turn where it stops to make a change of direction. The down strokes are a little firmer, or stronger, than the up strokes. This is because the muscles that control the movement of the hand toward the body are stronger than those that push the hand from it. Practice the copy freely, using a strong, rolling motion, and maintaining a fair rate of speed.

Copy 2.—*Capital O:* This letter is made twice as high as the small letter. It is shown in several difficult combinations—joining the *n* and connecting with the top of the *w*. Maintain a correct position of the body, observe the directions for holding the pen, and be careful of the movements. Practice the copy until the muscles respond quickly to the will.

Copy 3.—*Small c:* This letter appears at the beginning of a word, between two other letters, and two are shown in combination. Study these forms and be careful to get a uniformity of the down strokes.

Copy 4.—*Capital C:* Begin with a short, vertical, straight line; then trace the circle a few times with a free, smooth motion, but not necessarily rapid. Finish the letter at the headline. Make the single *C*, and write the words with

LESSON 1



1 o o on one none moon

2 O O One Own Over Owen

3 o c cone come ocean occur

4 C C Cone Corner Concern

5 g g q gun gage gang gong

6 G G Great men are workers.

7 d d due dine dime dinner

8 D D Do not fail to improve.

9 a a an am crane cream

10 A A Are you in earnest?

11 e e ear are ever exceed

12 E E Eunice Emma Emerson

the same strong but smooth motion, observing carefully the uniformity of the small letters.

Copy 5.—*Small g:* Begin by tracing a circle; then add the loop or the straight, vertical line, according to the style of letter used in different parts of a word. Write the words with a strong, pulling, rolling motion, giving fulness to the round turns at the top and the bottom of the letters.

Copy 6.—*Capital G:* The first part of the letter is like *C*, and should be drilled on in the same way. Study carefully the up stroke of the loop, and give special attention to the liberal spacing between the words in the sentence.

Copy 7.—*Small d:* This letter is formed from the circle. Make and trace the circle as indicated in the copy, to gain a free control of the muscles as well as to get a clear idea of the form, and then apply it to the *d* in the words, keeping constantly in mind the smooth touch of the pen that will lead to quality of line in the letters.

Copy 8.—*Capital D:* The shape of this letter and the motion in making it should be studied with great care. Observe the small loop at the bottom of the letter and the oval at the top. Drill especially on the oval by tracing it as shown in the copy and apply it to the beginning of a sentence. Notice every new combination of small letters and try to show improvement in the general appearance of the sentence.

Copy 9.—*Small a:* This letter appears at the beginning of a word and between other letters. It should be treated practically the same as the small *d*. Inasmuch as this copy is composed of short and not difficult words, there should be freedom of action. This will result in a good quality of line.

Copy 10.—*Capital A:* This letter is similar in shape to the small *a*, the difference being largely in the proportion. The capital is based on an oval and the small *a* on the circle. Practice the letter in the same way as before, and then write the sentence as if you were asking the question. This will enable you to make your writing practical—thought and act will become one. Study carefully the shape of the interrogation point.

Copy 11.—*Small e:* This letter is shown as it appears alone, at the beginning of a word, and between other letters, giving its formation from the base and from its connection with the top of another letter.

Copy 12.—*Capital E:* Make a short, vertical, straight line and then trace the circle both at the top and at the bottom, finishing at the headline, the upper circle being a little smaller than the lower one. The loop joining the upper and the lower parts of the *E* should be horizontal. The *E* joined to the small *u* and another to the *m* gives different combinations. These should be practiced until the combinations can be made with ease.

LESSON 2

Copy 13.—*Small n:* Observe that this letter occupies three squares, as indicated by the dotted lines, and that it appears in the copy in different parts of a word. This copy is not difficult and should be practiced with the view of freedom of motion, keeping in mind the uniformity of the up and the down strokes.

Copy 14.—*Capital N:* This letter is one of the "reversed motion" class of letters, the motion being distinctly opposite to the *O*, *C*, *G*, *D*, *A*, and *E* in the last lesson. Begin by making a short, vertical, straight line, which gives a little momentum to the hand, and then trace the circle several times until you can make a full, round turn, and reverse the motion at the bottom. Following this, make the *N* and write the sentence, observing carefully the spacing and the difficult combinations.

Copy 15.—*Small m:* This letter occupies four spaces in width, or one more block than the *n*. The same instructions apply to the two letters.

Copy 16.—*Capital M:* Practice making the *M* in the same manner as the *N*, as indicated, and then write the rest of the copy, observing the punctuation marks. Study carefully the form of the figures and the general appearance of the whole line.

LESSON 2

13 n n nine run men man

14 N N No time should be lost.

15 m m mum mine manner

16 M M Monday, March 2, 190 .

17 x x z z ox mix zero zinc

18 X X Xerxes Z Z Zero Zone

19 w w win new renew review

20 W W W Waste makes want.

21 q q qu quire quit quire.

22 Q Q Quick come, quick go.

23 v v vim vine over oven

24 V V Vigorous work wins.

Copy 17.—*Small x:* Notice that this letter is based very largely on the direct and indirect circle, and that the first part of the *x* and of the *z* are very much alike.

Copy 18.—*Capital X:* It will be seen that the upper part of the *X* and also of the *Z* is based on a larger circle than *N* and *M*. Work for results in form and easy motion.

Copy 19.—*Small w:* This letter is the *u* doubled, and each part is practically the same width as a *u*. The connection between *w* and *i* should have attention and careful practice.

Copy 20.—*Capital W:* The instruction given for the practice of *N* and *M* applies equally well to the *W*, except that the point where the pen stops to make the sharp turn is at the bottom of the *N*, while in the *W* it is at the top.

Copy 21.—*Small q:* This letter is made with the same motions as the *g*, except that the down stroke of the loop turns to the right instead of to the left. The loop should be the same length in both letters, and the distance between the *q* and *u* should be practically the same as between the down strokes of the *u*.

Copy 22.—*Capital Q:* The instructions in regard to the drill on the circle in the capital *X* and *Z* applies equally well to the *Q*. The loop at the base should be horizontal.

Copy 23.—*Small v:* In writing a word containing a *v*, great care should be given to the connection with other letters.

Copy 24.—*Capital V:* Drill on the *V* according to the instructions given for the *W*, and write the sentence, thinking of what you write as well as of how you make the form of the letters and the combinations.

LESSON 3

Copy 25.—*Small u:* This letter is two spaces wide and is made the same as *i*. Different combinations are shown.

Copy 26.—*Capital U:* Drill on the circles in the *U* until confidence is felt in control of the hand, and then make the single letter and follow it by writing the sentence, giving attention to both form and free control of the hand.

Copy 27.—*Small y:* Study carefully the shape of the first part of this letter, which is the same as the last part of an *n*. The loop crosses on the base line and finishes at the headline, and is one and one-half spaces long below the base line. The final *y* is finished with a straight, vertical line extending one space below the base.

Copy 28.—*Capital Y:* Practice tracing the circle; make the loop below the base the same length as that in the small *y*, and then write the sentence, observing the punctuation and uniformity of the up and the down strokes.

Copy 29.—*Small i:* This letter, without the dot, should fill one square block, as indicated by the dotted lines. Drill on the words in the copy to learn the different combinations.

Copy 30.—*Capital I:* Study carefully the lower part of the letter and observe that it is one-half of a circle. Drill by tracing the circle and follow this by making an *I* and writing the sentence.

Copy 31.—*Small j:* Observe that the loop is the same as in the *y*. Make the *j* and write the words with the different combinations.

Copy 32.—*Capital J:* Trace the letter as it appears in the copy, working for free control of the muscles as well as for shape of the letters. Make the single *J*, and write the name and date, observing the punctuation marks and shape of the figures.

Copy 33.—*Small h:* This letter is two spaces high and three wide. It is shown at the beginning and at the end of a word.

Copy 34.—*Capital H:* Trace the letter as indicated in the copy, and study the shape and the movement that produces it. Make the single *H*, and write the sentence as if you were giving advice.

Copy 35.—*Small k:* This letter is so nearly the shape of an *h* that the same instruction will apply except that the last part is a little higher and requires more care.

LESSON 3

25 u u urn run ruin union

26 U U Use the best materials.

27 y y y you year way say

28 Y Y Yours, with great hope.

29 i i in win vim aim rain

30 I I I am bound to succeed.

31 j j join joint rejoin ajar.

32 J J John. I. James, June 17, 190

33 h h he hem hush high

34 H H He that is wise is honest.

35 k k kin kind ink kink

36 K K Keep your own counsel.

LESSON 4

37 s s sun sum cases miss

38 S S Scorn every injustice.

39 l l line will alone illness

40 L L Let wisdon. be thy guide.

41 t t tin tint tent let letter

42 T T Think before you act.

43 f f fun fife of off offer

44 F F Few words are best.

45 p p pin pen pun up upper

46 P P Pay all of your debts.

47 b b ban able rub rubber

48 B Be not vexed over trifles.

Copy 36.—*Capital K:* Observe that the tracing exercise of the *K* has both the direct and the reverse motion of the circle. The loop that joins the parts together should be horizontal. Write the sentence with a graceful movement.

LESSON 4

Copy 37.—*Small s:* Observe carefully the reversed motion of the circle that produces the main part of the *s*. Drill on the tracing. Make the single *s* a little higher than the other short letters and write with care the letter in the different combinations.

Copy 38.—*Capital S:* The lower part of the capital *S* is also based on the circle, and should be practiced until the muscles will produce it with ease. Make the single *S*, and write the sentence, which has some difficult combinations.

Copy 39.—*Small l:* The loop of the letter should cross three-fourths of a space above the base and be two spaces high, and two spaces wide. Make the *l* and write the words with *l* combinations.

Copy 40.—*Capital L:* Make a short vertical line at the beginning of the letter, and follow it with a small circle, tracing it several times; then descend with a vertical compound curve to the base line, forming a loop, and finishing at the headline with a horizontal compound curve. Write the sentence with a firm, strong motion.

Copy 41.—*Small t:* Observe that this letter is not quite so high as the loop letters, and that the crossing is one space long and equal on the two sides of the down stroke. The final *t* should be without the crossing. The final up stroke is used as a substitute for it, to make the letter more simple, so that it can be written rapidly.

Copy 42.—*Capital T:* This letter begins with a vertical straight line made very short, joined with a horizontal compound curve to a vertical compound curve, which should be made nearly straight. It is finished with a dot at a point a little to the left of the beginning. Practice the *T* with a free, graceful movement.

Copy 43.—*Small f:* This letter should extend two spaces above the base line and one below. The pen should be removed at the bottom of the final stroke. The width is two spaces. Observe that the *f* is used in different combinations.

Copy 44.—*Capital F:* The instructions given for *T* apply to the *F*, except that this letter is finished with a horizontal straight line about one space long and half the height of the letter. Drill as indicated in the copy.

Copy 45.—*Small p:* The first part of the *p* is a little less than two spaces above the base line and one below. Remove the pen at the bottom and finish the letter with a reversed circle, terminating with a dot. Practice as shown in the copy.

Copy 46.—*Capital P:* This letter begins with a slight curve, merging into a vertical straight line. Remove the pen at the bottom and finish with a reversed circle, terminating with a dot. The best results will be obtained by a systematic drill on the copy.

Copy 47.—*Small b:* Observe that the shape of the first part of the *b* is the same as the first part of the *l*, and the last the same as the final part of the *w* and the *v*. Practice with the view of mastering the difficult *b* combinations.

Copy 48.—*Capital B:* The first stroke of the *B* is the same as in the *P* and the rest is based on the two reversed circles of about equal size. Practice this with the same end in view as in the small *b*.

LESSON 5

Copy 49.—*Small r:* This letter is a little higher than the *i* and should be made with three simple curved lines and one straight down stroke. Do not make a compound curve in the down stroke. The words in the copy involve difficult combinations of the *r*, and should be practiced with the view of mastering them.

Copy 50.—*Capital R:* This letter is made in the same way as the *B*, except that the loop is a little shorter and that the final down stroke is a vertical compound curve. Practice according to the plan in the copy, and do not scatter your energy.

LESSON 5

49 r r run or error runner

50 R R Read only the best books.

51 O.H.King. E.G.Case. A.D.Hunter.

52 T.F.Loomis. N.Miner. X.Z.Quinn.

53 P.B.Rice. U.V.Wilson I.J.Sims.

54 L.W.Lewis L.V.New. V.U.Young.

55 1 2 3 4 5 6 7 8 9 0 # $ ¢ a/c c/o % & &c &c

56 # 9 8 7 6 5 4 3 2 1 0 $1.4,679,012,358.00

Work

57 "Work that is not finished is not work at all; it is merely a botch, a failure."

LESSON 6

Conquest

58 "The conditions of conquest are
always easy. We have but to toil
a while, endure a while, believe
always, and never turn back."

Think for thyself.

59 "Think for thyself - one good idea,
But known to be thine own,
Is better than a thousand gleaned
From fields by others sown."

Copy 51-54.—*Review:* The purpose of these lines is to review different capitals, including all of the direct oval letters and also *H* and *K*. Another purpose is to show how these letters may be formed into a signature, which is a very important thing to know. There is nothing more interesting and useful for practice. Observe that some of the capitals are joined together, which will be suggestive to the student in joining together the initials in his name. It is advisable for every one to make a study of his own signature, and when he has found one that he likes, one that is perfectly legible, he should make it a practice to drill on it until the result becomes a habit. There is but one way by which this can be accomplished—by always trying to write the combination the same way.

Copy 55.—*Figures and Characters:* The figures should be made a very little higher than the small letters. Make a careful study of the shape of the figures and characters. The figures, with the exception of the *6* and the flourish on the *8*, are all of the same height; the *7* and *9* descend a little below the base line. This copy should receive much practice and should be made with a fair rate of speed.

Copy 56.—*Review:* Observe that this is a review of the figures and two of the characters. They should be written with greater speed than the former.

Copy 57.—*Paragraph:* The purpose of this copy is to teach the mechanical form of a paragraph, which begins about $\frac{3}{4}$ inch to the right of the beginning of the lines, and also to express thought in well-formed letters.

LESSON 6

Copy 58.—*Indention:* The purpose of this copy is not to teach how to write a word or a sentence, but to show the indention of a paragraph. A person may be able to repeat the same sentence and make a creditable page, but may not be able to write a pleasing page composed of several sentences. This copy is composed of two sentences, with a number of punctuation marks, which should be made with care. First, read and study the copy carefully from the standpoint of the whole, not from that of a single letter or word; then write it with confidence and with a free, smooth motion, which will produce a pleasing effect. When the work is examined and criticized, it will be judged by its appearance as a whole.

Copy 59.—*Stanza:* This copy contains another difficulty of mechanical arrangement, every other line being indented from ½ to ¾ inch. Study the copy carefully and write it with the same points in view as in writing the previous paragraph.

LESSON 7

Copy 60.—*Receipt:* Business forms are of such importance in every-day life that their use in connection with the subject of penmanship requires no explanation.

A *receipt* is a written or printed statement or acknowledgment of the receipt of money or other value. The most common kinds are on account and in full of account. *On account* simply means that a part of the indebtedness has been received; *in full of account* means that the whole indebtedness has been canceled. A receipt may be given for rent, for a note, to apply on a partial payment of a note, for tuition, and for various other causes.

The writing of a receipt involves mechanical arrangement, capitalization, punctuation, good writing, and the thought expressed. One of the secrets of success in this work is in learning how to look at the copy to gain the essential points, and how to sift them from the non-essential.

Copy 61.—*Due Bill:* This copy is also a common business paper. It should be studied with the same thoughtful care as the receipt.

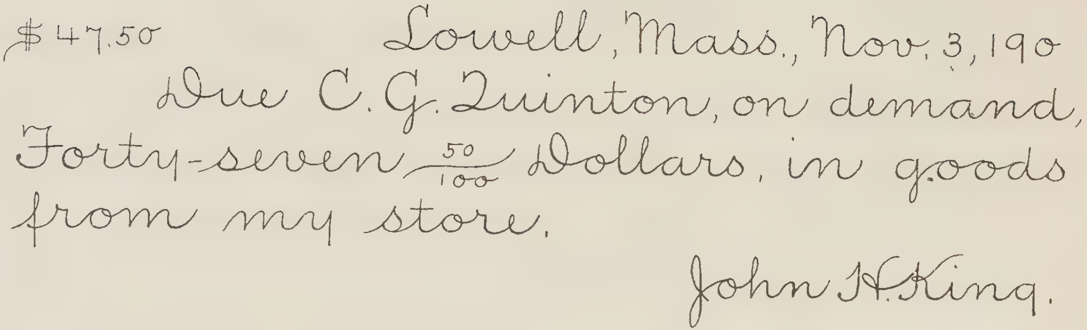
A *due bill* is a written acknowledgment of debt, and is usually payable in merchandise, but it can be made payable in money. It is more common to make it payable on demand than at some stated time. If it is desired to make it transferable to another person, or negotiable, the words "or order" should be inserted the same as in a promissory note.

LESSON 7

60.

$960.20 Albany, N.Y., April 1, 190
Received of Wilson E. Thomas
Nine hundred sixty $\frac{20}{100}$ Dollars,
in full of account.
V. W. Utley.

61



$47.50 Lowell, Mass., Nov. 3, 190

Due C. G. Quinton, on demand, Forty-seven $\frac{50}{100}$ Dollars, in goods from my store.

John H. King.

LESSON 8

62 $2699.10 Scranton, Pa., May 4, 190 .

On demand I promise to pay L. L. Brown, or order, Twenty-six hundred ninety-nine $\frac{10}{100}$ Dollars. value received.

Wm D. Hunter.

63

Atlanta, Ga., June 1, 190

Mr. U. V. Wilson,

Bought of P. B. Rounds.

May	26	1000 bu. Oats,	.55	550	—		
"	27	250 " Wheat,	.61	152	50	702	50

Paid.

P. B. Rounds.

LESSON 8

Copy 62.—*Note:* This copy is one of the most common business forms and is called a promissory note. It should be studied in every detail and practiced until its form is impressed on the memory and the writing of it is a physical equipment of the student.

A *promissory note* is a written promise to pay some designated person a specified amount of money at a certain time. It may be made negotiable by using the words "or order" following the name.

Notes are indorsed on the back in the following ways: In blank, in full, receipt, and without recourse. Indorsing a note *in blank* means simply that the person in whose favor the note is drawn writes his name on the back of the paper, about one-fourth the distance from the end, directly opposite the beginning of the note; *in full* means that the person in whose favor the note was originally drawn writes "Pay to the order of" a third person; a *receipt* indorsement is one in which the holder of the note writes the words "Received on the within note," stating the sum, and writes his name below it; *without recourse* is simply writing "Without recourse" and below it the signature.

Great care should be given to the writing as well as to the other essential points.

Copy 63.—*Bill:* This is another very important business form. The one shown in the copy is receipted. Study every detail of it with great care. This is an excellent copy on figures, writing, punctuation, and signature.

LESSON 9

Copy 64.—*Business Letter:* We have now reached a full-page unit composed of two paragraphs and arranged in the most approved manner on letter paper 8 inches by 10 inches. There is nothing of more importance than to know how to write a creditable business letter. Many a position has been lost because the person has not been able to come up to a fair standard in this simple, every-day matter of writing a letter. Others have received their first recognition on account of methodical, painstaking work in this line, and have risen to positions of trust and influence.

There are three elements that constitute a good letter: First, the ability to express the thoughts in simple, concise language; second, in being able to arrange sentences and paragraphs in such a manner as will conform to custom and good taste; third, in having power over the muscles so as to be able to execute a good handwriting. The copy has been arranged with these points in view, and it remains with the student to master them.

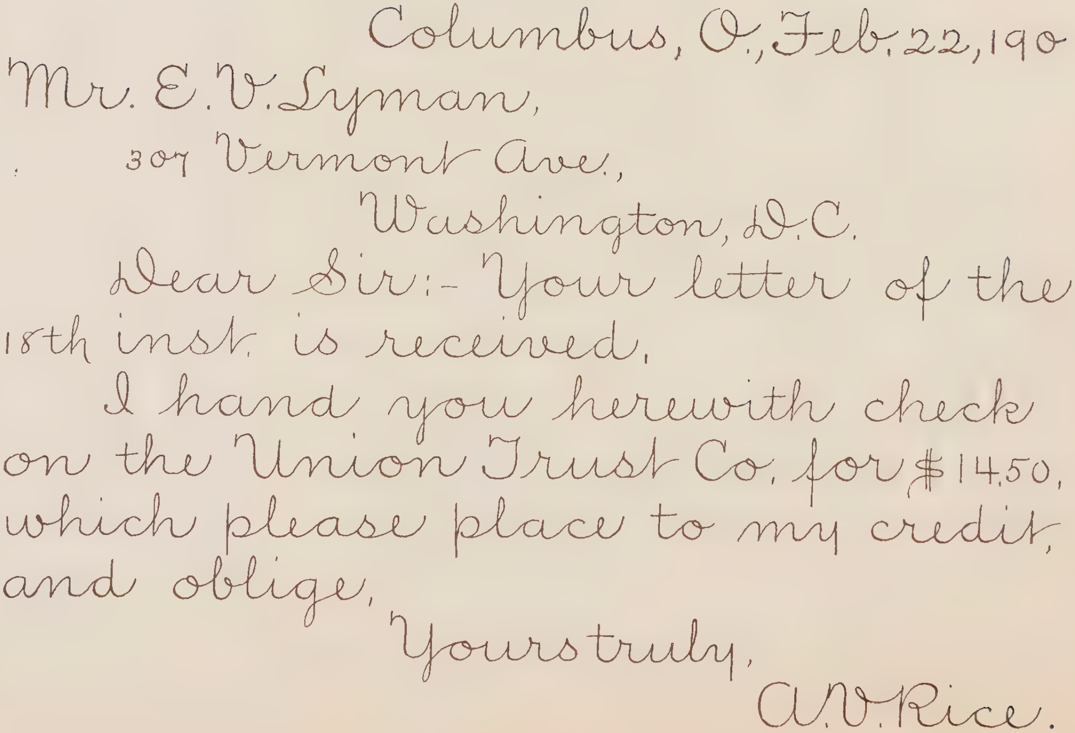
As a rule, the name of the place should begin practically in the middle of the line. On letter paper, the name of the person should begin $\frac{3}{4}$ inch to the right of the edge of the paper, and a little less on note paper. This arrangement leaves a liberal margin. The address under the name should be so arranged as to break the space and avoid having two lines begin or end at the same point. The words "Dear Sir" and all paragraphs should begin at a point $\frac{3}{4}$ inch to the right of the beginning of the name. The margin at the right should not be as much as at the left. "Yours truly" should be practically in the middle of the line, and the name should finish at a point directly under the finishing of the full lines.

Write this letter several times with a free, strong movement.

Copy 65.—*Superscriptions:* While these copies seem very easy to write, they will be found quite difficult when they are to be placed on an envelope and arranged so that the space will be broken in a manner pleasing to a cultivated taste. Three different superscriptions are given, which are arranged to illustrate the most common kinds in use, and also a visiting card. The name should be located in the middle of the envelope, from left to right, and also from top to bottom. The space below the name is divided into as many equal parts as there are lines, and of two successive lines neither the beginning nor the ending of the second is in a vertical line with the beginning or ending of the first. Inasmuch as

LESSON 9

64



Columbus, O., Feb. 22, 190

Mr. E. V. Lyman,
307 Vermont Ave.,
Washington, D.C.

Dear Sir:- Your letter of the 18th inst. is received.

I hand you herewith check on the Union Trust Co. for $14.50, which please place to my credit, and oblige,

Yours truly,
A. V. Rice.

65

SUPERSCRIPTION

Miss Lucy L. Sims,
Hampton,
Washington Co. N.Y.

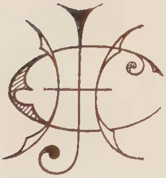
Mr. R. K. Zaner,
404 Main St.,
Buffalo,
N.Y.

Hon. A. E. Bryant,
Weston,
Vt.

W. H. Worthington.

LESSON 10

66



Dear William:

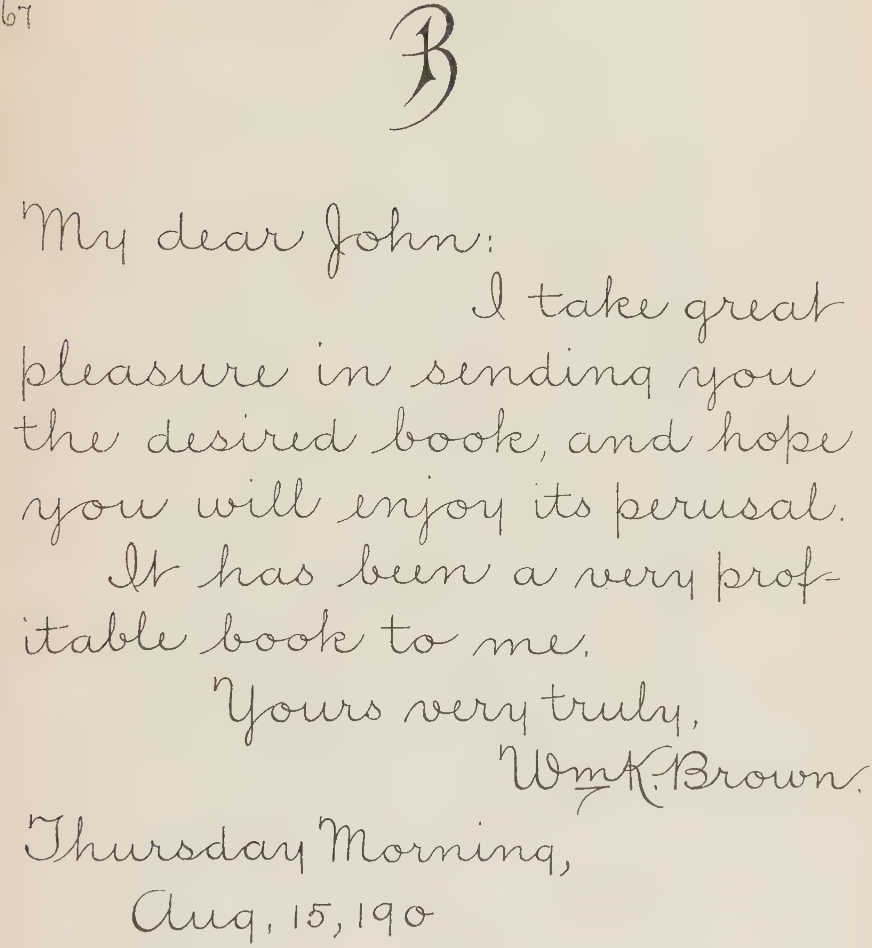
Will you be so kind as to loan me, for a few days, "Outlines for the Study of Art"? By so doing you will greatly oblige,

Your friend,
Jno. H. Case.

Wednesday,
Aug. 14, 190

LESSON 10—(Continued)

67



B

My dear John:

I take great pleasure in sending you the desired book, and hope you will enjoy its perusal.

It has been a very profitable book to me.

Yours very truly,

Wm K. Brown.

Thursday Morning,

Aug. 15, 190

LESSON 11

Review

68

A B C D E F G H I J K L M

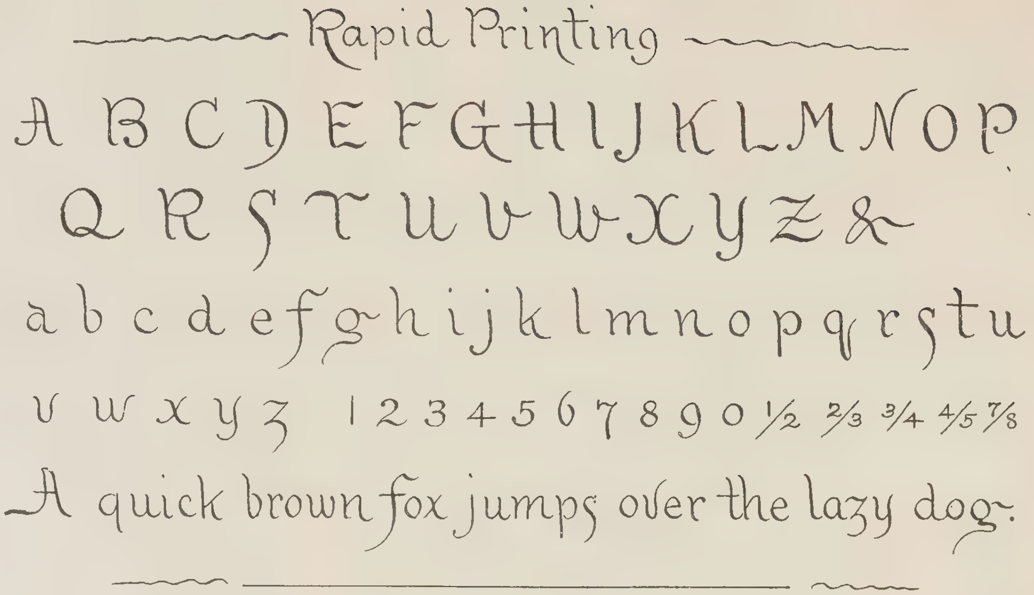
N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m n

o p q r s t u v w x y z &c.

A quick brown fox jumps over the lazy dog.

69



Rapid Printing

A B C D E F G H I J K L M N O P

Q R S T U V W X Y Z &

a b c d e f g h i j k l m n o p q r s t u

v w x y z 1 2 3 4 5 6 7 8 9 0 ½ ⅔ ¾ ⅘ ⅞

A quick brown fox jumps over the lazy dog.

there are no two superscriptions that are exactly alike, it requires a good eye and much practice to adapt the matter to the space.

Write these copies with great care.

LESSON 10

Copy 66 and 67.—*Friendly Notes:* The purpose of these copies is to teach how to write a friendly note, how to answer one, and also how to arrange them on commercial note paper. First, observe the appearance of the whole page of each and then carefully analyze the arrangement of the different lines; second, study the language that expresses the thought; third, study the letter forms and combinations.

Such notes are written on commercial note paper or on the note paper that comes in boxes, with envelopes to match. The ordinary note paper is 5 inches by 7½ inches, and is folded twice to fit the envelope, while that which comes in boxes varies in size and is folded once to fit the envelope; a popular size is 5½ inches by 7 inches.

Write the note of request several times.

It will not be necessary to make the monogram shown at the top of the note.

LESSON 11

Copy 68.—*Review:* This copy is a review of all the capital and small letters separately, and gives a sentence containing all of the small letters of the alphabet in nearly all of the different combinations. Drill on each letter separately before practicing the entire copy. These specimens should show a quality of work superior to the others, both in form and movement, and they will be a test of the student's perseverance. Every line will be a picture of the touch and motion that produced it, and each letter and combination will be closely examined and criticized in a spirit of justice and with a view of benefit to the student.

Copy 69.—*Rapid Printing:* With the exception of the specimen page, the final copy of the series has now been

LESSON 12

70

International
Correspondence Schools.
Scranton, Pa.
Gentlemen:- This is a specimen of
my writing after completing your
course of lessons in penmanship.
Yours truly,

reached, which is a few lines of rapid printing in a style well adapted to business use. The forms and proportions are the same as in writing. It is necessary, in printing, to acquire a firm, steady touch of the pen, keeping the lines uniform in width, spacing, and height, and as nearly as possible of the same strength.

LESSON 12

Copy 70.—*Specimen:* This copy is nearly like the letter the student was asked to write before beginning practice. The work on this specimen should show that the eye has been trained to see better and the muscles to act better, which means greater skill.

Study this letter carefully and observe the arrangement of the different parts, paying particular attention to those to be added. Give it conscientious practice, for it should be an improvement over the first specimen.

PENMANSHIP

GENERAL INSTRUCTIONS

PRELIMINARY REMARKS

1. Good writing may be briefly defined as writing that is easily read and easily and rapidly written. Legible writing depends on a correct idea of form. Easy writing depends on movement. Rapid writing depends on simplicity of form combined with ease of movement.

Easy, rapid, graceful, tireless writing is the result of *muscular*, or *arm*, *movement*. Users of the arm movement never have writers' cramp, which results from gripping and the use of the small muscles of the hand and fingers that tire easily. The large muscles of the arm and shoulder, which are tireless so far as their use in writing is concerned, should be made to do the work. The function of the hand and fingers is simply to hold the pen, not to propel it.

Two things are necessary to the acquirement of a good handwriting: *study* and *practice*. Study is the only means by which the correct forms of the different letters can be learned, and practice is necessary to gain the control of the muscles required for the execution of those forms when once fixed in the mind.

Since the ability to write a fine hand is an acquirement and not a natural gift, any person not positively deformed who is willing to work, who is intelligent enough to work in the right way, and who will persevere for a reasonable time, can become a good writer—not an expert, perhaps, for that requires talent

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as well as effort, but able to write a good, plain business hand, which is all that is needed or desired by the average person.

The time is past when writing was an accomplishment possessed by a few. Today every one is expected to be able to write, and although writing may be regarded as a fine art, and is considered as such in the penmanship of those who excel in it, yet it is not too much to expect all to be able to write legibly and well. To ask a man to spend his valuable time and exercise his ingenuity in trying to read illegible writing is nothing short of imposing on his good nature.

In commercial schools more attention has been given to the teaching of penmanship than in public schools, simply because a good hand is a qualification that business men insist that a bookkeeper or clerk shall have. Penmanship is, therefore, an extremely important subject for one who expects to enter the commercial field, and the greatest effort should be made to master it.

2. Foundation of Good Writing.—In the system of writing set forth here, the first task will be to learn to use the arm movement. This is in a sense the foundation, though, if satisfactory progress is made, movement and form must advance together. Attention given to form only means cramped writing; to movement only, scribbling.

Correct movement depends so much on correct position that the position may be said to be for a time the most important thing to consider. It is the key to easy and rapid writing. Do not be content to read the description of the position once and to glance at the illustrations today with no thought of ever going back to them; refer to them often and study them so closely that they will become fixed in your mind to stay. With every lesson let the first thought on beginning daily practice be of your position and the manner of holding your pen. In time you will learn to assume the right position without special care, for it is in reality the easiest and most natural one that can be taken. But until the correct position has become a habit you must watch yourself closely.

POSITIONS

3. Position of Body.—Observe attentively the position shown in Figs. 1 and 2. These illustrations show the correct position more plainly than it can be explained in words. The feet are squarely on the floor and separated, and both arms are

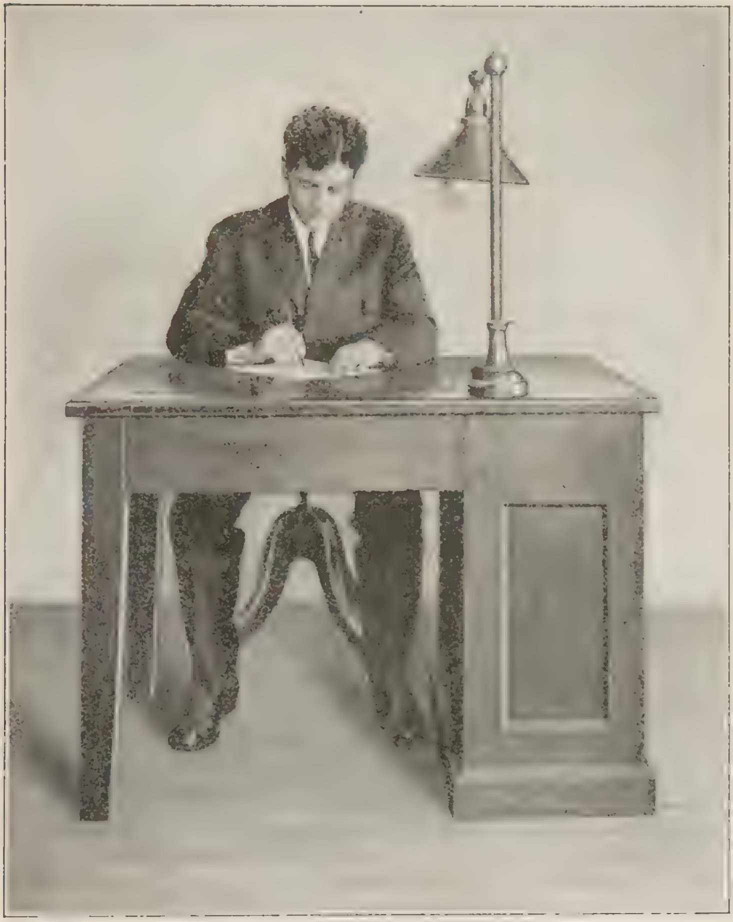


FIG. 1

on the table, so that the body is in what is known as the *front position,* which keeps the shoulders even. The body leans forwards over the table only as much as the bending from the hips throws it forwards naturally. If the sight is normal this brings the eyes at the proper distance from the writing. Thus, the position of the body is natural, easy, and healthful. Writing will not make you round-shouldered or your shoulders uneven if your position is right.

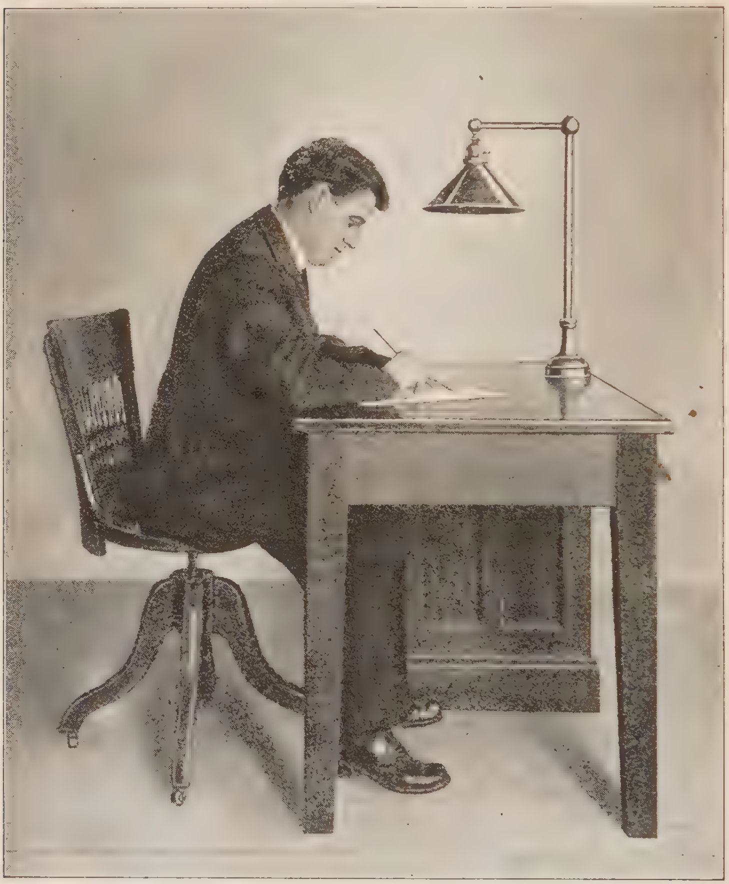


FIG. 2

4. Position of Paper, Etc.—Fig. 3 shows the relative position of desk, paper, arms, and pen. Observe that the paper is placed diagonally across the desk but that it is not slanted enough for the edge to be parallel with the right forearm; that the right and left arms are at right angles to each other; that the elbows project just a trifle over the edge of the desk; and that the penholder points in a direction not quite over the right shoulder but between shoulder and elbow.

A blotter should be placed under the right hand and be held in place by the left. The hand glides on this and the paper is protected from being soiled by perspiration from the hand.

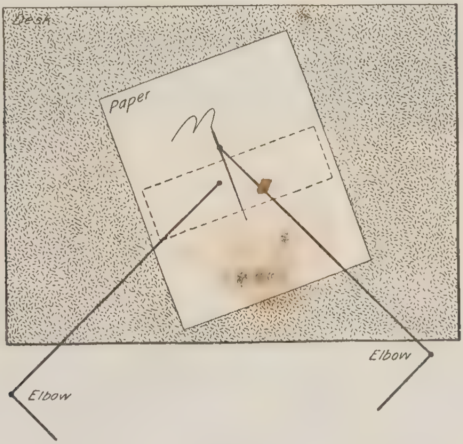


FIG. 3

The right elbow should be shifted from two to four times in writing across a page and the sheet must be moved forwards as progress is made toward the bottom.

5. Position of Arm and Hand.—Figs. 4 and 5 show the position of the arm and hand. Note that the arm rests on the fleshy part just in front of the elbow and that the hand rests and glides on the nails of the third and fourth fingers. Some penmen prefer to curl the fingers under a little more so that the gliding rest is on the joint of the little finger. Which of these two positions is the better depends on the length of the fingers. *The* *hand must not be turned over to the right enough so that the fleshy part touches the table.* Neither should the wrist quite touch.

The illustrations show the penholder crossing the hand at the knuckle joint. Some persons prefer to hold the pen in a slightly more upright position so that it crosses the hand just in front of the knuckle, others let it cross behind the joint. There is no set rule regarding this, although it is generally agreed by penmen that the penholder should incline at an angle of about 45°, which is half way between horizontal and perpendicular.

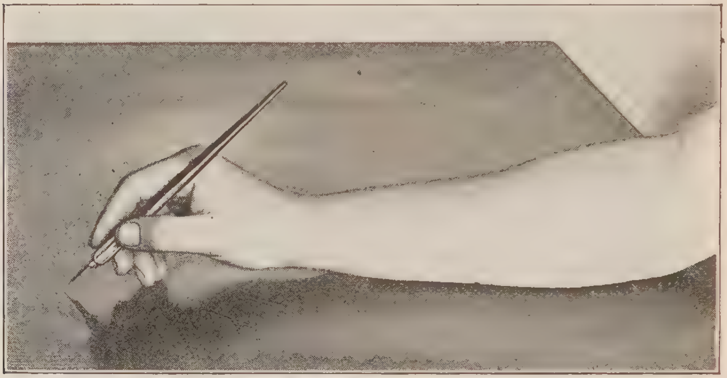


Fig. 4

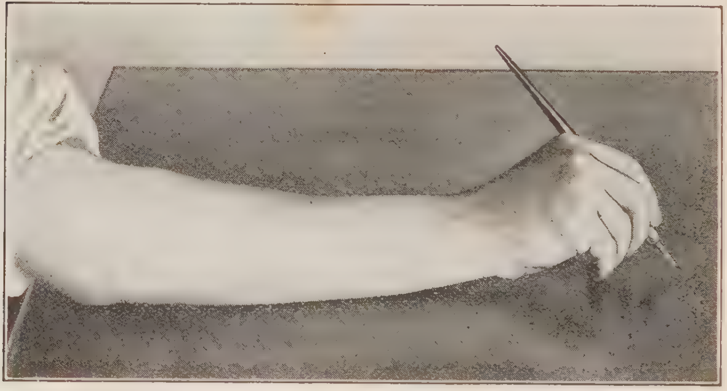


Fig. 5

The pen should be grasped firmly but lightly. *Guard against gripping it tightly; this cramps the muscles, tires the arm, and prevents freedom.*

The clothing on the arm and shoulders should be loose and light. Tight sleeves make freedom impossible. Stiff cuffs are also a hindrance.

The full weight of the right arm should fall on the muscle, as shown, but any weight from the body that is made to fall on the arms can be thrown entirely on the left so as to allow the right perfect freedom.

No part of the hand should touch the paper except the tips of the third and fourth fingers.

6. Drill for Position and Arm Movement.—To get into position for writing, relax all muscles and let both arms fall lifelessly to the sides; swing them backwards and forwards a few times, then raise them entirely above the desk and let them drop into place. The fingers will curl under naturally so that the hand will be about half closed and in just the right position for holding the pen. Do this for practice a number of times.

A drill for muscular action is the following: Place the arm in the position shown by Fig. 4, but, instead of taking the pen, close the hand in a tight fist, then move the arm rapidly forwards and backwards on the muscle as far as you can without sliding the sleeve. The movement should be in and out of the sleeve, or if you wish you may practice with the sleeve rolled up as shown in the illustration.

7. Requisites for Free-Arm Movement.—The requisites for free-arm movement may be briefly summarized as follows:

1. Correct position of body, arm, and hand, which allows unrestricted action of the muscles.
2. Loose clothing on the arm and shoulders.
3. No gripping of the pen.
4. Relaxation. Not a muscle in the entire body should be tense.
5. The use of the forearm muscle as the center of motion.

MATERIALS

8. Do not try to economize on writing materials. Poor materials mean poor work. Use plenty of paper and pens but do not waste them. Utilize every inch of your practice paper.

The **paper** should be ruled and have a firm and smooth, but not glossy, surface. A good quality of foolscap is to be preferred.

Pens about as fine as Gillott's No. 604, Tec. No. 1, or Spencerian No. 1 are recommended. Moderately fine pens produce the best results for practice work. They compel the cultivation of a light, elastic touch, one of the essentials of good writing; they bring out the defects of the writing more plainly than others, thus allowing closer criticism; and they make necessary the exercise of care and attention to details.

If your touch is naturally very heavy and you find it impossible not to shade when you use a fine, flexible pen, try Gillott's No. 603 or 601. These are fine but stiff.

The **penholder** should have a cork or a rubber tip or be entirely of wood. Metal penholders are not good because they are usually so small near the point as to cause a tendency toward gripping.

The **ink** needs to be of a quality that will flow freely and evenly. A blue-black fluid ink is much better for ordinary writing than a jet-black ink, the latter being too heavy and thick. Do not use ink that is old, pale, or filled with sediment. Stafford's, Carter's, or Arnold's inks are of a satisfactory quality.

THE LESSONS

DIRECTIONS FOR STUDY AND PRACTICE

9. This Section on penmanship consists of twenty lessons and each lesson is divided into a number of exercises, there being, in all, one hundred and twenty-one exercises. Neither a lesson nor an exercise of a lesson should be thought of as a day's work, but at least one day's practice period should be devoted to each exercise, and after all the exercises of a lesson have been practiced the lesson should be reviewed as a whole and extra time devoted to exercises that are especially hard. The aim should be to get everything possible out of a lesson and not to get over it in the shortest time.

In order to become a good penman, it is necessary to practice intelligently. Little or nothing can be accomplished by careless practice, and every hour of scribbling will make you a poorer rather than a better penman. Before beginning practice on any exercise, you should study the copy carefully in order to learn the correct form of each letter. Observe the parts of each letter closely, noting each curved, each straight, and each retraced line, each angle and each turn. The closer the distinctions that are made between curved and straight lines and between angles and turns the better the writing will be. Your own work should also be studied and compared with the copy in order to find the faults and their remedy. If any of your letters are badly made, compare each one with the copy and practice on it until you can make it correctly.

The correct forms of letters are not hard to learn, but much practice is required in order to gain the skill necessary to execute the forms correctly and to secure such control of the muscles as will enable you to write with an easy, rapid, flowing motion.

The small muscles of the hand and fingers are easily controlled but they tire easily. The large muscles of the arm and shoulder are so far removed from the point at which the writing must be done that they are difficult to control, but they can be used for hours without fatigue. It follows that finger-movement writing is easy to learn but hard to use, and that arm-movement writing is hard to learn but easy to use.

It will be unnecessary to be concerned about the matter of slant of the letters if the manner of practicing is right. Uniformity in, rather than any particular degree of, slant is what is desired. The paper must be properly placed and the position of the body, hand, and arm be correct. Down strokes are produced by drawing the hand back on the muscle of the forearm, and if they are all made to take a direction *toward the center of the body* they will slant properly.

To be well executed, the writing must be neither too fast nor too slow. Slow writing shows weak, tremulous lines and is nearly always done with a cramped movement; a too rapid motion results in scribbling and does not allow of proper letter formation. The speed should be so regulated as to allow the motion to be easy, uniform, and graceful. Light, firm, clean-cut lines will be produced if the rate of speed is correct.

10. Before beginning practice on any exercise, read the instructions regarding it and follow the directions carefully. Practice regularly and send lessons for criticism at intervals of about 10 days or 2 weeks. By the time you have practiced as long as that on any lesson you need criticism. Perhaps you may not have succeeded in getting your work up either to your own standard of excellence or to what we require for a passing grade, but send it anyhow. If you are not improving, it shows that you need help. Work faithfully and do not get discouraged or feel sensitive if your lesson is returned with a request for additional work. Monotony may be avoided by taking up the study of some other subject in connection with penmanship.

Never send in more than one penmanship lesson at a time. Lessons on different subjects may be sent in the same envelope.

LESSON 1—EXERCISES 1 TO 8

11. Exercise 1.—The first exercise is an oval drill. Before beginning it give careful attention to your position to see that you have it exactly right, and practice the drill given in Art. **6** for securing muscular relaxation and arm movement.

The direct oval drill begins with a down stroke on the left-hand side as shown by the arrows. It is distinguished from the reverse, or indirect, oval which will be used later and which is made with the opposite motion, having an up stroke on the left.

The movement for this exercise comes from the same muscles that are used in the drill for arm movement described in Art. **6**, but a circular motion is required, the arm rolling on the muscle in front of the elbow.

Keep the fingers passive. Remember that the force which propels the pen is above the elbow. *Do not let the hand turn over on its side.*

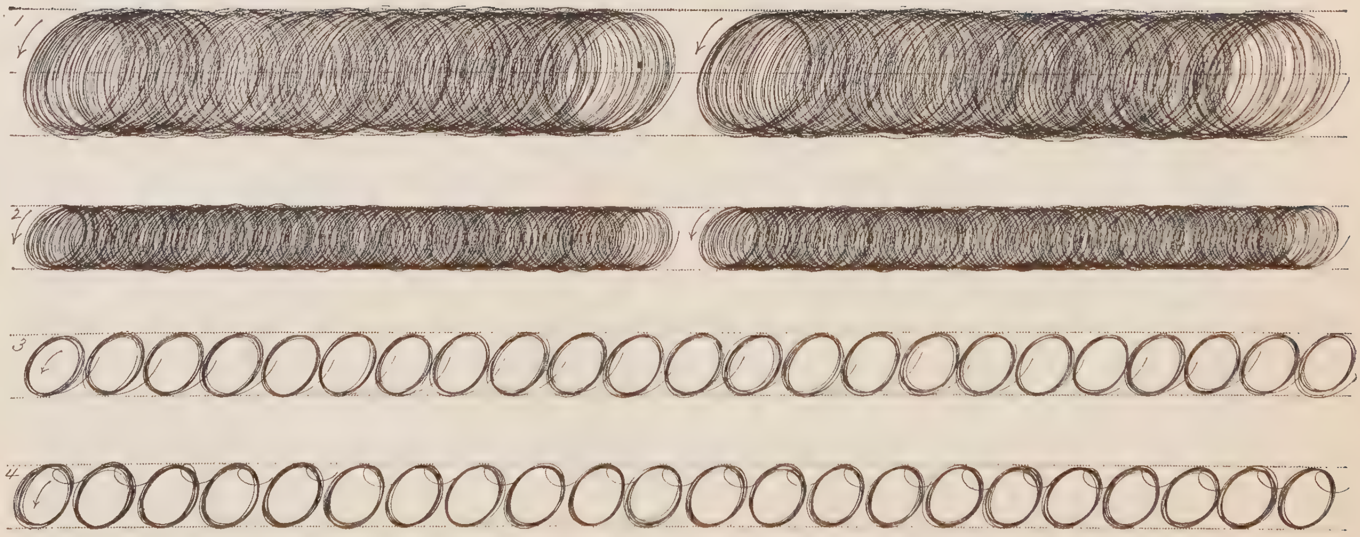
The height of the oval should be twice the space between two lines on the paper and the exercise should be in two parts, each half a line in length. Each half line should represent about two hundred revolutions and be written in about 65 or 70 seconds, a rate of about three revolutions a second, or 180 revolutions a minute. You can regulate your speed and secure a rhythmic motion by counting as follows: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 2, 3, 4, 5, 6, 7, 8, 9, 20, 1, 2, 3, 4, 5, 6, 7, 8, 9, 30, etc. Do not pause after 10, 20, and 30 but keep the count up steadily. Be sure to time yourself until you get the correct speed.

Strike out boldly, freely, and forcibly, keeping in mind that the objects of this exercise are *position*, *movement*, and *speed*, not perfect forms. The forms will come in time if your method of practice is right and your mental picture of the drills correct.

Aim for light, fine lines, and try to see how far you can write with one penful of ink.

It will pay you to put all of your practice for several days on Exercise 1 and the drills for position and arm movement.

LESSON 1



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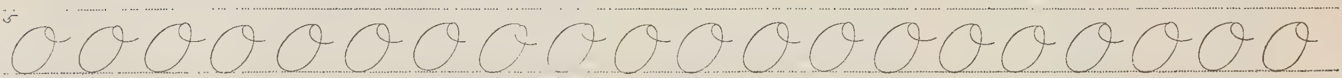

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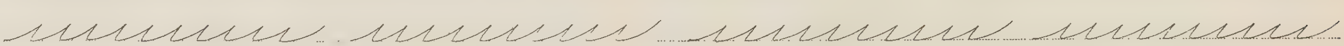

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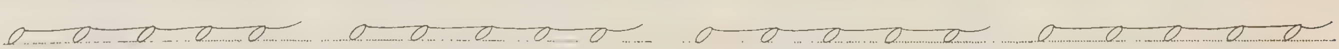

12. Exercise 2.—Exercise 2 is like Exercise 1, except that the oval is only half as high. Two hundred revolutions in this drill should be made in about 1 minute. Count as in Exercise 1.

The general direction of the down strokes should be toward the center of the body. If your paper is properly placed and your position is right but your drills have a backward slant, it is because you are making the down strokes toward the right shoulder; if they have too much forward slant, you are making the down strokes toward the left shoulder.

13. Exercise 3.—Write lightly and retrace each oval five times. Count 1, 2, 3, 4, 5. Do not stop the movement between the ovals; keep up a continuous motion with the pen raised from the paper, make one or two revolutions with the pen poised in the air, and begin the next oval with the pen at full speed. Observe the compactness of the line of ovals.

Each oval should be about two-thirds as wide as high. Make ovals, not circles.

14. Exercise 4.—Notice how the capital *O* is developed from the oval drill. Count 1, 2, 3, 4, 5, and finish. The ending stroke must turn upwards. If you allow it to drop downwards you are in danger of making the *O* too much like an *A*.

15. Exercise 5.—Count 1, 2 for each letter. Write with a smooth continuous movement; do not pause or check the motion in getting from one letter to the next. The spaces between the letters should be as nearly equal as possible. Try to get as many letters on a line as there are in the copy.

Make the capital *O* smaller than in the preceding drills—a little more than two-thirds of the height between the lines. About sixty *O's* per minute is a good speed.

16. Exercise 6.—The object of the drill of Exercise 6 is to develop a sliding, left-to-right motion for use between letters. The elbow should act like a hinge. The movement must *not* be from the wrist. Count 1, 2, 1, 2, or right, left, right, left.

17. Exercise 7.—Observe that in Exercise 7 the up strokes are curved and the down strokes straight and that an angle is made at the top and a turn at the bottom. Observe whether your work has any faults such as curves or loops at the top, curves in the down strokes, or angles at the base. Write carefully but vigorously and with the arm movement. Count 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

18. Exercise 8.—Begin now to use the same freedom in writing letters as in writing the oval drills. The sooner you begin to make practical application of the arm movement to your writing the sooner you will begin to improve.

Make the *o* with a circular motion and give a good curve to both the down and the up strokes. Close it at the top. Connecting strokes should be very slightly curved. If they curve too much your *o's* will resemble *a's*. Use a swinging motion between the letters. Write eighty *o's* a minute.

19. Do not forget the importance of study as well as practice. Your mental picture of a letter must be correct, for the motions of your pen follow the lines of the letter as it is in your mind rather than as it is in the copy.

After you have practiced the preceding eight copies, prepare your first lesson for criticism. Send several lines of each copy. One or two lines is not enough.

Arrange the work in a neat, compact, and orderly manner. Lack of neatness is one of the principal causes of poor writing.

After mailing Lesson 1, begin work on Lesson 2, but do not prepare your specimen of Lesson 2 for criticism until Lesson 1 has been returned with a percentage mark, unless, of course, you live at a very great distance from the Schools so that the corrected work will not reach you for some weeks. In that case you need not wait for the return of your work but may send the lessons when you feel that you have practiced them thoroughly.

LESSON 2—EXERCISES 9 TO 16

20. Exercise 9.—Get the correct position and practice a minute or two on the preliminary drill in Art. **6.** Then practice Exercise 9. This exercise is to serve as a review and should be practiced *each day* at the beginning of your practice period. Aim to grow skilful in making the ovals. Work for fine, clean-cut lines.

Do not try to write the entire line without lifting the pen. Stop and shift either the elbow or the paper two or three times.

21. Exercise 10.—In practicing Exercise 10, count 1, 2, 3, 4, 5 on the retraced oval and 1, 2 on each capital *C*. Try both styles of *C*. In the first style, you may need to guard against too great a curve in the down stroke. In the second, you should try to get the loop at the beginning proportionately the correct size. Notice that it begins with a curve toward the left, not a straight down stroke. The body of the letter, you will observe, is exactly like the oval. Keep up a steady, even motion in writing the entire line. The pen should be going at full speed when you begin a letter and there should be no pause when it is lifted at the end.

Space the writing so that you get the same number of ovals and letters on a line that there are in the copy. Do not leave a margin on the right-hand side of the page. Use a speed that will allow you to write easily and gracefully, and have care as to form. At least two lines a minute should be made.

22. Exercise 11.—Count 1, 2, 3 for the capital *E*. The first count is for the dot at the beginning, the second for the loop, and the third for the finish. The beginning of this letter differs from that of the *C* in that there must be a pause as you place the pen on the paper. The upper part of the *E* should be only a little smaller than the lower. Both should be very rounding. The little loop in the middle should point downwards. Finish with the pen still in motion.

You will perhaps have to work hard on this capital. Compare your work with the copy and study it critically. Ask yourself, Do I proportion the letter correctly? Does the central loop point downwards or upwards? Is my slant correct? Does my work lack gracefulness and freedom?

23. Exercise 12.—Use a purely arm movement and write the *A* with a great deal of freedom but do not write it hastily. Study it closely. Note how much greater is the curve in the down stroke than in the up stroke. Aim to close every *A* at the top. The finishing stroke is slightly curved and ends just below the base line. Do not make it too long. Guard against a loop where this stroke joins the up stroke.

Count 1, 2, 3 for the *A*. Swing along at a good rate, making between fifty and sixty a minute.

Make your capitals small enough so you can get as many on a line as there are in the copy. Small capitals will make your writing appear neater and the speed will be greater.

24. Exercise 13.—If you have thoroughly mastered the form of the capital *A*, the small *a* will not trouble you. Make each group of four without lifting the pen. The stroke that connects the preceding letter should be carried to the top of the *a* and retraced carefully with the down stroke. Avoid forming a loop where these two strokes join. Do not leave the *a* open at the top. Make the last down stroke straight and see that it slants properly.

Keep a good position. Write freely and at a moderate speed, making at least sixty letters a minute. Count 1, 2, 3, 4 on each group.

25. Exercise 14.—Make the groups of *e's* with a rolling motion but give considerably more curve to the up strokes than to the down strokes. Remember that down strokes must go toward the center of the body, not toward the right shoulder. Count 1, 2, 3, 4. The *e's* can be written more rapidly than the *a's*; 120 a minute is an easy speed for them.

26. Exercise 15.—Review Exercise 7 as preparatory work for the *i's*, *u's*, and *w's* in Exercises 15 and 16. Remember that the important points in *i* are: a curved up stroke, a very sharp angle at the top, a straight, slanting down stroke, and a

LESSON 2

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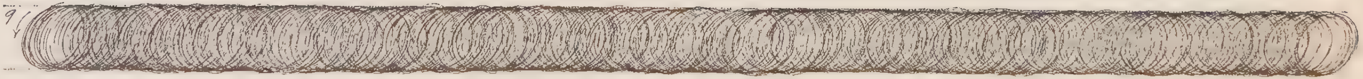

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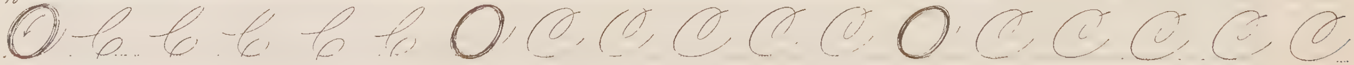

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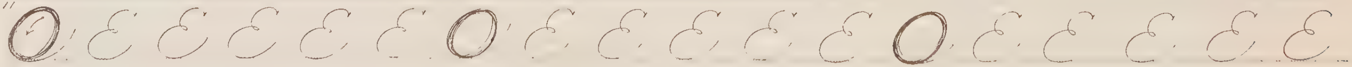

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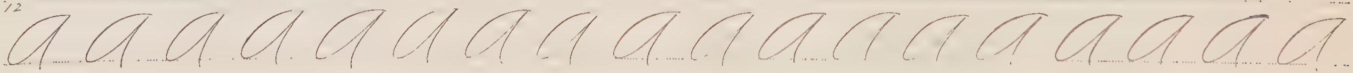


13

a aaaa aaaa aaaa aaaa aaaa

14

e eeee eeee eeee eeee eeee eeee

15

i iiii iiii iiii iiii iiii iiii

16

u uuuu uuuu w wwww wwww

round turn at the base. Place the dot carefully. Count 1, 2, 3, 4, dot, dot, dot, dot. Write at the rate of from sixty-five to seventy-five a minute.

27. Exercise 16.—Use close enough spacing between the parts of the *u* so that each letter stands out distinctly. Every *u* should be sharp at the top, round at the bottom, have the up strokes curved, and the down strokes straight.

Count 1, 2 on each letter. Write from sixty to seventy a minute.

Notice the little retrace, called a shoulder, on the last part of the *w*. This is the feature that distinguishes *w* from *u*, and pains must be taken to make it show plainly. Do not sprawl this letter out; space closely between the parts so that it will have a compact appearance.

Count 1, 2, 3 for each letter and pause a little as you make the shoulder. Do not write hastily, but form each letter with care. Make fifty or more a minute.

28. After you have practiced all the copies of Lesson 2, review the entire lesson thoughtfully, giving your work critical study for its weak points. Then prepare your copy for criticism. Endeavor to have each lesson you send show as much improvement as possible over the preceding one.

LESSON 3—EXERCISES 17 TO 24

29. Exercise 17.—Before beginning on Exercise 17, practice a short time on the large oval exercise in Lesson 1. Examine your position to see that it is correct. Do not let the hand fall over on its side. Keep the penholder pointing almost over the right shoulder.

Use a vigorous push-and-pull movement in the straight-line part of Exercise 17, writing at a rate of about 200 down strokes a minute. The motion in this is exactly the same as that described in Art. 6.

The *t's* are made with the same motion, but make curves at the bottom and space between the strokes. Observe that the *t* should be retraced about half its length. Do not leave it open too far, or loop it, or make the last stroke sharp at the base. Cross each *t* neatly and carefully. Do not place the cross-stroke above or at the side of the letter.

The correct height for *t*, *d*, and the upper part of *p* is about half the space between the lines, or about twice the height of the letter *i*.

Fifty to sixty letters a minute is a high enough speed for the groups of *t's*.

30. Exercise 18.—Write freely but strive for uniformity. The *d* differs from *a* only in the up stroke being carried two spaces above the line to form the stem. Retrace this stem carefully to the point where it meets the *a* part. This is one of the difficult letters. Put plenty of work on it.

The groups should be written with care and at the rate of about twelve groups a minute.

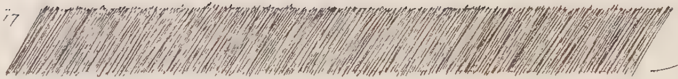
31. Exercise 19.—The beginning stroke of *p* must curve. The down stroke must be perfectly straight and should end firmly, and there should be a slight pause before the pen is lifted to complete the letter. This down stroke must be rather short. Notice that the last part of *p* is exactly like the last part of *n* and that it contains no angles. A common fault is to get this part of the letter sharp at the top.

Aim for about thirteen or fourteen groups a minute.

32. Exercise 20.—Write several lines of each word, then several pages of the entire line. Stop often to look over what you have done and criticize it as severely as you know how. The most important thing in learning to make correct letter forms is *to have exactly the correct forms in your mind.* If your mental picture of a letter is correct you will naturally strive for the right form whether you are conscious of it or not.

The entire line can be written three times in a minute. Your speed on this or any of the other copies may be a little greater or a little less than that designated. If you find that you can write a little more rapidly and still give care and thought to the forms, you may do so. Good writers will be

LESSON 3

17 

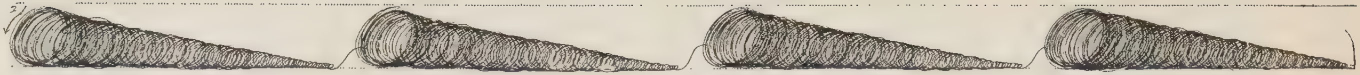
t ttt tttt tttt

18 *t tititi tititi d dididi dididi*

19 *p pppp pppp pppp pppp pppp*

20 *dip top pipe dot puppet pippin p*

21



22 c cccc cccc cccc cccc ccccc

23 r rrrr s ssss rrrr ssss rrss

24 rustic dice wipe cure tide wait rw

able to work up a higher speed, though we would not advise a great deal of deviation either way, and we warn you particularly against writing so very slowly in your effort for perfect forms that you cramp your movement and make tremulous lines. The quality of the lines is as worthy of your attention as the shape of the letters. Writing may be good in form but lacking in freedom; it then shows weak, tremulous lines. Or, it may be good in freedom but poor in form; it is then illegible or unpleasing in appearance. Make it your aim to advance in both freedom and form. You can if you work in the right way.

33. Exercise 21.—Your practice up to this time should have given you good control of your movement for making the tapering oval. Try for uniformity in taper while writing as freely and rapidly as in previous drills.

34. Exercise 22.—Make the hook and little dot at the top of the *c* distinct enough that you do not get the letter like *i*. Retrace the strokes carefully at the top. Do not make a loop.

Write about eighty *c's* a minute.

35. Exercise 23.—Observe the *r* carefully. Note the curve in the first stroke and the little retrace at the top, followed by the very small down stroke, or shoulder, that differentiates *r* from *i*. Since this is the distinguishing feature of this letter, you must take pains with it. Make the shoulder as plain as you can but do not make two angles.

Make the up stroke of *s* a decided curve and you will find it easier to get the angle at the top. The down stroke must be very fully curved and the letter closed carefully at the base. Avoid forming a loop where you close it. Make the four *s's* without lifting the pen.

Both *r* and *s* are just a trifle higher than the other small letters.

Write the *r's* at a rate of seventy or seventy-five and the *s's* at about eighty-five letters a minute.

36. Exercise 24.—The writing of words is the practical test of how well you have mastered the various letters. Write with due care as to the forms, but freely and rapidly.

37. After reviewing the entire lesson you should be ready to prepare Lesson 3 for criticism.

It is a good plan to go back and review the preceding lessons occasionally, and also to read over frequently the general directions at the beginning of the Section. You cannot be too particular to have your method of practice right. Your improvement depends as much on how you practice as on how much you practice.

LESSON 4—EXERCISES 25 TO 32

38. Exercise 25.—In Lesson 4 are presented letters that are formed by the use of the *reverse*, or *indirect*, *motion*. The indirect oval drill of Exercise 25 must, therefore, be practiced. The directions of the arrows and ending strokes show the difference between this exercise and Exercise 2. *Do not make it like Exercise 2.* It will perhaps be a little harder for you than the direct oval. Work hard on it. Count as in Exercise 2 and write at the same speed.

39. Exercise 26.—The oval of Exercise 26 is also written with the reverse movement. Count 1, 2, 3, 4, 5 on each oval and make the ovals close together. Watch your position. The freedom of your movement depends on having it correct.

40. Exercise 27.—Write with a smooth, flowing motion. Be sure to get curves at the top and angles at the bottom. Retrace the strokes where the parts join. Taper with uniformity.

41. Exercise 28.—The *m* and *n* must be well rounded at the top. The angles where the parts join must be sharp and distinct and the strokes retraced a little. Do not make *n* like *u*. Avoid getting the last stroke of *m* and *n* sharp at the base. Fifty letters a minute for the *m's* and sixty-five for the *n's* is a good speed.

42. Exercise 29.—Make *x* exactly like the last part of *n* and cross it with an up stroke beginning at the base line. The letter will be easy for you if you follow this rule. Write about sixty *x's* a minute.

LESSON 4

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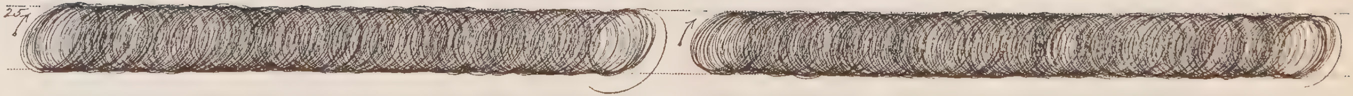

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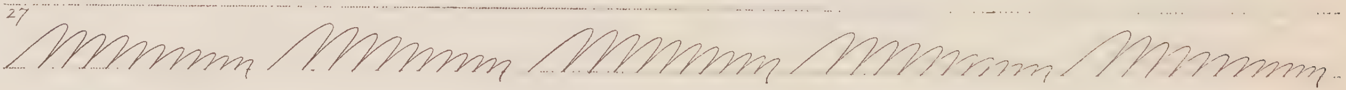

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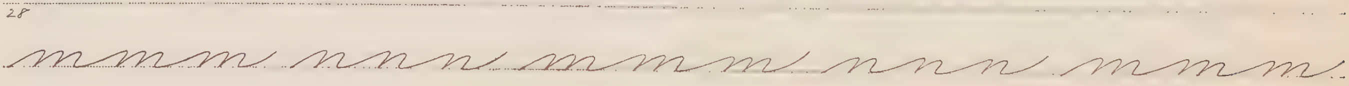


29 x xxxx v vvvv xxxx vvvv xxxx

30 D D D D D D D D D D D D D D D D

31 L L L L L L L L L L L L L L L L

32 Lima Dominion Luxor Dividend L

The first part of *v* is also like the second part of *n*. The shoulder on the last part is like the one on *w*. Guard against making the first part of *v* sharp or letting the shoulder fall too low. These faults make it look like *u*. Do not slant the second up stroke too much; this will make the letter too wide. Write from sixty-five to seventy *v's* a minute.

43. Exercise 30.—Get the capital *D* well in mind before writing it. Note the curve in the down stroke, the size of the loop at the base, the double curve along the bottom of the letter, and the little loop at the top. Use a purely arm movement in all capitals. You cannot get graceful lines if you use any finger action.

Count 1, 2, 3 for each *D* and write at a speed of from fifty to sixty a minute.

44. Exercise 31.—The down stroke and the loop at the base of the *L* are much the same as in *D*. Be careful that you do not make the beginning and ending strokes long and awkward. Do not pause between the letters but begin and end each one with the pen in motion.

Count 1, 2, 3, and write at about the same speed as for *D*.

45. Exercise 32.—Give the words in Exercise 32 thorough practice and criticize the various letters on the points that have been drawn to your notice as these letters were presented singly. Put special work on words containing letters that are particularly hard for you. Notice whether you are producing firm, smooth lines and whether the writing as a whole is uniform and pleasing in appearance.

46. Send several pages of your very best work on Lesson 4. Make it as much better than Lesson 3 as you can.

LESSON 5—EXERCISES 33 TO 40

47. Exercise 33.—The capital stem of Exercise 33 is used for all of the capital letters of Lesson 5, and in order to make these capitals easily the stem must be carefully studied and thoroughly mastered. Observe the size and slant of the little loop at the top and the slant of the down stroke. Do not give the down stroke more curve than it has in the copy and do not let the lower part of it reach too far to the left. End it firmly rather than with the pen in motion.

Count 1, 2, 3, 4, 5 on the tracing exercise and 1, 2 on the capital stem. The reverse movement is required for both.

48. Exercise 34.—Notice that each part of the capital *M* is lower than the preceding part. Notice how far the strokes retrace where the parts join. The spacing between the strokes must be close enough to give the entire letter a compact and well-proportioned appearance. The last stroke must have the same slant as the preceding down strokes and end just below the line.

Uniform spacing between letters is necessary to neat work. Count 1, 2, 3, 4, and write at the rate of thirty-five to forty letters a minute.

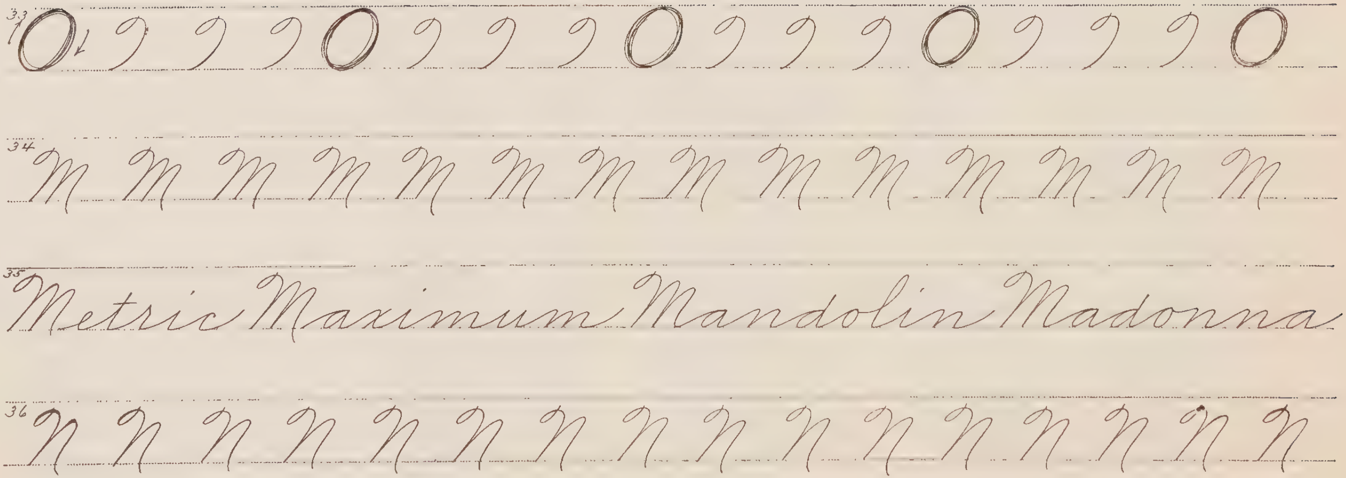
49. Exercise 35.—The separated capital as in *Metric* and *Maximum* allows greater freedom and requires less perfect control than the joined capital as in *Mandolin* and *Madonna.* The latter is, however, made more rapidly and has a smoother appearance.

Write, criticize, and rewrite. Give due attention to your position and movement.

50. Exercise 36.—Keep the down strokes parallel. Watch the loop in the beginning stroke. Do not make it too large or incorrect in slant. Retrace the strokes where the parts of the letter join.

Count 1, 2, 3 for each letter, and write at the rate of from forty-five to fifty a minute.

LESSON 5


33
34
M M M M M M M M M M M M M M
35
Metric Maximum Mandolin Madonna
36
N N N N N N N N N N N N N N N N

37 Neurotic Newman Narcissus Nation

38 U U U U U U U U U U U U U U

39 Upton Uniontown Uncas Undine U

40 V V V V V V V V V V V V V V V

51. Exercise 37.—The letters in the words of Exercise 37 should be as skilfully executed as in the groups of single letters. They give you work of a more practical nature. Endeavor not only to make good, legible forms and to write with an arm movement but give thought to the size of the writing, and to the spacing between letters, parts of letters, and words. Write a page, then criticize it as to general appearance.

52. Exercise 38.—Make a round turn at the base of the *U* and make the angle very sharp where the last stroke joins. These are the features that distinguish *U* from *N* and on which the legibility of the letter depends.

Count 1, 2, 3 and write at a speed of forty-five to fifty letters a minute.

53. Exercise 39.—Practice Exercise 39 in the same way as Exercises 35 and 37, stopping often to study and criticize. Write pages of every copy.

54. Exercise 40.—A common fault in making *V* is to slant the up stroke too much. Keep it parallel with the down stroke and curve the end gracefully. The first part should be a little higher than the second. Avoid an angle at the base of the letter.

Count 1, 2 and write at the rate of nearly sixty letters a minute.

LESSON 6—EXERCISES 41 TO 48

55. Exercise 41.—In practicing Exercise 41, think about the points that have been drawn to your notice in the various letters. Be sure to think good forms and you will write good forms. Write with care always.

56. Exercise 42.—Begin *W* like *M* or *N*. Pause at the bottom of the first stroke and make a decided angle. Curve the next stroke enough to make just the right space between the strokes. It is necessary to guard against either too much or too little space. This second stroke is higher than the first; the third stroke is almost straight; and the last one is curved and shorter than any preceding it. The *W* is a difficult letter to make well. Work faithfully on it. A common fault is to curve the first stroke too much and let the lower part of it reach too far to the left.

Count 1, 2, 3, 4, pausing after the first count, and write from forty to forty-five a minute.

57. Exercise 43.—Do not make the spaces between the words too great. Try to see how neat and compact you can make your page.

58. Exercise 44.—Try both styles of *X*. The second permits of joining to the following letter. Aim to have the parts just touch without crossing.

Count 1, 2 on the *X* and make from forty to forty-five a minute.

59. Exercise 45.—*Y* begins like *U* but ends with a loop below the line. The *Y* will be easy if you have mastered the preceding letters with the capital stem. The loop should extend only one-half the space between two lines.

Count 1, 2, 3 and write at a speed of at least forty a minute.

60. Exercise 46.—Note the double curve used in joining *Y* to *e* and *Y* to *u*.

Do not lift the pen before *a* when it occurs in the middle of a word.

Spend a short time daily in reviewing the oval exercises. You cannot grow too skilful in making them.

61. Exercise 47.—Observe that a small loop, not an angle, is used where the upper and lower parts of *Z* join. A short curved stroke in a horizontal direction connects the lower loop. Do not make this horizontal curve too long or extend it upwards too far, as in either case the letter will have a clumsy, humpbacked appearance. The loop below the line is curved on both the up and the down strokes. In this it is unlike the loop in *Y*, which has a straight down stroke.

Count 1, 2, 3 and write at the same rate of speed as for *Y*.

The beginning of the small *z* is exactly like the first part of *n*. It is finished much like the capital but with a slight retrace instead of a loop where the upper and lower parts are joined.

LESSON 6

41 Vance Vernon Vixen Vespers Vermont

42 W W W W W W W W W W W W W W W

43 Winona Winter Warsaw West Wax

44 X X X X X X X X Xavier Xantippe

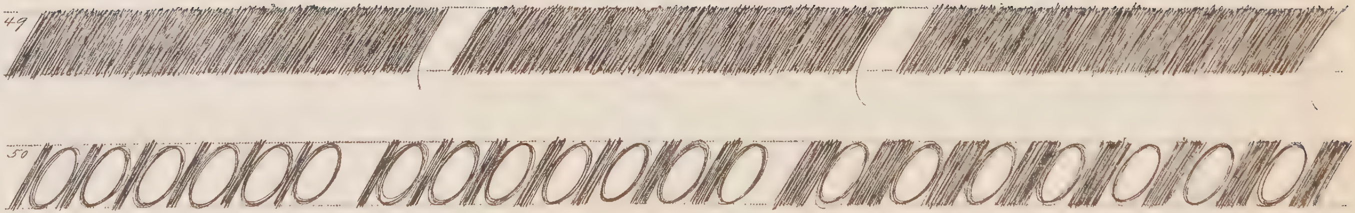
45 Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y

46 Yates Yeoman Yucatan Yemen Y

47 Z Z Z Z Zanzibar Zacher Zouave

48 A C E D E L O M N U V W X Y Z

LESSON 7



51 l l llll llll llll llll lll

52 lull little lisle island toll lot

53 b bbbb bbbb bbbb bbbb bbbb

54 bibber bubble blubber bill battle

55 h hhhh hhhh hhhh hhhh

56 Observe how each letter is made.

62. Exercise 48.—Space the letters of Exercise 48 evenly and make them equal in height and uniform in slant. It is not easy to write this line of capitals and have it present a pleasing appearance.

Remember that arm movement is absolutely necessary in the capitals.

Review Exercises 41 to 48 and prepare Lesson 6 for criticism.

LESSON 7—EXERCISES 49 TO 56

63. Exercise 49.—You have had the drill of Exercise 49 before, but you need to review it as preparatory work for the loop letters. You should do well on it this time. See that your position is correct. If you are not sure review the directions and look at the illustrations.

64. Exercise 50.—Use the direct motion for making the ovals. Count 1, 2, 3, 4, 5 for both the straight-line and the oval drill. Glide smoothly from one to another without a pause.

65. Exercise 51.—Note how curved is the up stroke of *l* and how straight is the down stroke. Pause a little at the bottom so that the turn at the base of the letter is quite decided but is neither too angular nor too rounding. The strokes should cross at the height of the letter *i* above the line. The upper loops should be the same height as capitals. *Do not make them too high.*

Count 1, 2, 3, 4 on the groups of *l's*, and write at least eighty letters a minute.

66. Exercise 52.—If your loops slant badly examine them to see whether you are getting the curves in the right places. When the up strokes are straight instead of the down strokes it causes too much slant.

67. Exercise 53.—The little retrace, or shoulder, at the end of the *b* is important. Make it show distinctly and do not push it over toward the loop so far as to close the lower part of the letter, thus making it look like a badly proportioned *f* resting on the line. A slight finger action is sometimes necessary in the loop letters.

Count 1, 2 for each *b* and write fifteen groups a minute.

68. Exercise 54.—Strive to get the *b's* and *l's* equal in height. Write freely and gracefully. Aim to have all down strokes parallel.

69. Exercise 55.—The *h* begins like *l* and ends like *n*. Note the angle where the parts join. Be careful to make the second part round at the top and to give the last down stroke the correct slant. This letter turned upside down should be exactly like *y*.

Count 1, 2 to each letter and write fifteen groups a minute.

70. Exercise 56.—Follow the advice given in the copy. Observation and care must be combined with persistent practice to make you a good penman.

Put several hours on this lesson before you prepare it for examination.

LESSON 8—EXERCISES 57 TO 64

71. Exercise 57.—Write these words at a speed of at least fifteen a minute. If you do not quite fill a line by writing the copy once, repeat the first word or two. A wide margin on the right-hand side is unsightly. The spaces between the words should be only twice as great as between letters.

72. Exercise 58.—The point of difference between *k* and *h* is the little knob at the top of the second part of *k*, which makes this part of the letter much like a very small capital *R*. Keep the last down stroke parallel with the preceding down strokes.

Count 1, 2 to each *k* and write at least forty-five a minute.

73. Exercise 59.—If you find certain letters are especially hard for you, those letters and the words containing them should have special attention. To see your own fault is the first step toward correcting it. A mistake may be pointed out to you many times, but until you yourself are able to see the difference between the right and the wrong form you will

LESSON 8

57 hill hale happen heath loth

58 k kk kkkk kkkk kkkk kkkk

59 kink kicker kind kill kindle

60 g gggg gggg gggg gggg gggg

61 going ginger gangway gull gage

62 jj jjjj jjjj jjjj jjjj jjjj

63 joiner jump junket jibe juice

64 Continue to practice the movements.

succeed in getting the right one only by some lucky chance. The eye needs training as well as the hand.

74. Exercise 60.—Begin *g* like *a* and finish with a loop under the line. This loop must not reach more than half the space between two lines. The down stroke is perfectly straight except for the curve at the very bottom. The up stroke has a good full curve and the strokes cross at the base line.

Make the *g's* at about the same speed as the *k's*.

75. Exercise 61.—Do your *g's* slant too much? Perhaps you are curving the down strokes. Look over your writing to see whether you are closing all the *g's* and *a's* at the top. Do not patch the letters or lift the pen where one of these letters is connected with a preceding letter.

76. Exercise 62.—The upper part of *j* is like *i*, the lower part like *g*. Note the double curve that joins the letters in the groups.

Count 1, 2, 3, 4, dot, dot, dot, dot, and write from forty-five to fifty letters a minute.

77. Exercise 63.—Dot *j* and *i* with precision. Do not make the dot like a dash. Watch the *b* and *k*. The style of *t* in *junket* should always be used at the end of a word. It does not require crossing.

78. Exercise 64.—Heed the advice in the copy. Your daily practice until you have entirely completed the penmanship lessons should begin with a drill on the movement exercises.

Send us your very best work on Lesson 8.

LESSON 9—EXERCISES 65 TO 72

79. Exercise 65.—The right position ought to be easy and natural for you by this time. You need to think about it occasionally, however. Begin *z* like *n*, then make a slight retrace and a good full curve to form the down stroke of the loop. This is the only loop letter that is curved on both the down and the up strokes.

Count 1, 2 on each letter and make at least fifty a minute.

80. Exercise 66.—Don't get discouraged if the lessons seem hard and your improvement comes slowly. Practice faithfully, following the instructions intelligently, and you will surely succeed; but remember that working in the right way or working in the wrong way makes all the difference between success and failure.

81. Exercise 67.—The *y* is a combination of the last part of *n* with *j*. Faults to be avoided are making the first part sharp either at the top or the bottom and making a curve instead of an angle where the parts join.

Count 1, 2 to each letter and make sixty or more a minute.

82. Exercise 68.—Look over your *y*'s to see whether you are getting the first part round or sharp. Note whether they slant correctly and whether the down stroke of the loop is straight or curved.

Notice whether you are closing the *a* part of *d* and whether you are retracing the stem neatly. It is not incorrect to loop the stem of *d* and it is better to do so than to make it clumsy by reason of a failure to retrace the strokes, or to cramp the movement in the attempt to retrace them.

Examine your *k* as to the spacing between the first and second parts and the slant of the last down stroke.

83. Exercise 69.—The point that differentiates *q* from *g* is the manner of making the loop. Turn to the right at the bottom, instead of to the left as in *g*, then curve the up stroke, ending it at the base line and making it meet the down stroke at an angle.

Count 1, 2, and write at a rate of fifty letters a minute.

84. Exercise 70.—Give the down strokes in all loops the same slant. The *q* is usually made just a trifle shorter than the other loops.

85. Exercise 71.—The upper part of *f* is like *l*. The lower part is like a small *o* made double its usual length. Do not curve the down stroke except a very little at the top and bottom.

Make fourteen groups a minute, counting 1, 2 to each letter.

LESSON 9

65 z zzzz zzzz zzzz zzzz zzzz

66 zip dizzy zigzag lazy ozone zero

67 y yy yyyy yyyy yyyy yyyy

68 yawl yonder yearly yours yank

69 q qq qqqq qqqq qqqq qqqq

70 quiet quibble quickly quaint quip.

71 f ffff ffff ffff ffff ffff

72 Always study a copy before practicing it.

LESSON 10

73 fairly filling fulfill fear fight

74 I I I I I I I I I I I I I I

75 Issue Inman Innocent India Ill.

76 J J J J J J J J J J J J J J J

77 Jip Jupiter Jeremiah James John

78 H H H H H H H H H H H H H H

79 Hanover Hamilton Howe Harper H

80 Improvement follows careful practice.

86. Exercise 72.—Try to make a fine page of Exercise 72. It ought to just fill a line nicely, but if you are using a smaller style than that in the copies you may begin the following sentence where the first one ends, thus making the sentence end at a different point each time. The entire page should have a compact appearance.

LESSON 10—EXERCISES 73 TO 80

87. Exercise 73.—Do not fail to observe that the up stroke of the lower loop of *f* should meet the down stroke at the base line, not above the line, and that the strokes should just meet, not cross. Many awkward looking *f's* are due simply to these two faults.

Compare the size of your upper and lower loops. They should not be equal. Long under loops will conflict with the loops and capitals on the following line and detract a great deal from the neatness of your page.

88. Exercise 74.—The first style of *I* ends with the pen at a complete standstill. The second requires only a slight pause to make the point at the left and ends with the pen still in motion. This form has the advantage of being easily joined to a following letter. Try both and adopt the one that seems to be best suited to you.

89. Exercise 75.—A common fault in making the capital *I* is to give it too little slant. This can be avoided by care in regard to the first stroke, which should not curve to the left too much but go directly upwards.

90. Exercise 76.—Make the first stroke in *J* like the first stroke in *I*. Never begin the capital with a horizontal curve at the base line. Notice that the down stroke is almost straight. The common mistake of curving it throws the slant out. It is not a hard letter, but many persons have trouble with it simply because of curving the down stroke. Notice that the first and last up strokes cross at the base line and that the lower part of the letter is considerably smaller than the upper.

91. Exercise 77.—Aim for enough freedom and control of your movement to enable you to join the capital *J* as in *James* and *John*. The easier way, if you are at all lacking in freedom, is to separate it, for the new impetus taken after lifting the pen permits of more force to the movement.

Be sure to make the down stroke in *p* parallel with the down stroke in *J*.

92. Exercise 78.—Try to get a graceful curve in the second part of the *H* and make this part of the letter a little higher than the first. Observe particularly that the strokes connecting the first and second parts form a triangle and that there is no loop. The spacing in this letter must not be too wide.

93. Exercise 79.—Study your *a's*, *o's*, *v's*, *w's*, and *r's*. Look for your weak points.

94. Exercise 80.—An entire page of Exercise 80 should be written in about 12 minutes.

Do not neglect your daily practice on the movement exercises.

LESSON 11—EXERCISES 81 TO 88

95. Exercise 81.—Observe that the beginning of the second part of *K* is a left and not a direct downward stroke. Keep this in mind and you will find it easier to get the curves correct. Note the size and position of the small loop. It is a little above the middle of, and at right angles with, the down stroke.

Make the same number of *K's* to a line as there are in the copy and be sure to space them evenly.

96. Exercise 82.—Are you sure that you are using the arm movement? Look to see whether your thumb joint moves. If it does you are introducing some finger action. Clean, sharp, smooth, and graceful lines require good pens, ink, and paper, and a light touch and a free-arm movement.

97. Exercise 83.—The capital *Q* resembles a very large figure 2. See how freely and gracefully you can write it.

LESSON 11

81 K K K K K K K K K K K K K K K K

82 Kenton Kensington Korea Kismet

83 Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q

84 Quinton Quinine Query Quince

85 S S S S S S S S S S S S S S S

86 Susan Samson Saint Summit

87 G G G G G G G G G G G G G G G

88 Keep the copies before you when practicing.

98. Exercise 84.—Examine the turns at the bases of the letters to see whether you are making them round enough. One of the things most necessary to legibility is to get curves and angles in the correct places.

99. Exercise 85.—A common fault in making *S* is to give the first stroke so little curve that the letter has entirely too much slant. Compare yours with the other capitals to see whether your slant is right.

Count 1, 2, 3, and make at least fifty letters a minute. You can write sixty or more if your movement is steady. The varying speeds, if the movement is right, result not so much from the actual speed of the pen while on the paper as from the habit on the part of many persons to stop and take aim or go through more or less preliminary motion before touching the pen to the paper.

100. Exercise 86.—Watch the *m's* and *n's* in Exercise 86. Make curves, angles, and retraced strokes in the proper places. Observe and criticize your work constantly and practice always for improvement in some particular point.

101. Exercise 87.—Do not make the beginning stroke in *G* any longer than the one in the copy. If you see that you have a tendency to get it too long try making it a little shorter. Care as to beginning and ending strokes has much to do with the general neatness of the writing.

Practice both styles of *G* and adopt the one you like better.

102. Exercise 88.—Do not let yourself become careless about *how* you practice. Have you the correct position of the body? Is your paper properly placed? Are you holding the pen correctly or do you let the hand fall over on its side? Do you succeed in producing smooth, graceful lines? The quality of the lines tells whether you are using arm movement and whether your speed is as high as it should be. If you are not getting good lines you need more work on the movement exercises.

LESSON 12—EXERCISES 89 TO 96

103. Exercise 89.—The words of Exercise 89 should prove to be easy to write smoothly and gracefully. Make sure that every letter is legible and do not forget that legibility means that a letter be so well made that it can be read without the aid of the context and that it is in itself perfectly plain and could not be mistaken for any other letter if it were standing alone.

104. Exercise 90.—Practice the indirect oval drill as preparation for the *T* and other capitals in this lesson. Study the curves in the stem of *T*. Begin with a curve toward the left and end the stroke with a dot. Keep the curve at the base of the stroke wide enough to be graceful. The top should be a compound curve and it should not be allowed to touch the stem.

Count 1, 2, and write at a speed of from thirty-five to forty letters a minute.

105. Exercise 91.—If you find it easier to end *T* like *S* and join it to the following letter it is not wrong for you to do so. Capitals should be neat and plain in form and have good style. Beyond that the forms that are chosen may be those best suited to an individual. Whenever more than one style of capital is given it is that you may have the advantage of choosing the one which seems best adapted to your hand.

106. Exercise 92.—Observe the loop in *d* in the word *Good*. This style allows greater freedom than the standard *d* and is often found to be easier, especially at the end of a word. Study the *o*, *s*, *a*, *r*—any letter, in fact, that is unusually hard for you. The *n's* and *u's* are not difficult letters, but you must not make them alike.

107. Exercise 93.—The capital *F* is exactly like *T* except for the horizontal stroke and dot. Write this to a count of **1**, **2**, **3**, and aim for nearly thirty a minute.

LESSON 12

89 Gaming Gamewell Giant Going.

90 T T T T T T T T T T T T T T T

91 Training Tammany Trenton Tip.

92 Good movement is the true foundation.

93 F F F F F F F F F F F F F F F F

94 Fremont Fairman Frye Farmington.

95 P P P P P P P P P P P P P P P P

96 Train the eye to see and the hand to act.

108. Exercise 94.—Take care in writing the combination *ye* to have the up and the down stroke of the loop cross at the base line. It is necessary to use a compound curve in connecting the two letters. The combination *gt* in *Farmington* is also somewhat difficult. Aim to become able to write it well and at the same time freely.

109. Exercise 95.—The *P* begins with a stroke like the stem of *T* or *F* but continues in the direction of the oval and ends with a down stroke, clasping the first half of the beginning stroke. This is a much easier capital than the two that have just preceded, both because of its simpler form and because it can begin and end with the pen in motion. Write the entire line of *P's* with an even, rhythmic motion, counting 1, 2, 3 at the rate of fifty-five or sixty a minute. Get the hand to moving rhythmically and you will find that your writing will be more uniform in slant and spacing. Often when the slant is bad it is because of the introduction of a little finger action.

110. Exercise 96.—The ending *d* as in *and* and *hand* may finish below the line. This permits of more freedom, as it is easier than the upward ending stroke.

Try to prepare some exceptionally fine pages of Lesson 12.

LESSON 13—EXERCISES 97 TO 104

111. Keep up the practice on the indirect oval daily throughout Lesson 13.

112. Exercise 97.—Watch your *a's* and *c's* with special care in practicing Exercise 97. Close the *a* and make a distinct hook on the *c*. Do not lift the pen immediately before either of these letters. Sometimes they are found to be hard to connect, but the continued exercise of care soon makes it a habit to connect them.

113. Exercise 98.—The *B* is to be made without lifting the pen, the up stroke retracing the beginning stroke. Keep the oval parts of the letter rounding and about equal in size. Use either ending you prefer. Count 1, 2, 3 and write at a speed of from forty-five to fifty letters a minute.

114. Exercise 99.—Make the shoulder on the *w* very plain. This is a letter that many persons do not make legibly. The *m's* and *n's* occur frequently. Do not make the too common mistake of getting them sharp at the top.

115. Exercise 100.—Write freely and forcefully. Make several pages of the sentence in Exercise 100. Criticize your work in regard to appearance, strength of line, uniformity, and letter formation. Be a severe critic of your own work and you will find that your instructor will criticize you less severely.

116. Exercise 101.—You will like the *R* in Exercise 101. It is an easy letter. It begins much like *B* and ends like *K*. Write the *R's* to a count of 1, 2, 3, and make about fifty a minute. Space evenly between the letters.

117. Exercise 102.—Practice Exercise 102 with care and attention to details, and let us say at this point that to make good progress in penmanship you must write carefully *always*, not simply when you are practicing on these lessons. An hour's careful work on the penmanship lessons will in a large measure be lost if you do several hours of careless writing on something else. You cannot afford to be in so great a hurry that you do not do your best at all times.

118. Exercise 103.—We hope that you do not need the gentle reminder contained in the copy of Exercise 103, but it will be well to make sure that you do not. Correct position is important in more ways than in making you a better writer. It has to do with your health, and you cannot afford to neglect it for that reason if for no other.

119. Exercise 104.—The stub *g* used in *preceding* is legible and can be easily and rapidly made. End it with a firm stroke, not with the pen in motion. Make it the same length as the lower part of *p*.

Look over your *s's* to see whether you curve the up strokes enough. Avoid lifting the pen immediately after *s*.

LESSON 13

97 Panama Puritan Practice Peter Pie

98 B B B B B B B B B B B B B B B B

99 Bowman Barnum Banner Blake

100 Practice the movement drills frequently.

101 R R R R R R R R R R R R R R R R

102 Running Rambler Randall Revere

103 Be sure to assume the proper position.

104 Review preceding lessons often as possible.

LESSON 14

105 A. B. Carver D. E. Fuller G. H. Innes J. K. Lee

106 M. N. Olney P. Q. Reid S. T. Ulmer V. W. York

107 Watch your position and movement closely.

108 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0

"Belive me when I tell you that the thrift of time will repay you in after life with a usury of profit beyond your most sanguine dreams, and that the waste of it will make you dwindle, alike in intellectual and in moral stature, beneath your darkest reckonings."

LESSON 14—EXERCISES 105 TO 109

120. Exercise 105.—Space the capitals evenly and try to get them equal in height and uniform in slant. Place periods after the initials. Note the style of capital *F* used in the second name.

121. Exercise 106.—A signature has no context. It must be legible in itself, but often signatures are almost or quite unreadable either because of the carelessness of the writer, who apparently forgets that every one is not so familiar with his name as he is himself, or because of an evident desire for something fanciful in the way of a combination of the capitals. Individuality in handwriting is all right provided it does not interfere with legibility. It is often possible to combine capitals artistically and still have them perfectly plain. To this there can be no valid objection if it makes the name easier to write or more pleasing in appearance. Yet we want you to note the beauty and simplicity of the names used in the copy. You can hardly do better than to follow this example and use a style of equal simplicity in writing your own name.

122. Exercise 107.—Follow the advice in the copy of Exercise 107, practice much, criticize closely, and you will steadily improve.

123. Exercise 108.—Make figures with a quick, precise motion. A little finger action is permissible. Every figure must be perfectly plain, for the context never helps in reading.

Figure *1* is a straight, slanting line; figures *2* and *3* are alike in their beginning strokes; *4*, *5*, and *6* each begin with a straight line; *3* and *5* end alike, and *7* and *9* end alike. The horizontal stroke of the *5* should be made to touch the beginning stroke; *4* must not cross the line; *7* and *9* must cross it; *7* begins with a short down stroke; the first part of *9* is exactly like the body of *a*; *6* is higher than the other figures. The strongly curved stroke of the *8* is its beginning stroke; do not make this figure backwards. The *0* must be closed at the top; it is like the main part of the letter *o*.

Write several pages of figures, keeping them in vertical columns. Do not let them run diagonally across the page, as you perhaps will find there is a natural tendency to do. To keep the columns vertical you must begin each figure a little to the right of the bottom of the one above it. For easy adding, columns must be vertical. Therefore, practice making them so. If you find it hard, turn the paper and make them between the lines, placing two figures between each two lines. Small figures are the best and most practical for use in bookkeeping work.

124. Exercise 109.—The kind of writing shown in Exercise 109 will test your real skill. A page of solid writing executed freely and rapidly, yet so as to have a fine appearance by reason of being uniform in size, slant, spacing, and touch, and the letters made with reasonable correctness so that perfect legibility is insured, may be called good business penmanship. Do your best on this.

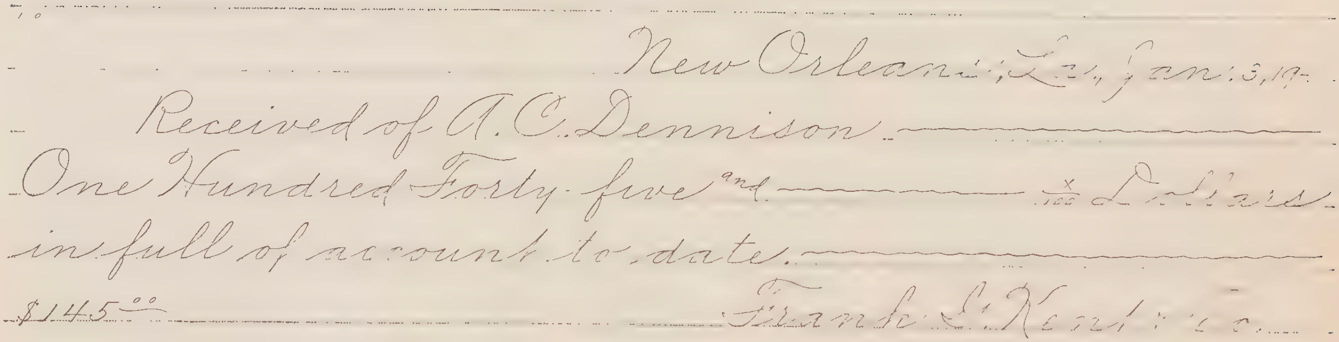
LESSON 15—EXERCISES 110 AND 111

125. Exercise 110.—Exercise 110 is a business paper called a **receipt,** which is an acknowledgment of the payment of value and may be either a written or printed form. The form may vary according to the kind of value received and the reason it is given. Thus, one may be given to apply on account, in full of all demands, for rent, to apply on a note, for a note, to executor for payment of a bequest, for instruction, and for many other things. A receipt should state plainly and fully for what the payment was made. A receipt is not certain proof of payment; it may be invalid because of mistake or fraud and is open to explanation or contradiction.

Note the position that each part of this business form occupies, and study the capitalization and punctuation closely. Do not make the capitals too large, but be sure to make them with a quick, dashy movement so as to get smooth, clean-cut lines.

As you write this copy, notice the arrangement and general appearance of your work. Study to find your errors and then do your best to eliminate them. A constant striving for betterment will do much to improve your writing. You may

LESSON 15



New Orleans, La., Jan. 3, 19—

Received of A. C. Dennison

One Hundred Forty-five and $\frac{x}{100}$ Dollars

in full of account to date.

$145.00 Frank L. Kent & Co.

111

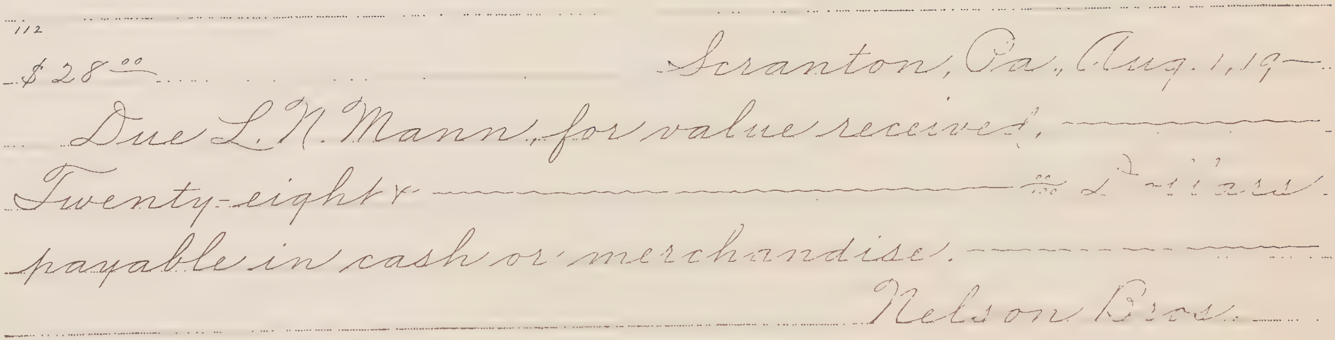
$72 40/100 Scranton, Pa., Jan. 6, 19

Ten days after date I promise to pay to the order of J. B. Landon & Sons Seventy-two and 40/100 Dollars at I. C. S. Bank.

Value received.

Due Jan. 16, 19— Thomas E. Wilson.

LESSON 16



112

$28.00 Scranton, Pa., Aug. 1, 19—

Due L. N. Mann, for value received,

Twenty-eight & 00/100 Dollars,

payable in cash or merchandise.

Nelson Bros.

113

$85⁰⁰ Scranton, Pa., May 15, 19—

At thirty days' sight pay to the order of R. J. Underwood & Co., —

Eighty-five & — $\frac{x}{100}$ Dollars

Value received and charge to account of

To John Wilson,

Carbondale, Pa.

V. I. Young

receive ever so much instruction, but the real writing must be your own work produced by your own efforts. Some of the finest writers have had the most difficulties to overcome. The policy of indomitable pluck has carried many persons to success, and in these days of business competition it pays to write well. Do not allow yourself to become careless because of the poor writing of some famous men, or to consider that you are like them because you write in like manner.

126. Exercise 111.—The copy of Exercise 111 is a **promissory note,** which is a written promise to pay a certain sum of money at a specified time, or on demand, to a person therein named, or to his order or assigns, or to the bearer. A note may pass from one man to another by indorsement, and in effect be the same as a bank note. It differs from a bank note only in that it is transferred by indorsement and matures at a stated subsequent time, and that the indorsers are liable to the holder in case the maker refuses payment. The person who signs the note is the *maker*, and the person to whom it is made payable is called the *payee*. The person who writes his name across the back of the note is an *indorser*. A note is *negotiable*, that is, transferable from one person to another by indorsement, when it reads "pay to the bearer" or "pay to the order of."

Write these business forms many times. Aim for constant improvement and submit your best work for criticism and additional instructions.

LESSON 16—EXERCISES 112 AND 113

127. Exercise 112.—At an adjustment of claims between parties, a **due bill** may be issued; that is a written acknowledgment of debt and may be payable in merchandise or money. It may be made payable on demand or at a future date, and by the insertion of the words *or order* it becomes negotiable the same as a promissory note.

Keep reviewing past lessons, always working for a light touch and good control of the pen. Business writing must be done quickly and should be dashy and clean-cut in appearance.

The only way to learn to write well is through earnest and well-directed practice. There is no royal road. We want your best work on this copy.

128. Exercise 113.—A **draft** is an order directing a second person to pay to a third person, or to any one he may designate, a certain sum of money.

The original parties to a draft are three: the *drawer*, who signs the draft; the *drawee*, to whom it is addressed and who has to pay it; and the *payee*, to whom it is made payable.

In the draft used in the copy of Exercise 113, Young is the drawer, Wilson the drawee, and Underwood & Co. the payee. This draft is drawn "at thirty days' sight" and is called a *time draft*. If worded "at sight" it would be a *sight draft* and payable immediately on presentation. The time on a draft which reads "At——days' sight" does not begin to run until the draft has been *accepted*, which means that the drawee writes across the face, "Accepted," the date, and his signature. If it reads "Thirty days from date" the time would run from the date of the draft, not from the date of acceptance. Some states allow days of grace on sight drafts, which makes them require acceptance and they become due three days after presentation. The draft used as the copy is of no value until it has been presented to Wilson and he writes his acceptance across its face. If he chooses to refuse to accept it, he may do so. If acceptance is refused the draft is said to be dishonored.

LESSON 17—EXERCISES 114 AND 115

129. Exercise 114.—A **check** is an order drawn on a bank or banker for the absolute payment of a certain sum of money to a certain person, or to his order, or to bearer, and is payable on demand.

A check is a kind of draft and has the same three parties, the drawer, who signs it; the drawee, who is always a bank or banker; and the payee, to whom it is made payable. Of course in actual business most papers like notes, checks, receipts, etc. are made out on printed blanks. It is well, however, to know

LESSON 17

114

No. 167 Cleveland, O., Feb. 24, 19—

Bank of Commerce

Pay to the order of L. B. Powell & Co.

Nine Hundred Twenty-one and $\frac{60}{100}$ Dollars

$921\frac{60}{100}$ James E. Robinson.

115

Warren E. Smith.

Pay to O. L. Isaacs.
Warren E. Smith.

Pay to the Order of
O. L. Isaacs.
Without recourse.
Warren E. Smith.

Pay to the Order of
O. L. Isaacs.
Warren E. Smith.

For Deposit only.
Warren E. Smith.

how these papers should read so that if you were required to draw one up on blank paper, you could do it correctly and without hesitation. It is said that the largest check ever drawn by Jay Gould was written in pencil on the back of an old envelope. Such copies as these are, therefore, valuable not only as penmanship lessons but also as samples of business forms.

130. Exercise 115.—Exercise 115 consists of **indorsements.** An indorsement is the memorandum made on the back of a note, check, or draft, usually for the purpose of transferring it from one person to another. The one who indorses a paper is called the *indorser*, and the one to whom it is transferred is the *indorsee.* There are various kinds of indorsements. The most common are:

1. *In blank.* An indorsement in blank is simply the name of the payee (or indorsee) in his own handwriting. This makes the paper payable to any person into whose hands it may fall.

The indorsement should be written on the left-hand end of the paper and if the first one is in blank it should be about 1 inch from the end. The new holder may now, if he wishes, for the sake of his own safety in case he loses the paper, change it to a *full* indorsement, which is not transferable without being again indorsed.

The first signature, "Warren E. Smith," is a blank indorsement.

2. *In full.* An indorsement in full mentions the name of the person to whom or to whose order it should be paid. This requires the indorsement of the indorsee before it is again transferable. It is a good form to use when sending negotiable paper through the mail.

The form "Pay to the order of C. L. Isaacs," is a full indorsement.

3. *Without recourse.* By adding the words "without recourse" the holder simply assigns the paper, relieving himself of responsibility for its payment. There are, however, certain things that every indorser warrants, and by the very act of transferring the instrument he engages that it is what it purports to be, the valid obligation of those whose names are

upon it, while specially declining to assume any responsibility as a party to the instrument.

4. *Restrictive.* A restrictive indorsement limits the negotiability of the paper. "Pay to C. L. Isaacs" makes the paper payable to C. L. Isaacs only. "For deposit only" is also a restrictive indorsement.

LESSON 18—EXERCISES 116 AND 117

131. Exercise 116.—A nicely written letter is appreciated even in the rush of business. Often a good-paying and responsible position is secured through the applicant's ability to write a good letter. There are employers who judge of a young man's ability from his letter of application, in preference to a personal interview. A great many marks of character are expressed in a letter—neatness, arrangement, penmanship, expression of thought, and attention to details, all of which go to make up a model business letter. It is worthy of mention that some of the best clerical positions in the world are for work requiring the best penmanship.

There are certain matters in connection with all letters that are of importance. These are form, arrangement, and punctuation. A letter may be said to consist of seven principal parts, namely: (1) the *heading*, including the date; (2) the *address;* (3) the *salutation;* (4) the *body;* (5) the *complimentary close;* (6) the *signature;* and (7) the *superscription.*

The heading occupies the top line and consists of the address of the writer and the date. When paper of the commercial letter size ($8\frac{1}{2}$ in. × 11 in.) is used, a heading containing no street address may occupy a single line. It should be so arranged as to end at or very near the right-hand edge of the paper. The parts of the heading, which are the name of the city or town and the state, the day of the month, and the year, are separated by commas. All abbreviations are followed by periods.

The address, which is the name and address of the person to whom the letter is written, and which in the case of Exercise 116 is "L. B. Powell & Co., Washington, D. C.," requires

LESSON 18

Cleveland, O., Feb. 24, 19—

L. B. Powell & Co.,

Washington, D. C.

Gentlemen:

I enclose herewith check for [illegible], which please place to my credit and oblige;

Yours truly,

James F. Robinson.

Miss Lucy Garland,
Wyoming,
Box 84. Pa.

Mr. J. B. Dunn,
416 Spruce St.,
Scranton, Pa.

Myers & Landon,
Summit,
N. J.

Mr. F. P. Marsden,
Windsor,
c/o Gunn & Co. N. Y.

LESSON 19

118

Franklin, Pa., July 6, 19—

The Deering Harvester Co.,
Chicago, Ill.

Gentlemen:

Please ship us at once by fast freight 20 Deering harvesters.

Yours truly,
Spencer & Loftus.

119

Chicago, Ill., July 8, 19—

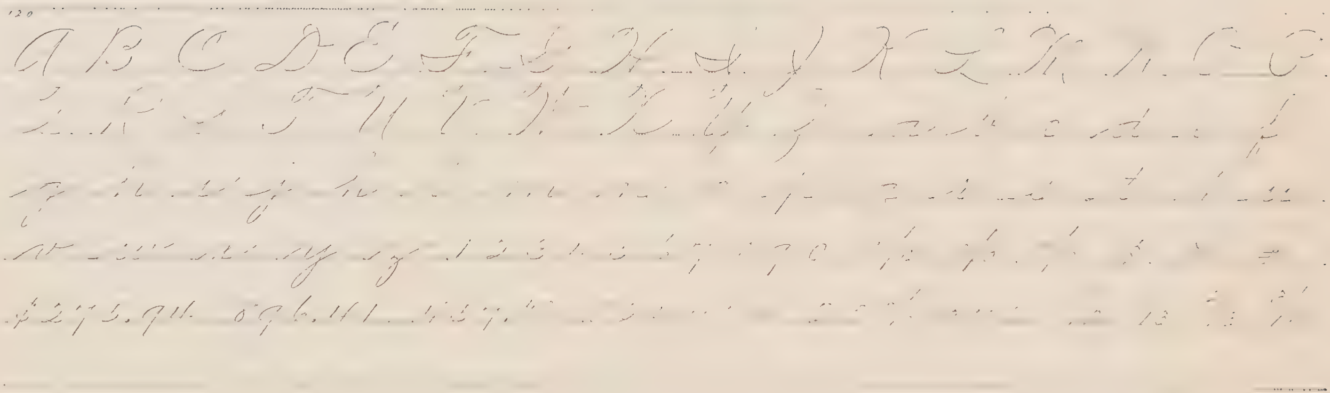
Spencer & Loftus,
Franklin, Ia.

Gentlemen:

We have today received your order of the 6th inst., for which accept our thanks. We will ship the harvesters tomorrow, the 9th, at the latest.

Yours respectfully,
The Deering Harvester Co.,
per C. C. Mills.

LESSON 20



The Inter. Corres. Schools.

Gentlemen: This is a specimen of my best writing after completing your course of lessons in penmanship.

Yours truly,

two lines. If the street address were given it would require three. The parts of the address should be followed by commas except the last, which is followed by a period. The second line is indented about $\frac{3}{4}$ inch from the left margin.

The salutation is the greeting, "Gentlemen," "Dear Sir," "My dear John," etc. with which it is customary to commence a letter. This begins at the margin and is followed by a colon or a comma. The colon is considered a little more formal. No dash is needed after the colon unless the letter is begun on the same line with the salutation.

The body of the letter is the actual communication. The first line of this and the first line of every new paragraph should begin directly under the beginning of the second line of the address, making an indention of about $\frac{3}{4}$ inch for all paragraphs. Throughout the entire letter there should be a left-hand margin of about $\frac{3}{4}$ inch, and this should be kept as even as possible.

The complimentary close, "Yours truly," or some similar expression, occupies a line by itself and begins at about the middle of the line. Only the first word of this should be capitalized. It should always be followed by a comma.

The signature should contain the full name of the writer. This should end at the right margin and be followed by a period.

132. Exercise 117.—The superscription is the address appearing on the envelope. The first line of the superscription contains the name and the title, as Mr., Miss, Mrs., Hon., Dr., etc. This line should be placed at the middle or a little below the middle of the envelope. Each line of the superscription should begin farther to the right than the line preceding; the last one, which contains the name of the state or country, should end at a distance of about $\frac{1}{2}$ inch from both the bottom and right-hand edges of the envelope.

When the superscription contains a street address the name of the city and state may occupy a single line, as in the case of "Scranton, Pa.," in Exercise 117. If there is a box number or the letter is in care of some one this memoranda may appear in the lower left-hand corner.

It is customary to use a comma after each part of the superscription, but this is no longer considered essential and many people omit it except when two parts of the address appear on a single line.

LESSON 19—EXERCISES 118 AND 119

133. Exercises 118 and 119.—The copies of Exercises 118 and 119 need no special explanation. Keep in mind the points drawn to your notice in Lesson 18. Keep in mind, also, the things you know to be necessary to good execution, position, movement, study, and care.

LESSON 20—EXERCISES 120 AND 121

134. Exercise 120.—Write the capitals and small letters of Exercise 120 in your best style. In the characters ℀, ℅, %, make the letters smaller than usual. Make the down stroke firmly and on the main slant. Give the figures a thorough review. Read Art. 123 again.

135. Exercise 121.—Exercise 121 is the last one of the Course, and we have made it a copy of a letter you are requested to write, not only because of the excellent practice it affords but also because it will give you a specimen, prepared under instruction, to compare with your writing at the beginning of the lessons. Note carefully the arrangement of the different parts of the exercise and endeavor to follow this arrangement in your work.

Please send us two or three pages of this last lesson in order that we may retain one as a final specimen of your writing.

SPELLING

INTRODUCTION

1. Most persons of the present generation are very poor spellers—a deficiency that seems to be due to the fact that they have not developed the *spelling instinct;* that is, the ability to recognize at a glance whether a word is spelled correctly or not. This quality is found to be natural in some persons, but those in whom it is not inherent should exert every effort to acquire it, as its possession is of much importance. Any person who will study carefully and who will learn to note words minutely will find that after a time he can distinguish at a glance whether the letters composing a word are the proper ones. He should train the eye to see immediately the letters contained in a word and should fix in the mind a vivid mental picture of the word, so that the way it looks may be recalled without difficulty. Noting words in this manner produces a much more permanent impression than is produced by the mere repetition of the letters of a word. Proficiency in spelling may be acquired by practice along these lines, and any person who wishes to increase his spelling instinct will find that his perceptive faculties are growing keener if he practices noting words in this manner.

2. In the expression of thought, words are used in two important ways—*in speech* and *in writing.* In speech, the pronunciation and the meaning of a word are the important elements; in writing, the spelling and the meaning are the essential points. It will therefore be seen that to know merely the spelling of words is not sufficient; the meanings that words may have and the ways in which they are pronounced must be understood also. This twofold aim of the subject of spelling

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can be effected only through a careful study of the pronunciation and the meanings of words, in addition to the learning of the letters of which words are composed.

3. To augment the study of words as it is here presented, a good dictionary should be consulted frequently. In the lessons that follow, the correct spelling of all words contained in them and the pronunciation and the meanings of many of the words are given; however, a dictionary is the only book that contains the correct spelling, the pronunciation, and the meanings of all English words. The "dictionary habit," that is, the habit of consulting the dictionary freely, should be acquired. Few persons realize the value of a good dictionary and do not know that it is a regular storehouse of general and technical information. No other book is a more complete or more convenient reference book. For these reasons, a good dictionary should be owned by every one and should be referred to frequently. Familiarity with its use will soon show its value and will prove how indispensable it is.

SPELLING LESSONS

LESSON 1

DIACRITICAL MARKS

4. In most dictionaries, spellers, and books pertaining to words, many of the letters of words have certain marks above, below, or through them, as in the words *āte*, *ĕnd*, *ärm*, *ēat*, *ovẽr*, *ġem*. Such marks are known as **diacritical marks,** and they are used to indicate the exact sounds of letters. Many letters have two or more sounds, and it is by the aid of diacritical marks that such sounds can be determined; thus, it is not a difficult matter to pronounce words correctly when one has learned what the marks mean.

The distinction between vowels and consonants must be understood if diacritical marks are to have their full significance. Five letters, *a*, *e*, *i*, *o*, and *u*, are pronounced by the vibrations of the vocal cords and are known as *vowels;* the other letters of the alphabet are *consonants.* Each of the consonants represents a sound that, in utterance, is usually combined with a vowel.

In every dictionary some system of diacritical marks is used to bring out the sounds of the letters. The sounds denoted by the various marks are usually shown in a table, or key, given at the beginning of the dictionary. Frequently, these sounds are also illustrated at the top or the bottom of each page of the dictionary. If these illustrations are studied thoughtfully, no difficulty should be experienced in pronouncing words in which the same sounds occur.

In the table of diacritical marks that follows and in the exercises in which diacritical marks are shown, the system used in the latest edition of Webster's Collegiate Dictionary has been followed.

Table of Diacritical Marks

ā as in ale
ă as in am
â as in care
ä as in arm
ȧ as in ask
ē as in me
ĕ as in end
ẽ as in her
ī as in ice
ĭ as in ill
ĩ as in sir
ō as in old
ŏ as in not
ô as in orb
ū as in use
ŭ as in up
û as in urn

o͞o as in food
o͝o as in foot

ȼ, like k, as in cat
ç, like s, as in cent
ȼh, like k, as in chorus
çh, like sh, as in machine
ch, unmarked, as in child
ḡ, hard, as in go
ġ, like j, as in age
n̲, like ng, as in ink
ş, like z, as in has

5—*ā* LONG, AS IN *ale*

āte
day
ray
hay
cage

fāme
make
jail
bait
take

plāce
trade
chain
pray
frail

bāthe
slate
slave
frame
graze

clāim
paste
faint
praise
plague

6—ă SHORT, AS IN *am*

lăd
apt
flat
glad
băck
bran
sash
sang
hăve
crash
drank
stamp
cătch
scalp
badge
scamp
scrătch
plank
hatch
sprang

7—ē LONG, AS IN *me*

ēve
eat
tea
here
need
chēat
near
scene
shears
sphere
blēak
leave
clean
thief
grief
crēak
theme
sneak
preach
shield
bēast
please
scheme
smear
seized

8—ĕ SHORT, AS IN *end*

cĕnt
step
lend
help
tent
dĕaf
next
debt
when
hemp
mĕant
dread
chest
smell
sweat
dĕnse
tempt
fence
flesh
scent
brĕast
wealth
pledge
death
thread

9—ī LONG, AS IN *ice*

mīce
time
nice
high
sign
knīfe
prize
price
white
wire
spīne
bribe
quite
blithe
strife
heīght
flight
while
writhe
bright
guīde
shine
tribe
blind
right

10—ĭ SHORT, AS IN *ill*

pĭn
sing
quit
milk
knit
lĭve
zinc
niche
print
stiff
clĭck
think
rinse
width
which
brĭng
still
whist
stitch
script
glĭmpse
bridge
switch
fringe
kissed

11—ō LONG, AS IN *old*

fōe
comb
torn
loan
yolk
foam
tōast
wrote
throne
scold
forge
coach
ghōst
rogue
mourn
broke
though
coarse
cōax
known
hoarse
growth
clothe
throat

12—ŏ SHORT, AS IN *not*

fŏg
dog
knob
spot
rod
dŏll
rock
lodge
drop
song
blŏnde
prompt
block
throng
stock
flŏck
copse
knock
dodge
wrong

13—ô AS IN *orb*

fôrm
corn
morn
sort
lôrd
fork
chord
horse
nôrth
scorn
torch
stork
stôrm
thorn
morgue
corpse
scôrch
short
horned
gorge

14—ū LONG, AS IN *use*

tūne
cube
mule
cue
suit
jūice
deuce
cute
feud
sued
glūe
pure
fuse
duke
cube
plūme
flute
sluice
glue
fugue

15—ŭ SHORT, AS IN *up*

bŭd
jump
much
numb
gun
yoŭng
touch
crumb
fudge
fund
scrŭb
crust
struck
grudge
flush
dŭnce
thumb
crutch
plunge
lunch

16—â AS IN *care*

râre
lair
fair
tare
âir
wear
bear
hare
fâre
chair
snare
spare
stâre
square
flare
scare
sweâr
scarce
prayer
glare

17—ä AS IN *arm*

fär
half
part
scar
cälf
start
parse
starve
pärk
star
charm
large
sälve
psalm
march
sparse
jäunt
starch
heart
haunt

18—ȧ AS IN *ask*

gȧsp
fast
mass
last
chȧnt
dance
grasp
clasp
pȧss
lance
path
past
chȧnce
mast
cast
branch
bȧsque
flask
glance
graft

19

The sound of *ẽ* in *her*, *ĩ* in *sir*, and *û* in *urn* is identical. The following words illustrate this sound:

fẽrn
herb
jerk
verse
pert
earl
heard
dĩrt
girl
dirge
birth
firm
smirch
shirred
slûr
urge
spurt
curve
nurse
surge
spurn
lẽarn
nẽrve
squĩrm
thĩrst
lûrch
scoûrge
twĩrl

20—SYLLABLES AND ACCENT MARKS

All the words given thus far are words of *one syllable;* that is, each of the words is of such nature that it is pronounced by a single vocal sound. However, the majority of words in the English language consist of *two* or *more syllables*. Correct pronunciation depends very largely on the proper separation of a word into syllables, and for this reason an understanding of the syllables of which a word is composed is necessary.

All words of two or more syllables are accented more strongly on one of the syllables than on the others. In order to indicate this fact, an *accent mark*, which is made thus ′, is placed at the end of the syllable that is to receive the greatest stress. For example, when the word *cap′tain* is pronounced, more emphasis is given to the first syllable than to the second. This is shown by the use of the accent mark, which occurs immediately after the first syllable.

In words of three or more syllables, two accent marks are very often used. One of them is known as the *primary accent* and the other as the *secondary accent*. The primary accent marks the syllable that is to receive the stronger emphasis, and the secondary accent follows the syllable that is to receive the weaker accent. These marks are distinguished from each other by the fact that the primary accent is a heavy mark, and the secondary accent,´ a light accentual mark. These marks are illustrated in the word *in′di ges′tion*, where the primary accent is placed on the third syllable *ges* and the secondary accent on the first syllable *in*.

In the exercises that follow, many words of two or more syllables are given. In these words only the primary accent is indicated, but the syllables are shown in every case.

21

peel
core
seeds
stem
pulp
rind
tree
fruit
eyes
nose
head
chin
ears
teeth
cheek
mouth
bones
el′bow
knee
lungs
heart
feet
arms
fin′gers
just
earn
blue
bus′y
hand′ful
trib′al
cough
lose
your
were
worse
broad
col′or
read′y
brown
man′y

22

trunk
round
their
claws
world
through
board
green
mean
light
quilt
gone
howls
called
curled
smok′y
po′ny
phlegm
ci′der
can′dy
cor′ner
pea′nuts
stock′ing
cheer′ful
e nough′
dread′ful
ven′ture
skip′ping
sur′face
char′coal

23—THE CLOCK

face
hour
hands
wheel
key
glass
brass
case
wood
weights
i′ron
di′al
fig′ures
min′ute
ham′mer
pen′du lum

24—RELATIVES

pa′pa
fa′ther
moth′er
par′ents
mam′ma
sis′ter
son
aunt
niece
wife
un′cle
cous′in
neph′ew
broth′er
chil′dren
daugh′ter
hus′band
kin′dred
kins′man
grand′fa ther
an′ces tors
grand′daugh ter
son′-in-law
moth′er-in-law

25—FOOD AND EATING

pies
buns
pork
game
beef
lamb
cake
eggs
bread
ba′con
liv′er
sug′ar
tap i o′ca
sau′sage
oat′meal
pud′ding
break′fast
din′ner
sup′per
lunch′eon

26—THE HOUSE

floor
porch
par′lor
kitch′en
nurs′er y
clos′et
gar′ret
fire′place
chim′ney
cup′board
cel′lar
hearth
man′tel
fur′nace
door′step
at′tic
en′try
pan′try
di′ning-room
stair′case

27—IN THE SCHOOLROOM

desks
class
pens
seats
globe
shelf
chart
pa′per
ru′ler
cray′on
pu′pils
teach′er
schol′ars
pic′tures
pro′gram
shut′ters
black′board
at′las
ink′stand
pen′-wi per
waste′-bas ket
tran′som
reg′is ter
dic′tion a ry

28

toil
moat
arch
frown
sell
pawn
heat
jest
joke
pile
stray
moves
filth
gape
space
quaint
blast
quoth
ache
ruse
scrub
scour
trick
yawn
scowl
dressed
trench
fu′el
jol′ly
stick′y
wan′der
di rect′
frol′ic
la′bor
be fore′
mid′dle
of′ten
or′der
din′gy
sor′ry

29

lean
lamp
bold
noise
ripe
south
dude
on′ly
o′pen
mer′ry
mor′tal
sev′en
wag′on
dug′out
mis take′
my self′
par′ing
naugh′ty
rum′ple
quar′rel
roast′er
jin′gle
par′cel
crip′ple
tor ment′
shad′ow
re cluse′
sul′len
spo′ken
un less′
try′ing
un til′
tum′ble
er′ror
ca reer′

30

taste
teach
stood
wake
thought
speech
im part′

worth
wreck
whose
move
beast
doubt
a′bly

up set′
up′per
sto′ries
writ′ten
sel′dom
alm′ond
o′gre

shiv′er
gal′lon
sal′low
slang′y
whisk′ers
in ter rupt′
foe′man

pa′gan
lo′cal
trip′le
pref′ace
fu′sion
drilled
post′age

31

prism
loaf
came
mill
scum
beard
sprite
bonds

groan
strength
tow′er
reach
joints
leaves
dry′ad
o′pi um

re fine′
rip′en
boiled
hogs′head
mo las′ses
liq′uid
bo′gus
dudg′eon

cot′ton
tun′nel
blos′som
sev′er al
talk′a tive
plan ta′tion
pro te ge′
blind′ness

32 THE HOUSE

tongs
ba′sin
pok′er
shov′el
dip′per
buck′et
coal′-hod
grid′i ron

sieve
stove
broom
ket′tle
scut′tle
dust′pan
skim′mer
sauce′pan

buf′fet
chi′na
plates
sau′cer
gob′let
tea′cup
tu reen′
plat′ter

fork
spoon
ta′ble
cru′et
cast′ers
nap′kin
tum′bler
side′board

33—ŏ SHORT, AS IN *not*

cŏf′fin
for′est
bot′tle
gloss′y
bon′net
tri′pod
con vey′

cŏn′cert
pos sess′
sor′rel
cof′fee
rock′et
nos′tril
prob′lem

trŏt′ted
nov′el
pop′py
trol′ley
com plaint′
froth′y
ob serve′

hŏm′i ly
pock′et
of fense′
pov′er ty
pros′e lyte
vol can′ic
lon′gi tude

34—THE HOUSE

soap
bowl
tow′els
bol′ster
pitch′er
scis′sors
nee′dles
thim′ble

sheets
lounge
pil′lows
bu′reau
blank′ets
mat′tress
bed′stead
cov′er let

vase
car′pet
mir′ror
cur′tain
has′sock
book′case
arm′chair
ot′to man

so′fa
stool
ta′ble
mu′sic
pi an′o
por′trait
cab′i net
cush′ions

35—BIRDS

owl
dove
crow
lark
gull
swan
thrush
ruff

wren
hawk
quail
stork
grouse
crane
snipe
plov′er

ra′ven
ea′gle
con′dor
lin′net
cuck′oo
spar′row
starl′ing
pet′rel

par′rot
os′trich
pig′eon
her′on
pea′cock
swal′low
gros′beak
pheas′ant

o′ri ole
vul′ture
par′tridge
blue′jay
bob′o link
night′in gale
wood′peck er
pel′i can

36—WHAT BIRDS DO AND HAVE

coo
caw
chirp
cheep

soar
whir
poise
perch

car′ol
hov′er
war′ble
twit′ter

whis′tle
mi′grate
wings
beak

tal′on
pin′ion
feath′ers
plu′mage

37—ANIMAL SOUNDS

purr
hum
low
grunt
squeal

yelp
howl
quack
growl
mew

bleat
cluck
neigh
croak
snort

squeak
roar
scream
buzz
screech

gob′ble
cack′le
whin′ny
bel′low
chir′rup

38—o͞o LONG, AS IN *food*

mo͞on
bloom
goose
booth

spo͞ol
root
moor
snooze

ho͞od′lum
sham poo′
roost′er
boot′black

to͞oth′ache
broom′stick
mon soon′
car toon′

39—o͝o SHORT, AS IN *foot*

wo͝ol
book
stood
shook

co͝op
soot
wood
look

wo͝od′bine
brook′let
foot′ball
crook′ed

ho͝od′wink
good by′
rook′er y
cook′y

40—LAND

plain
o′a sis
val′ley
des′ert

cape
is′land
con′ti nent
pen in′su la

isth′mus
head′land
moun′tain
prom′on to ry

prai′rie
pla teau′
vol ca′no
ta′ble-land

41—WATER

creek
stream
strait
brook
frith
fiord

marsh
o′cean
riv′er
cas cade′
chan′nel
la goon′

har′bor
tor′rent
break′ers
del′uge
foun′tain
whirl′pool

mael′strom
cat′a ract
es′tu a ry
riv′u let
trib′u ta ry
res′er voir

42—INSECTS

bee
wasp
flea
lo′cust
hor′net
shrimp

gnat
moth
roach
bee′tle
crick′et
scar′ab

ant
wee′vil
mos qui′to
glow′worm
silk′worm
scor′pi on

drag′on-fly
bum′ble bee
but′ter fly
ka′ty did
grass′hop per
ta ran′tu la

43—OCCUPATIONS

tai′lor
ba′ker
doc′tor
law′yer
cob′bler
sto′ker

flo′rist
join′er
gro′cer
bank′er
mer′chant
pur′ser

farm′er
weav′er
build′er
butch′er
drug′gist
stew′ard

mil′li ner
min′is ter
gar′den er
car′pen ter
black′smith
boat′swain

44—HOMONYMS

Frequently two or more words are of different origin and meaning, but have the same spelling or the same pronunciation. For example, *lēad*, meaning to conduct, and *lĕad*, a metal, are spelled alike, but are pronounced differently, while *cent*, *sent*, and *scent*, are similar in sound, but not in spelling. Such words are known as **homonyms.** Because of their similarity, words of this kind are often confused and used incorrectly. An understanding of such terms is necessary if one wishes to avoid errors in their use, and for this reason the exercises on homonyms should be studied very carefully, so that the distinction between the terms given in them may be understood.

1 cord, *a small rope*
chord, *the string of an instrument; a combination of musical tones*

2 made, *finished*
maid, *a maiden; a girl*

3 throne, *the seat of a king*
thrown, *cast*

4 bread, *food made of flour*
bred, *brought up*

5 flour, *ground wheat*
flow′er, *a blossom*

6 knight, *a title of honor*
night, *time of darkness*

7 know, *to understand*
no, *a word of denial*

8 hall, *a large room*
haul, *to drag by force*

In the sentences that follow, the *homonyms* are printed in Italic. Reference to the preceding list will show that these words are used correctly.

(*a*) Use this *cord* for your package.
(*b*) Where are you going, my pretty *maid*?
(*c*) The *knight* was *thrown* from his horse.
(*d*) We went into the reception *hall*.
(*e*) She *made* the *bread* from wheat *flour*.
(*f*) Did you *know* that he came home last *night*?
(*g*) *No*, you cannot have this *flower*.
(*h*) The king sat on his *throne*.
(*i*) He is a well *bred* man.
(*j*) *Haul* this tree to the bank of the river.
(*k*) That *chord* of music is beautiful.

45—ē LONG, AS IN *eat*

co hēre′
im pede′
fe′male
de′pot
e′qual
sen′ior
trēa′ty
con vene′
ju′bi lee
the′o ry
need′y
su preme′
fēa′si ble
de scribe′
e′qui nox
trea′tise
sea′port
fea′ture
mē′di um
pre′mi um
weak′ly
mean′ing
free′dom
crea′ture

46

so′lar
lu′nar
tar′tar
stel′lar
sec′u lar
vin′e gar

nec′tar
lat′ter
an′ger
bet′ter
or′der
tem′per

dro′ver
can′ker
la′ter
cof′fer
mourn′er
pep′per

dif′fer
hin′der
gan′der
fil′ter
plun′der
tick′er

lar′der
blis′ter
blub′ber
mem′ber
nei′ther
ped′dler

47

where as′
wheth′er
here to fore′
so′journ
in′stance
per turbs′
al to geth′er

doff
oc curred′
cash ier′
u′sing
in dulge′
ex panse′
sol′dier ly

bar′ring
gra′ded
lim′it ed
sham′rock
them selves′
en a′bling
mid′dle

im ag′ine
dis pense′
as cer tain′
thank′ful
peace′ful
per tain′ing
piece′meal

48—PERTAINING TO NUMBER

first
three
third
twice
fifth
eighth

twelve
twelfth
for′ty
nine′ty
fif′ty
four′teen

sin′gle
dou′ble
hun′dred
mil′lion
tri′ple
thir′teen

thir′ti eth
for′ti eth
thou′sand
eight′i eth
nine′ti eth
hun′dredths

49—TREES

fir
elm
oak
pine
palm
larch
beech
birch
bal′sam

ce′dar
ma′ple
wal′nut
hem′lock
cy′press
pop′lar
wil′low
hol′ly
ban′yan

spruce
lin′den
dog′wood
hick′o ry
chest′nut
red′wood
rose′wood
but′ter nut
se quoi′a

bass′wood
syc′a more
ca tal′pa
box′wood
pal met′to
mag no′li a
sas′sa fras
ma hog′a ny
per sim′mon

50—WE SHOULD BE

good
frank

cor′dial
sin cere′

help′ful
thought′ful

civ′il
o blig′ing

prompt
hon′est
no′ble
po lite′
lov′ing
truth′ful
care′ful
hope′ful
ge′ni al
stu′di ous
pa′tient
cour′te ous
gen′er ous
o be′di ent
tem′per ate
in dus′tri ous

51—WE SHOULD NOT BE

mean
curt
proud
la′zy
sulk′y
sau′cy
stin′gy
cru′el
self′ish
un kind′
fret′ful
sul′len
rude
sur′ly
care′less
haugh′ty
de ceit′ful
tat′tling
im po lite′
dis hon′est
cow′ard ly
quar′rel some
dis hon′or a ble
dis o be′di ent

52—WINDS

gale
gust
breeze
squall
zeph′yr
tem′pest
whirl′wind
hur′ri cane
cy′clone
si moon′
ty phoon′
tor na′do
trade′-wind
an′ti trade
si roc′co
mis′tral

53—BOATS

barge
yawl
sloop
yacht
do′ry
gon′do la
frig′ate
steam′er
ca noe′
cut′ter
schoon′er
ves′sel
dread′nought
sub ma rine′
man-of-war′
cruis′er

54—FRUITS

lime
grape
quince
pear
peach
plum
lem′on
ap′ple
cher′ry
or′ange
rai′sin
cit′ron
cur′rant
grape′ fruit
rasp′ber ry
pine′ap ple
huc′kle ber ry
straw′ber ry
a′pri cot
goose′ber ry
ba na′na
wa′ter mel on
pome′gran ate
can′ta loupe

LESSON 2

55—NOUNS

Many words are used as the name of some person, place, thing, quality, or idea. Such words are known as **nouns**. Each of the words in the following list is a noun.

cloak
rib′bon
er′rand
bal loon′
kit′ten
jack′et
sleeve
fields
type
win′dow
scare′crow
mus′tard
pen′knife
Hen′ry
Bos′ton
mes′sage
bride′groom
cask′et
loaves
mar′bles
Hud′son
Mon′day
tres′tles
coun′tries

56—SINGULAR NOUNS

Nouns that denote only one person, place, thing, quality, or idea are **singular nouns**, or nouns in the *singular number*. Each of the nouns that follow is in the singular number.

skull
stamp
creed
let′ter
lad′der
span′iel
cra′dle
wa′fer
swim′mer
in ven′tion
shin′gle
pol′i cy
les′son
ur′chin
cab′in
cit′i zen
ex cur′sion
mis′sion
vil′lain
bun′dle
han′dle
horse′shoe
marks′man
snow′shoe

57—PLURAL NOUNS

Nouns that denote more than one person, place, thing, quality, or idea are **plural nouns**, or nouns in the *plural number*. Each of the nouns that follow is in the plural number.

choirs
tri′als
slip′pers
bridg′es
sal′ads
nov′els
nos′trils
ra′zors
or′phans
drag′ons
In′di ans
vi′sions
jour′neys
wo′men
pau′pers
crutch′es
guin′eas
tis′sues
stock′ings
tongues
sal′ar ies
hatch′ways
he′roes
pi o neers′

58—ARITHMETIC

rate
terms
prime
dig′it
ze′ro
a′cre
naught
cu′bic
fac′tor
frac′tion
al′i quot
dis′count
ex am′ple
in′ter est
in′te ger
dec′i mal
av oir du pois′
cal cu la′tions
com pos′ite
nu′mer a tor
de nom′i na tor
a mount′
prin′ci pal
di vis′i ble
in sur′ance
bro′ker age
per cent′age
mul′ti ple

59—HOMONYMS

1 pair, *two*
pare, *to cut off*
pear, *a fruit*

2 their, *belonging to them*
there, *in that place*

3 cap′i tal, *chief town; stock*
cap′i tol, *building*

4 hear, *to listen*
here, *in this place*

5 meat, *animal food*
meet, *proper; to come together*
mete, *to measure*

6 as cent′, *a rising*
as sent′, *agreement*

7 pane, *a plate of glass*
pain, *an ache*

8 rye, *a grain*
wry, *crooked*

In the following sentences, the numbers enclosed in marks of parenthesis refer to words in the group of the same number. Write each sentence correctly by using, in place of the number in it, the correct word from the preceding group of words. For instance, in sentence (*a*), the word *there* should be used in place of the number (2). This sentence will then read: *There is nothing new under the sun.—Bible.*

(*a*) (2) is nothing new under the sun.—*Bible.*
(*b*) Can you (1) a (1) with a (1) of scissors?
(*c*) When we visited the (3) of the United States, we saw the (3), a white marble building.
(*d*) When shall we three (5) again?—*Shakespeare.*
(*e*) Our guide said that the (6) of Mont Blanc is full of danger.
(*f*) (4) rests his head upon the lap of earth.—*Gray.*
(*g*) (2) lives were in danger when that large (7) of glass fell.
(*h*) He would not give his (6) to the proposal.
(*i*) (8) grows in cold countries.
(*j*) She suffered great (7) after the operation.

60

de clare′
col lect′
al lay′
ap pall′
at tach′
pac′i fy
de fy′
can′cel
a ver′
a mass′
as suage′
af fright′
ap pend′
pit′y
ap pease′
a bol′ish
as sert′
con cede′
man′age
con cern′
ca jole′
con sent′
re voke′
dis cuss′
af firm′
al low′
at tract′
dis cern′
se cede′
a gree′
an nul′
ar′gue
se cure′
an noy′
re volt′

61

nerve	un nerve′	ech′o	re ech′o
spell	mis spell′	e lect′	re e lect′
spend	mis spend′	le′gal	il le′gal
step	mis step′	sev′er	dis sev′er
en gage′	re en gage′	mor′tal	im mor′tal
sat′is fy	dis sat′is fy	sim′i lar	dis sim′i lar

62

ro tund′
doub′le
punc′ture
mat′ting
val′u a ble
pen′sion
brick′bats
jew′el er
to mor′row
which ev′er
scen′er y
tel′e graph
i de′als
su pe′ri or
va ca′tion
twi′light
moun′tain
cam′e ra
myth
sleet
tacks
flu′ent
awn′ing
al fal′fa

63—THE FARM

soil
earth
ground
marl
loam
muck
mold
turf
silt
stone
sand
hu′mus
fal′low
fer′tile
ster′ile
pas′ture
mead′ow
wood′land
sub′soil
ver′dure
drain′age
ar′a ble
fruit′ful
or′chard

64—NAMES OF THE MONTHS WITH THEIR ABBREVIATIONS

Jan′u a ry	Jan.	Ju ly′	July
Feb′ru a ry	Feb.	Au′gust	Aug.
March	March	Sep tem′ber	Sept.
A′pril	Apr.	Oc to′ber	Oct.
May	May	No vem′ber	Nov.
June	June	De cem′ber	Dec.

65—WEARING APPAREL

hose
gloves
cra vat′
skirt
shawl

mack′in tosh
gai′ters
col′lar
trou′sers
shirt

veil
scarf
hood
ruf′fle
neck′tie

muff
boots
tip′pet
storm′coat
o′ver alls

66—ō LONG, AS IN *old*

vōt′er
ve′to
bo′nus
mo′tive
lo′cate
sloth′ful
o paque′
post pone′

clō′ven
gold′en
pro fuse′
thresh′old
o′ver seer
mo rose′
tel′e phone
gro tesque′

suf′fō cate
o′pen ness
be moan′
o′pi ate
ig no′ble
e mo′tion
no′ta ry
ho′li ness

vō′cal ist
ro′se ate
her′o ism
in′do lence
ret′ro spect
po lite′ness
be hold′er
tro′che

67—FOOD

veal
tarts
toast
gru′el
hon′ey
sal′ad

mut′ton
bis′cuit
noo′dles
crack′ers
waf′fles
catch′up

broth
cut′let
con′serve
muf′fins
por′ridge
chow′der

hom′i ny
cus′tard
dump′ling
dough′nut
sauer′kraut
om′e let

68

fools′cap
si′ren
mi rage′
pas′time
jus′tice
Cu′ban
des′pot
ton′sil

Bronx
cav′il
civ′ic
de bate′
sub′way
fer′ry
slo′gan
ex haust′

jun′ior
el lipse′
de spair′
um′pire
armed
de pos′it
ban′dits
a byss′

ven′ti late
des′o late
hap′pened
at ten′tion
em broid′er
fore gone′
sham′bles
sol′ace

69—PLURALS

Rule.—*The plural of many nouns is formed by adding s to the singular;* as, bride, bride*s*; girl, girl*s*.

feasts
sprites

hor′ses
but′tons

cel′lars
mit′tens

mag a zines′
fos′sils

Ro′mans
scarfs
cor′sets
bon′nets
an′kles
hel′mets
a′prons
gai′ters
ur′chins
rib′bons
er′rands
dra′mas
farm′ers
tel′e scopes
sched′ules
cab′i nets

70—PLURALS

Rule.—*Nouns ending in s, sh, x, z, ch soft, and some ending in o after a consonant, add es to the singular to form the plural;* as, class, class*es*; bush, bush*es*; fox, fox*es*; fuzz, fuzz*es*; watch, watch*es*; hero, hero*es*.

kiss′es
loss′es
pass′es
grass′es
cho′rus es
wit′ness es
rush′es
crash′es
lash′es
marsh′es
flash′es
brush′es
box′es
tax′es
lynx′es
frizz′es
fezz′es
to′paz es
ditch′es
ranch′es
birch′es
peach′es
church′es
wretch′es
car′goes
ech′oes
ne′groes
em bar′goes
mu lat′toes
man′goes

71—ḡ HARD, AS IN *go*

ḡos′sip
tar′get
girl′ish
ig nore′
gas′tric
gos′pel
ḡrav′el
drug′gist
gal′ler y
gal′lop
reg′u lar
lit′i gate
fru′ḡal
in trigue′
con′gre gate
au′to graph
reg′u late
grav′i tate
del′e ḡate
par′a gon
gov′er nor
al′li ga tor
ob li ga′tion
ig no min′i ous

72—HOMONYMS

1 air, *what we breathe*
e'er, *ever*
ere, *before*
heir, *one who inherits*

2 sail, *of a ship; to begin a voyage*
sale, *a selling*

3 ho′ly, *sacred*
whol′ly, *completely*

4 plain, *level ground; clear*
plane, *flat surface; tree; tool*

5 rain, *water from clouds*
reign, *rule*
rein, *for a horse*

6 coarse, *rough*
course, *way*

7 col′lar, *band for the neck*
chol′er, *anger*

8 dy′ing, *ceasing to live*
dye′ing, *coloring*

Write the sentences that follow, and in place of each number enclosed in marks of parenthesis, write the correct word, selecting the word from the group numbered the same as the number enclosed.

(*a*) They were having a (2) of lace (7)s.
(*b*) Russia is almost (3) a vast (4).
(*c*) That cloth is very (6).
(*d*) The (5) will purify the (1).
(*e*) The ship will (2) (1) daybreak.
(*f*) His (7) is easily aroused.
(*g*) We thought her (8) when she slept.
(*h*) The king will (5) with justice and wisdom.
(*i*) The Prince of Wales is (1) to the throne.
(*j*) That is a good material for (8).

73—FAR NORTH

seal
whale
sa′ble
wal′rus
er′mine
rein′deer
po′lar bear
ei′der duck
floe
Lapp
au ro′ra
gla′cier
moss′es
li′chens
Es′ki mo
ice′bergs

74—TOOLS

file
adz
rake
spade
plane
gouge
square
lathe
wrench
shears
le′ver
au′ger
gim′let
mal′let
pin′cers
chis′el
lev′el
trow′el
hatch′et
cork′screw

75

se rene′
com′ic
pu′trid
ea′ger
rud′dy
lav′ish
foiled
rus′tic
tac′it
tur′bid
stur′dy
som′ber
tim′id
be trothed′
waste′ful
fear′ful
ru′ral
plac′id
ar′dent
hard′y
de throned′
si′lent
gloom′y
mud′dy
flor′id
rot′ten
mirth′ful
shift′less

76—ā LONG, AS IN *ate*

en grāve′
a′gen cy
cham′ber
crāy′on
pa′tron
sta′tion
ef fāce′
con′cave
en gage′
dān′ger
ha′zy
ba′bel

brace′let
fa′vor ite
va′por
an′cient
bay′o net
dra′per y
be came′
pro fane′
dis place′
fa′ble
ha′tred
man′ger

77—WORDS LIABLE TO BE CONFOUNDED

Words that are liable to be confounded should receive careful study. Such words differ from homonyms in the fact that while they are closely related in spelling and pronunciation, they are not exactly the same in either of these two points. Because of the similarity in their form and pronunciation, they are frequently confused and used incorrectly. Take, for example, the words *precede* and *proceed*. *Precede* means to go before, and *proceed* means to advance; but how often do we hear such sentences as, *He* PRECEDED *on his journey* and *Lightning* PROCEEDS *thunder*. When the meanings of these two words are considered, it can readily be seen that the sentences should be given, *He* PROCEEDED *on his journey* and *Lightning* PRECEDES *thunder*.

A number of exercises on words of this kind are given to call attention to such mistakes and to aid in avoiding such errors. In the study of these words, careful thought should be given to their spelling, pronunciation, and meaning.

1 sculp′tor, *a carver of stone*
sculp′ture, *carved work*

2 proph′e sy, *to foretell*
proph′e cy, *a prediction*

3 pre cede′, *to go before*
pro ceed′, *to advance*

4 light′en ing, *to make light*
light′ning, *electric flash*

5 pa′tients, *sick people*
pa′tience, *endurance*

6 dis ease′, *illness*
de cease′, *death*

Write the following sentences, selecting the word that expresses the desired meaning:

(*a*) His (prophesy, prophecy) was fulfilled.
(*b*) Let us run with (patience, patients) the race that is set before us.—*Bible.*
(*c*) Enjoy the kingdom after my (disease, decease).
(*d*) The captain ordered the (lightning, lightening) of the ship.
(*e*) We saw many beautiful pieces of (sculptor, sculpture).
(*f*) The physician from Berlin cured many (patience, patients).
(*g*) The flashes of (lightning, lightening) were frequent.

(*h*) (Proceed, precede) on your journey and you will soon be with your friends.
(*i*) Phidias was a famous (sculptor, sculpture) of ancient Greece.
(*j*) Certain signs always (proceed, precede) certain events.
(*k*) The remedy is worse than the (disease, decease).
(*l*) They often (prophesy, prophecy) that evil will befall you.

78

ga rage′
ton neau′
chauf feur′
mo′tor ist
mo′tor ing

gas′o line
run′a bout
au to mat′ic
car′bu ret er
li mou sine′

au to mo′bile
lo co mo′bile
au to mo′bil ist
au to mo′bil ism
speed om′e ter

79—MILITARY TERMS

cap′tain
ma′jor
colo′nel
ser′geant
lieu ten′ant
gen′er al
en′sign

mi li′tia
ar′sen al
re doubt′
breast′works
bar′racks
gun′ner y
mar′shal

fur′lough
com mand′er
pa role′
dis′ci pline
de sert′er
can non ade′
cut′lass

cam paign′
ma neu′ver
com mis′sion
e quip′ment
fron tier′
em′is sa ry
e ques′tri an

80—REVIEW OF DIACRITICAL MARKS

clŏs′ĕt
ēa′gẽr
dĭ vōrce′
ā′prĭ cŏt
bȧ nä′nȧ

säun′tẽr
shĕr′ĭff
Tūes′day
trĭb′ūne
ăd drĕss′

fĭ năncé′
fĭn ăn ciēr′
cŏm′răde
ĕn vĕl′ŏp
cŏch′ĭ nēal

stū′dĭ oŭs
trăv′ĕl ẽr
drăm′ȧ tīze
ăc knŏwl′ĕdge
skĕp′tĭc

81

height
re sult′
con fess′
wrig′gle
fir′kin
anx i′e ty
bar′y tone

count′er
awk′ward
bus′i ly
re mark′a ble
men tal′i ty
u′su al ly
or′ches tra

la′zi ly
hes′i tate
spe′cial
u′ni verse
vil′lage
pro tect′
rap′id ly

ach′ing
sur prise′
quan′ti ty
syl′la ble
scold′ed
gos′sa mer
de li′cious

82

rai′ment
pan ta loons′
cod′i cil
u ten′sil
ve ran′da
beau′ti ful
won′der ful
the′o rize
south′ern er
myr′i ad
wiz′ard
griz′zly
ker′o sene
jew′el ry
tyr′an nize
au′thor ize
in dis tinct′
bach′e lor
par′lia ment
sep′a rate
ox′y gen
rep′tile
pat′ron ize
doc′ile
cor′nice
trans par′ent
for′eign er
sal′a ry
mam′mal
cru′ci fy
sol′em nize
rec′og nize
poor′house
wood′chuck
dra goon′
guard′i an

83—ç SOFT, LIKE *s*

çym′bal
ceil′ing
cis′tern
cen′tral
pac′i fy
plac′id
çyl′in der
cel′e brate
a cerb′i ty
so lic′i tude
pan a ce′a
per cep′tion
çin′der
cer′tain
nov′ice
cer′ti fy
or′i fice
mer′ci ful
çer′e mo ny
ce les′tial
cen ten′ni al
lu cid′i ty
mer′ce na ry
ne ces′si ty

84—*çh* SOFT, LIKE *ch*

çha rade′
chiv′al ry
cham′ois
cha grin′
mus taçhe′
cham pagne′
chat′e laine
chiv′al rous
ma çhine′
che mise′
chev′ron
cha teau′
çhev a lier′
Chey enne′
chan de lier′
at ta ché′

85—PLURALS

Rule.—*Nouns ending in y preceded by a vowel add s to form the plural;* as, monkey, monkey*s*; valley, valley*s*.

SINGULAR	PLURAL	SINGULAR	PLURAL
al′ley	al′leys	door′way	door′ways
es′say	es′says	don′key	don′keys
cov′ey	cov′eys	kid′ney	kid′neys
tur′key	tur′keys	chim′ney	chim′neys
pul′ley	pul′leys	at tor′ney	at tor′neys
vol′ley	vol′leys	rail′way	rail′ways
med′ley	med′leys	vice′roy	vice′roys

86—PLURALS

Rule.—*Nouns ending in y preceded by a consonant change the y to i and add es to form the plural;* as, city, cit*ies*; county, count*ies*.

SINGULAR	PLURAL	SINGULAR	PLURAL
ar′my	ar′mies	dra′per y	dra′per ies
par′ty	par′ties	so ci′e ty	so ci′e ties
a′gen cy	a′gen cies	mal′a dy	mal′a dies
pan′try	pan′tries	sem′i na ry	sem′i na ries
col′o ny	col′o nies	bound′a ry	bound′a ries
laun′dry	laun′dries	sec′re ta ry	sec′re ta ries
in′ju ry	in′ju ries	mis′sion a ry	mis′sion a ries
cav′i ty	cav′i ties	bal′co ny	bal′co nies
bat′ter y	bat′ter ies	cem′e ter y	cem′e ter ies
ca na′ry	ca na′ries	phar′ma cy	phar′ma cies

87—FLOWERS

tu′lips
as′ters
li′lacs
clo′vers
or′chids
pop′pies
lark′spurs
lil′ies
dai′sies
mar′i golds
haw′thorns
car na′tions
pan′sies
sweet peas
hy′a cinths
dan′de li ons
prim′ros es
gold′en rod
sun′flow ers
hol′ly hocks
vi′o lets
ge ra′ni ums
morn′ing-glo ries
chrys an′the mums

88

an′te rooms
rou tine′
mo ral′i ty
So′cial ism
U to′pi an
now′a days
pri′ma ry
mile′age
poul′try
fore cast′
con vened′
rear′-end
tex′tile
trans′it
age′less
re vers′es
typ′i fy
cur′so ry
hu mane′
ra′di um
plat′form
e lect′or ate
re voked′
post′ of fice
al li′ance
qui e′tus
moth′er hood
Dem o crat′ic
per son nel′
dis sec′tion
il lit′er ate
di′et a ry
in debt′ed ness
may′or al ty
can′o py
de gen′er ate

89—RELIGION

priest
pas′tor
dea′con
chor′is ter
chap′lain
bap′tism
Bi′ble
re vi′val
par′a ble
Christ′like
sac′ri lege
dox ol′o gy

bish′op
cler′gy
la′i ty
lit′ur gy
mosque
di vine′
dis ci′ple
cer e mo′ni al
rev′er end
con′se crate
sac′ra ment
chan′cel
con ver′sion
wor′ship er
re deem′er
pen′i tent
im mer′sion
tes′ta ment
ben e dic′tion
mon′as ter y
the ol′o gy
sur′plice
syn′a gogue
car′di nal

90—THE CITY

may′or
ten′ant
sew′er
gut′ter
pa trol′
av′e nue
curb
hy′drant
po lice′
pre′cinct
hos′pi tal
sa loon′
the′a ter
mu se′um
pave′ment
lamp′post
traf′fic
ten′e ment
col′lege
li′bra ry
sta′tion
me trop′o lis
trol′ley car
gar′bage
op′e ra
neigh′bors
high′school
fac′to ry

91—POSSESSIVES

Nouns are often used in the possessive form to denote possession over some person, place, or thing. In the singular number of a noun, the apostrophe and *s* (’*s*) are added to denote possession. If the plural form of a noun ends in *s*, the apostrophe alone is added; but if the plural form does not end in *s*, the apostrophe and *s* (’*s*) are added.

SINGULAR	PLURAL
The *girl’s* doll	The *girls’* dolls
The *child’s* excuse	The *chil′dren’s* excuses
That *he′ro’s* story	Those *he′roes’* stories
A *man’s* mother	*Men’s* mothers
The *ba′by’s* clothes	The *ba′bies’* clothes
The *wom′an’s* work	The *wom′en’s* work

SINGULAR	PLURAL	SINGULAR	PLURAL
rob′in’s	rob′ins’	la′dy’s	la′dies’
fish’s	fish′es’	calf’s	calves’
broth′er’s	broth′ers’	goose’s	geese’s
gi′ant’s	gi′ants’	ox’s	ox′en’s

ac′tor's	ac′tors'	wife's	wives'
com′rade's	com′rades'	deer's	deer's
horse's	hors′es'	mouse's	mice's

92

per′son
mis′er
fe′ver
ver′dict
pas′tel
ser e nade′
strag′gler
fer ment′

nib′bler
fi′er y
ser′mon
her′mit
ver′bal
in trud′er
glad′i a tor
e qua′tor

ru′mor
vis′or
lan′guor
tre′mor
hor′ror
fer′vor
pre cep′tor
splen′dor

vir′gin
fir′kin
e lix′ir
thirst′y
fur′ri er
sur′geon
hur rah′
tur′bu lent

93—ĕ SHORT, AS IN *end*

fĕs toon′
Hes′sian
sel′dom
fem′i nine
ven′i son

sĕn′a tor
cel′i bate
cen′ti ped
des′ig nate
read′i ly

in fĕc′tious
rep′ro bate
stren′u ous
des′po tism
sen′si ble

mĕn′ace
em′is sa ry
im pet′u ous
des′ti tute
po ten′tial

94

loi′ter
toi′let
poi′son
coin′age
ap point′
poign′ant

in′voice
re joice′
mois′ture
oint′ment
em broid′er
hoi′ty-toi′ty

al loy′
coy′ly
roy′al ty
an noy′
de coy′
em ploy′

roy′al ly
toy′shop
boy′ish
boy′cott er
foy′er
buoy′ant

95

cowl
prowl
clown
crown
drown
chow′der

crowd
bow′er
dow′er
cow′ard
drow′sy
down′cast

rouse
shout
crouch
mound
flounce
cloud′y

de vour′
foun′dry
scoun′drel
com′pound
pro nounce′
boun′ti ful

LESSON 3

96—FISH

eel
cod
carp
perch
bass
squid
trout
pike
shark
shad
smelt
salm′on
dol′phin
sar′dine
had′dock
suck′er
min′now
saw′fish
her′ring
mack′er el
pick′er el
stur′geon
hal′i but
tor pe′do

97—FOOD AND EATING

rec′i pe
sher′bet
vict′uals
gro′cer y
es′sence
sa′vor y
ed′i ble
ap′pe tite
pro vi′sions
di ges′tion
beef′steak
glut′ton y
ep′i cure
fes′ti val
stim′u lant
suc′co tash
mac a roons
mac a ro′ni
lem on ade′
nour′ish ment
spa ghet′ti
pal′a ta ble
vo ra′cious
rav′en ous
gor′man dize
ca′ter ing
bev′er age
ban′quet ing

98—HOMONYMS

1 mar′shal, *an officer; to direct, guide, or lead*
mar′tial, *warlike; of, or pertaining to, war*

2 serf, *a servant; a slave*
surf, *the swell of the sea*

3 can′non, *a great gun*
can′on, *a rule of law*

4 great′er, *larger*
grat′er, *a tool for rubbing off small particles*

5 coun′cil, *an assembly of persons convened for consultation*
coun′sel, *one who gives advice in legal matters; to admonish or instruct*

6 birth, *coming to life*
berth, *a sleeping place in a ship or in a railway car*

7 cru′el, *unkind*
crew′el, *a soft yarn*

8 less′en, *to make or become less*
les′son, *that which is learned or taught*

9 colo′nel, *an army officer*
ker′nel, *the central part of a nut*

10 ce′re al, *edible grain*
se′ri al, *a publication appearing in a series*

Write the sentences that follow, and in place of each number enclosed in marks of parenthesis write the correct word, selecting this word from the group numbered the same as the number enclosed.

(*a*) The (9) ordered the men to move the (3) to higher ground.
(*b*) The city (5) met and decided many important matters.
(*c*) Do you enjoy bathing in the (2)?
(*d*) The (1) endeavored to prevent the crowd from entering the building.
(*e*) What means this (1) array?
(*f*) The sale of (10)s is growing (4) every year.
(*g*) Will you engage an upper (6) for me?
(*h*) Many Mexicans are very (7) to their (2)s.
(*i*) That (10) has been appearing in the magazines for several months.
(*j*) You should engage Hon. Mr. Brown as your (5).
(*k*) This (8) is not very difficult.
(*l*) The manufacturers violate many (3)s.

99—TERMS USED IN BUILDING

build′ing
mor′tar
gird′ers
struc′ture
flag′stones
spli′cing
ce ment′
con′crete

col′umn
ma′son ry
fire′proof
Goth′ic
res′i dence
E gyp′tian
or′na ment
di′a gram

bal us trade′
el e va′tion
ves′ti bule
par ti′tion
ca the′dral
cor′ri dor
ve ran′da
a but′ments

bun′ga low
sta bil′i ty
sanct′u a ry
tab′er na cle
lav′a to ry
col os se′um
ter′ra-cot ta
ar′chi tec ture

100—PLANTS

bone′set
dah′lia
sor′rel
plan′tain
chard
li′chen
jon′quil

mul′lein
bur′dock
cel′e ry
su′mac
a lys′sum
daf′fo dil
musk′mel on

nar cis′sus
lav′en der
a nem′o ne
mi gnon ette′
ja pon′i ca
chic′o ry
ru ta ba′ga

glad i o′lus
he′li o trope
nas tur′ti um
rho do den′dron
pen ny roy′al
nec′tar ine
hy dran′ge a

101—GEMS AND PRECIOUS STONES

pearl
o′pal

ber′yl
to′paz

cam′e o
cat′s′-eye

sap′phire
em′er ald

ru′by
gar′net
ag′ate
o′nyx
cor′al
jas′per
blood′stone
moon′stone
tur quoise′
di′a mond
am′e thyst
car nel′ian

102

pros′per ous
rec′og nize
ref′er a ble
vig′i lance
wil′der ness
de bil′i ty
dis as′ter
plau′si ble
su per′la tive
un spot′ted
re tire′ment
in hab′i tants
path o log′ic al
di min′ish
vig′or ous
phi lan′thro pist
cu′ri ous
ver′ti go
knav′er y
val′u a ble
aux il′ia ry
mel o′de on
cor′du roy
in trin′sic
ob′sti na cy
ret′i cent
tre men′dous
re sus′ci tate
pen′ni less
ac cel′er ate
ar is toc′ra cy
os′tra cism

103—ô AS IN *orb*

fiôrd
tor′pid
hor′net
dor′sal
es cort′
form′al
thorn′y
land′lôrd
horn′blende
short′age
hail′storm
for′tune
green′horn
dor′mant
fôr′mu la
gor′mand
horse′man
dis gorge′
im mor′tal
or′gan ism
e nor′mi ty
ex ôr′bi tant
for′fei ture
in dorse′ment
sub or′di nate
for′ti tude
hor′ti cul ture
ex traor′di na ry

104—MEDICINES

qui′nine
cam′phor
ar′se nic
cre′o sote
ar′ni ca
cal′o mel
hore′hound
al′co hol
co′ca ine
lau′da num
glyc′er ine
sas′sa fras
par e gor′ic
strych′nine
chlo′ro form
am mo′ni a
pep′per mint
cop′per as
bel la don′na
sar sa pa ril′la
mer′cu ry
i od′o form
win′ter green
eu ca lyp′tus
as a fet′i da
sac′cha rine
dig i ta′lis

105—VOCATIONS

chem′ist
or′a tor
states′man
lin′guist
sci′en tist
phar′ma cist
cler′gy man
mu si′cian
man′i cure
phys′i cist
sten og′ra pher
so lic′it or
te leg′ra pher
e van′gel ist
re vi′val ist

dai′ry man
drafts′man
oc′u list
chor′is ter
op ti′cian
bot′a nist
el o cu′tion ist
as tron′o mer
gram ma′ri an

106—ġ SOFT, AS IN *gem*

a′ġed
gen′der
le′gion
ag′ile
din′gy
gyp′sy
stin′gy
lodġ′ment
herb′age
ges′ture
mar′gin
syr′inge
dun′geon
knowl′edge
in dul′ġent
gen′er ate
har′bin ger
trag′e dy
dil′i gent
gen′u ine
o rig′i nal
con ta′ġious
in fringe′ment
gen til′i ty
re lig′ious
mag′is trate
ger′mi nate
ger′mi cide

107—PLURALS

Write the plural form of each of the following nouns:

coach
con′cert
fai′ry
sen′a tor
vic′to ry
sand′wich
jock′ey
cav′al ry
mat′tress
man′sion
mu lat′to
leg′a cy
vol ca′no
for′mu la
dic′tion a ry
to ma′to
os′trich
of′fi cer
bul′rush
buf′fa lo
har′ness
min′is ter
tor pe′do
bat′ter y

108—DISEASES

cough
col′ic
mea′sles
asth′ma
ca tarrh′
chol′er a
scrof′u la
bron chi′tis
con sump′tion
neu ral′gi a
di ar rhe′a
pneu mo′ni a
er y sip′e las
rheu′ma tism
in flu en′za
hys ter′ics
diph the′ri a
lar yn gi′tis
ap′o plex y
jaun′dice
ep′i lep sy
hy dro pho′bi a
flat′u lence
ul cer a′tion

109—HOMONYMS

1 { mus′sel, *shell fish*
mus′cle, *an organ of the body*

2 { met′al, *iron; gold; etc.*
met′tle, *spirit; courage*

3 { ex′er cise, *to set in action*
ex′or cise, *to drive out evil spirits*

4 { climb, *to go up*
clime, *region*

5 serge, *a kind of cloth*
surge, *to rise; to swell; a swelling wave*

6 prin′ci pal, *chief*
prin′ci ple, *rule of action*

7 cur′rant, *fruit*
cur′rent, *a stream*

8 cym′bal, *a musical instrument*
sym′bol, *a sign*

9 gilt, *adorned with gold*
guilt, *crime; sin*

10 mi′nor, *one under age*
mi′ner, *one who works in a mine*

Write the sentences that follow, and in place of each number enclosed in marks of parenthesis write the correct word, selecting this word from the homonyms numbered the same as the number enclosed.

(*a*) The (10) found that using the pick developed his (1)s.
(*b*) At first, we floated with the (7).
(*c*) You should (3) every day if you expect to (4) those mountains.
(*d*) Her dress was made of (5) and was trimmed with (9) braid.
(*e*) A (10) cannot vote.
(*f*) The (8) was played by a girl dressed in Spanish costume.
(*g*) Grasp it like a man of (2).
(*h*) He is a man of good (6)s.
(*i*) The (7)s did not ripen.
(*j*) We found many (1)s in the gulf.

110

ac crue′
a chieve′
a ghast′
greas′y
ca det′
con geal′
fid′dle
vil′las
lik′a ble
a thwart′
vict′uals
cro chet′
bou quet′
bru nette′
yeo′man
mu′tu al
unc′tion
thwart′ed
con′quer
pag′eant
al′oes
anx′ious
brig′and
pic′nic
dic′tate
cri′sis
tex′ture
ath′lete
fau′cet
cal′lous
bur′glar
dul′cet
daunt′less
fa tal′i ty
rit′u al
no′bod y

111—ANIMALS

boar
goat
deer
bi′son
li′on
cam′el
badg′er
al pac′a
hy e′na
por′cu pine
kan ga roo′
jag′uar

wolf
moose
rab′bit
rein′deer
er′mine
bea′ver
ze′bra
ot′ter
mon′key
gi raffe′
wea′sel
go′pher
ser′pent
rac coon′
pan′ther
squir′rel
buf′fa lo
go ril′la
cham′ois
leop′ard
ti′ger
el′e phant
an′te lope
croc′o dile
rhi noc′e ros
drom′e da ry
hip po pot′a mus
ar ma dil′lo

112—*ă* SHORT, AS IN *am*

săd′dle
glad′ly
stag′ger
nar′row
gal′lon
fab′ric
gath′er
lăv′ish
tan′gle
can′cer
fath′om
pat′ent
dan′druff
ex act′ly
scăn′ty
ram′ble
prac′tice
pal′ette
brack′et
grav′i ty
es tab′lish
chăp′er on
ham′mock
baf′fle
pad′lock
grat′i tude
fab′u lous
mat′ri mo ny

113—HISTORY

writs
re sist′
mobbed
war′rant
un′ion
ep′och
hos′tage
re doubt′
bul′let
pow′der
ram′part
car′tridge
pa′tri ot
re pealed′
col′o nize
lib′er ty
strat′a gem
al li′ance
as sem′bly
pro hib′it
tax a′tion
tyr′an ny
op pres′sion
rep re sent′
in de pend′ent
dec la ra′tion
ca lam′i ty
vol un teer′
e vac′u ate
rev o lu′tion
al le′giance
gov′ern ment

114—MILITARY TERMS

sol′dier
com′pa ny
reg′i ment
bri gade′
di vi′sion
corps
in′fan try
cav′al ry
ar til′ler y
or′der ly
pla toon′
bat tal′ion
sword
sa′ber
bay′o net
pis′tol
car′bine
mus′ket
ri′fle
can′non
sen′try
pick′et
u′ni form
knap′sack
can teen′
ep′au let
ar′mis tice
cais′son
biv′ou ac
hav′er sack
siege
sut′ler
re cruit′
aid′-de-camp
gar′ri son
coun′ter sign
court-mar′tial
am mu ni′tion
for ti fi ca′tion
in trench′ment

115—NEWSPAPER TERMS

dai′ly
morn′ing
proof
sem i week′ly
week′ly
bi month′ly
month′ly
ex chang′es
e di′tion
pro pri′e tor
pub′lish er
print′er
ed′i tor
jour′nal ist
con trib′u tor
re port′er
cor re spond′ent
sub scrib′er
ad ver ti′ser
car′ri er
news′boy
ex′tra
ty po graph′ic al
lin′o type
ed i to′ri al
lead′er
ar′ti cle
i′tem
lo′cal
tel′e grams
no′tic es
gos′sip
fi nan′cial
a muse′ments
com pos′i tor
mon′o type

116—WORDS LIABLE TO BE CONFOUNDED

1 ac cept′, *to take*
ex cept′, *to leave out*

2 ad vice′, *counsel*
ad vise′, *to give counsel*

3 at tend′ance, the *persons who attend any services, etc.*
at tend′ants, *those who attend as servants, companions, etc.*

4 for′mer ly, *time past*
form′al ly, *in a formal way*

5 sta′tion a ry, *fixed*
sta′tion er y, *paper, pens, etc.*

6 pop′u lous, *full of people*
pop′u lace, *the people*

Write the following sentences, and in place of the numbers enclosed in marks of parenthesis use the correct word from the group of words numbered the same as the number enclosed.

(*a*) China is a (6) country.
(*b*) Will you (1) my (2)?
(*c*) The king had many brave (3).
(*d*) The (6) swarmed in the streets.
(*e*) A (5) engine drew the car to the top of the hill.
(*f*) The (3) was large at every lecture (1) the first.
(*g*) That (5) is of an excellent quality.
(*h*) The meeting was (4) opened by the president.
(*i*) (4) buffaloes roamed over the Great Plains.
(*j*) Your father will (2) you concerning the matter.

117—WORDS ENDING IN *cle* OR *cal*

i′ci cle
cu′ti cle
mir′a cle
bi′cy cle
ve′hi cle
clav′i cle
trea′cle
ves′i cle
pin′na cle
ob′sta cle
par′ti cle
pe dun′cle
man′a cle
spec′ta cle
bar′na cle
ven′tri cle
car′bun cle
tu′ber cle
sur′gi cal
op′tic al
clas′sic al
com′ic al
mag′ic al
med′ic al
prac′ti cal
trag′ic al
i ron′ic al
pe ri od′ic al
i den′tic al
crit′ic al
cler′ic al
cyn′ic al
ec o nom′ic al
spher′ic al
quiz′zic al
rad′i cal

118—PERSONAL QUALITIES

truth
hon′or
mer′cy
pa′tience
fe′al ty
loy′al ty
hon′es ty
sa gac′i ty
sanc′ti ty
hu mil′i ty
her′o ism
sto′i cism
fe roc′i ty
du plic′i ty
treach′er y
cow′ard ice
ret′i cence
lo quac′i ty
in′no cence
par′si mo ny
con′science
te mer′i ty
pre sump′tion
ar′ro gance
fas ci na′tion
com pla′cen cy
ge ni al′i ty
dif′fi dence

119—RAILWAY

en′gine
dan′ger
pi′lot
switch
sig′nal
pis′ton
boil′er
fire′man
road′bed
brake′man
wreck
safe′ty
bag′gage
cau′tion
rail′road
sched′ule
ex haust′
traf′fic
air′brake
throt′tle
pas′sen ger
au to mat′ic
con duc′tor
in jec′tor
cyl′in der
ec cen′tric
en gi neer′
col li′sion
ac′ci dent
lo co mo′tive
de rail′ment
e mer′gen cy

120—GAMES

whist
chess
craps
quoits
dice
golf
bowls
ten′nis
eu′chre
po′ker
pi quet′
hock′ey
crick′et
bil′liards
lot′to
pi′noc le
po′lo
check′ers
draughts
back′gam mon
base′ball
bas′ket ball
rou lette′
leap′frog
pa chi′si
foot′ball
cro quet′
crib′bage
sol i taire′
bac ca rat′
curl′ing
dom′i noes

121—ĭ SHORT, AS IN *ill*

ag′ĭle
viv′id
doc′ile
hos′tile
mis′sile
vig′il
rig′id
gui tar′
thĭm′ble
ac′tive
fes′tive
vi′cious
vic′tim
rep′tile
prin′cess
spir′it
crev′ĭce
im′age
jus′tice
den′tist
ser′vice
rig′or ous
vis′i tor
di gest′ive
fĭd′get
res′pite
fi del′i ty
ad he′sive
doc′trine
vi cin′i ty
ging′ham
crim′i nal

122—SYNONYMS

An important class of words in the English language is known as **synonyms,** which are words having almost the same meanings, but differing somewhat in the application of these meanings. A thorough understanding of the slight differences in the meanings of synonymous terms is essential if a person desires to express his ideas clearly, as each one of several synonyms conveys a slightly different impression to the mind than the others do, or is a more fitting descriptive term for some word than it is for any other. For example, the words *industrious*, *busy*, and *diligent* are synonymous terms, yet the meaning of each one of these words is such that it may be used where the other terms would not express the intended thought. *Industrious* implies constant application to any work or business. *Busy* applies to an activity that may be only temporary, while *industrious* refers to a habit of life. *Diligent* refers to the employment of oneself about a particular favorite object and implies free exercise of the mental, as well as the physical, powers. We may say, *The man is busy just now*, but not, *The man is industrious just now.* When we say, *She is an industrious woman*, we mean that she applies herself to her work at all times. *Diligent* shows its meaning in the sentence, *Mary is a diligent student.*

From these examples it can readily be seen that synonyms serve a twofold purpose. A knowledge of them helps a person to express himself clearly and concisely and enables him to avoid repetition in his expression. If one knows several words that express nearly the same idea, one will not have to repeat the same term several times in a sentence or in a paragraph.

It should be the aim of every person to familiarize himself with words that will express his meaning clearly and effectively. One way of accomplishing this is to make a careful study of synonymous terms. A book of synonyms is especially helpful in study along this line, and a dictionary also aids to a great extent if it is used frequently and properly.

In the study of the exercises on synonyms the spelling of the words should be learned first. When this is understood, the meanings of the words should be taken into consideration. Since many of these words have more than one meaning, a dictionary should be consulted very frequently and the various meanings noted.

pret′ty
hand′some
splen′did

mer′ry
jol′ly
hap′py

schol′ar
stu′dent
pu′pil

free′dom
lib′er ty
li′cense

ster′ile
bar′ren

for lorn′
lone′some

loft′y
sub lime′

dis cov′er
in vent′

123—THE FARM

ma nure′
ni′trate
phos′phate
gua′no
gyp′sum
pot′ash
com′post

scythe
har′row
reap′er
thrash′er
wag′on
clay′ey
bog′gy

ar′a ble
till′a ble
cul′ti vate
ir′ri gate
fer′ti lize
hoe′ing
har′vest

gran′a ry
corn′crib
grind′stone
dai′ry ing
reap′ing
plow′ing
sow′ing

124—DISEASES

croup
cramp
pal′sy
drop′sy
ab′scess
ear′ache
head′ache
ver′ti go
hic′cough
ty′phoid

grippe
ra′bi es
lep′ro sy
cho re′a
ec′ze ma
neu ri′tis
nau′se a
lum ba′go
pleu′ri sy
tooth′ache

di a be′tes
dys pep′si a
de lir′i um
ma la′ri a
cat′a ract
in som′ni a
tu ber cu lo′sis
men in gi′tis
hem′or rhoids
ton sil li′tis

125—*ŭ* SHORT, AS IN *up*

sŭf′fer
un pack′
rub′ber
cur′rant
hun′gry
shud′der

cŭl′prit
tough′én
shuf′fle
in dulge′
huck′ster
trou′ble

re lŭc′tant
cudg′el
un pleas′ant
vul′ner a ble
re duc′tion
in cul′cate

in dŭl′gence
hus′ki ness
in cum′bent
pul′ver ize
sub junc′tive
de struc′ti ble

126

ap prise′
flo til′la
suf′fer ance
par′a dise
per′qui site
pe ruse′
ut′ter ance
va ri′et y

can′on ize
con cise′
per ni′cious
tend′en cy
per′ti nent
fu′si ble
Pu′ri tan
hin′dered

res′o nant
pil′grim age
in vin′ci ble
per sist′ent
per′pe trate
il lu′mi nate
fore tell′
ly ce′um

aus tere′ly
in fat′u ate
u biq′ui tous
un but′toned
as sim′ i late
a cid′i ty
more o′ver
con′se quence

127—ȼ HARD, LIKE *k*

cow′ard
cut′ler y
com plete′
lu′cre
oc′cu pant

ȼom′rade
cour′age
cos′tume
e clip′tic
en core′

ȼlev′is
Cal′va ry
cour′te sy
noc tur′nal
rec′ti fy

de ȼo′rum
e con′o my
dec′o rate
la con′ic
en com′pass

128—*ȼh* HARD, LIKE *k*

ȼhron′ic
chem′ist
Chris′tian
ech′oes

ȼho′rus
cho′ral
chris′ten
cho′roid

ȼhar′ac ter
chem′i cal
mech′an ism
me chan′ic

ȼhron′i cle
lach′ry mal
tech′nic al
chrys′a lis

129

gen′er ous
lib′er al
boun′ti ful

po lite′
re fined′
cul′tured

fru′gal
thrift′y
sav′ing

fan′ci ful
gro tesque′
lu′di crous

hap′pi ness
fe lic′i ty
rap′ture

rich′es
for′tune
op′u lence

prod′i gal
waste′ful
ex trav′a gant

anx i′e ty
con cern′
wor′ry

130—REVIEW OF DIACRITICAL MARKS

ĕx prĕss′ĭve
ĕx tē′rĭ or
dĭ lĕm′mȧ
dē scĕnd′ȧnt
guärd′ĭ ȧn
ăc cĕl′ẽr āte
ĭn gē nū′ĭ ty
ŏc′tō pŭs
bŏt′ȧ nĭst
ĭn tŏx′ĭ cāte
ŏb′stĭ nȧ cy
rē cĕp′tȧ cle
rĕv′ē nūe
rē tăl′ĭ āte
săn ĭ tā′rĭ ŭm
cŏn′grē gāte
ē nôr′moŭs
cŏn sĕc′ū tĭve

131—WORDS USED AT ELECTIONS

polls
votes
booth
re turns′
cau′cus
bal′lot
bor′ough
of fi′cials
ru′mor
in au gu ra′tion
del′e gates
pol i ti′cian
af firm′a tive
con′gress man
mu nic′i pal
pre lim′i na ry
Re pub′lic ans
can′di da cy
a mend′ment
chal′lenge
as sem′blage
tax′a ble
treas′ur y
ma jor′i ty
pro tect′or
Dem′o crats
pre′cincts
par tic′i pants
ri′val ry
mi nor′i ty
ex clu′sive ly
e lect′ors
can′di date
ap point′ment
Pro gress′ives
non-par′ti san

132

tor′tured
sub′urb
out′skirts
dawn′ing
con tri′vance
san′i ta ry
su per se′ded
in ces′sant
mer cu′ri al
au′di ence
sump′tu ous ly
en thu′si asm
pros′per ous
a man u en′sis
in flu en′tial
sen sa′tion al ism
grat i fi ca′tion
lu′bri cate
de sir′a ble
o rig′i nate
lo cal′i ty
cit′i fied
mag′ni tude
com mu′ni ty
mys te′ri ous
ex u′ber ant
rec′ti tude

133—NAMES OF LANGUAGES

French
Greek
Welsh
Dutch
Lat′in
Eng′lish
Da′nish
Ger′man
Span′ish
Swe′dish
Chi nese′
Rus′sian
Po′lish
Fin′nish
Mag′yar
He′ brew
Celt′ic
Gael′ic
Slo vak′
Ar′a bic
San′scrit
Ar me′ni an
Sla von′ic
Jap an ese′
Por′tu guese
Hun ga′ri an
Nor we′gi an
Ha wai′ian
Ser′vi an
E gyp′tian
Bo he′mi an
Hin du sta′ni

134—BUSINESS TERMS

freight
ledg′er
leas′es
pan′ic
cou′pons
ex pense′
re bates′
leak′age
boy′cott
dis′count

a′gen cy
auc′tion
break′age
bri′ber y
div′i dend
draw′er
draw ee′
em ploy ee′
e quip′ment
in dict′ment

ref er ee′
val u a′tion
stock′hold er
com′pa ny
dam′aged
in stal′ment
de pos′i tor
in′ven to ry
or′gan ize
cap′i tal ize

in dorse′ment
in dus′tri al
com pet′i tor
as sign′ment
ac cept′ance
auc tion eer′
eq′ui ta ble
bank′rupt cy
in junc′tion
cor po ra′tion

135—GEOGRAPHICAL TERMS

Hol′yoke
E liz′a beth
Read′ing
Syr′a cuse
O′ma ha
Se at′tle
Pas a de′na
Sa van′nah
Wich′i ta
Fond du Lac′

Los An′ge les
Val pa rai′so
Hon o lu′lu
Fil i pi′nos
Por′to Ri′co
Ha wai′i
Man i to′ba
Ad i ron′dack
Ok la ho′ma
Tal la has′see

An tie′tam
Get′tys burg
Charles′ton
Car ib be′an
Car pa′thi an
Phil a del′phi a
Yo sem′i te
Ab ys sin′i a
Beth′le hem
Pe′ters burg

136—HOMONYMS

1 break, *to part by force*
brake, *a fern; for stopping wheels*

2 week, *seven days*
weak, *feeble*

3 waist, *part of the body*
waste, *a desert; to squander*

4 piece, *a part; a composition*
peace, *quiet*

5 hail, *frozen rain; to salute*
hale, *healthy*

6 wait, *to stay; to rest in patience*
weight, *heaviness*

7 heel, *part of the foot*
heal, *cure*

8 peal, *a loud sound*
peel, *to strip off the skin; the skin or rind*

Copy the sentences that follow, and in place of the numbers enclosed in marks of parenthesis use the correct word from the group of words having the same number as the number enclosed.

(*a*) A (4) of banana (8) should not be thrown on the pavement.
(*b*) The engine whistled "Down (1)s."
(*c*) Learn to labor and to (6).—*Longfellow.*
(*d*) The (8) of the church bells was heard by all.
(*e*) (5) has fallen nearly every day this (2).
(*f*) Let us have (4).
(*g*) John is (5), but Henry is (2).
(*h*) Her dress was parted at the (3) and her shoes were down at the (7).
(*i*) How did you (1) that machine?
(*j*) What is your (6)?
(*k*) You should not (3) your time nor your money.
(*l*) This ointment will (7) that sore on your arm.

LESSON 4

137—ū LONG, AS IN *use*

re dūce′
mu′sic
youth′ful
pre sume′
us′age
cu′rate
stu′dent
sū′i cide
use′less
de mure′
Yule′tide
u′ni ty
pu′ri fy
u′ ni verse
mū′ti ny
ju′bi lee
in cur′a ble
cu′li na ry
in duce′ment
u′til ize
hu′mor
com mū′ni ty
lu′cra tive
du′pli cate
ves′ti bule
U ni ta′ri an
nu′tri ment
re cu′per ate

138

un just′	un just′ly	re′al	re′al ly
lan′guid	lan′guid ly	meek	meek′ly
se cure′	se cure′ly	court	court′ly
se′ri ous	se′ri ous ly	an′nu al	an′nu al ly
spite′ful	spite′ful ly	in tent′	in tent′ly
un u′su al	un u′su al ly	for′mer	for′mer ly
dread′ful	dread′ful ly	fre′quent	fre′quent ly

139

grief	griev′ous	pore	po′rous
joy	joy′ous	ri′ot	ri′ot ous
vice	vi′cious	glo′ry	glo′ri ous
stud′y	stu′di ous	dan′ger	dan′ger ous
la′bor	la bo′ri ous	mur′der	mur′der ous
in′dus try	in dus′tri ous	ma la′ri a	ma la′ri ous
mis′chief	mis′chie vous	mal′ice	ma li′cious

140—WORDS LIABLE TO BE CONFOUNDED

1 pop′lar, *a tree*
pop′u lar, *liked by the people*

2 rad′ish, *a vegetable*
red′dish, *somewhat red*

3 trea′ties, *agreements*
trea′tise, *a formal essay*

4 def′er ence, *respect*
dif′fer ence, *disagreement*

5 pil′lar, *a column for support*
pil′low, *a cushion*

6 ei′ther, *one or the other of two*
e′ther, *the upper air; an anesthetic*

7 plain′tiff, *the complainant*
plain′tive, *mournful*

8 ge′nus, *class; kind; race*
gen′ius, *a person of superior intellect*

Write the sentences that follow, selecting from the words enclosed in marks of parenthesis the one that makes sense in the sentence.

(*a*) That man with (radish, reddish) hair is a man of great (genus, genius), and is very (poplar, popular) with the officials.
(*b*) The Secretary of State signed many (treaties, treatise).
(*c*) The (deference, difference) between the (plaintiff, plaintive) and the defendant was soon adjusted.
(*d*) The (either, ether) was administered to the patient.
(*e*) You may give me a (radish, reddish).
(*f*) The (pillar, pillow) may be of (either, ether) marble or granite.
(*g*) She embroidered a beautiful (pillar, pillow) for me.
(*h*) You must write a (treaties, treatise) on the subject assigned to you.
(*i*) The white (poplar, popular) belongs to the (genus, genius) *Populus*.
(*j*) The (plaintiff, plaintive) notes of the birds were heard by all.

141—PLURALS

Many nouns that have been adopted from a foreign language retain their foreign plurals.

Singular	Plural	Singular	Plural
ax′is	ax′es	er ra′tum	er ra′ta
ba′sis	ba′ses	ver′te bra	ver′te bræ
da′tum	da′ta	ter′mi nus	ter′mi ni
ge′nus	gen′e ra	a nal′y sis	a nal′y ses
the′sis	the′ses	syn op′sis	syn op′ses
cri′sis	cri′ses	phe nom′e non	phe nom′e na
o′a sis	o′a ses	pa ren′the sis	pa ren′the ses
neb′u la	neb′u læ	hy poth′e sis	hy poth′e ses

142—PLURALS

Some nouns adopted from a foreign language have two plural forms. They retain their foreign plural and they also make a plural by adding *s* or *es*.

Singular	Plural	Singular	Plural
beau	beaux beaus	ban′dit	ban dit′ti ban′dits
in′dex	in′di ces in′dex es	for′mu la	for′mu læ for′mu las
fo′cus	fo′ci fo′cus es	ra′di us	ra′di i ra′di us es
cher′ub	cher′u bim cher′ubs	ser′aph	ser′a phim ser′aphs
fun′gus	fun′gi fun′gus es	ap pen′dix	ap pen′di ces ap pen′dix es
mem o ran′dum	mem o ran′da mem o ran′dums	stra′tum	stra′ta stra′tums
ros′trum	ros′tra ros′trums	spec′u lum	spec′u la spec′u lums

143

pep′per
wor′ship
grue′some
cam paign′
del′i cate
a gree′a ble
pen′u ry
rem′e dy
el′e ment
gib′bous
spe cif′ic
def′i nite
pov′er ty
ped′a gogue
cow′ard ice
ex pi ra′tion
ma ter′nal
pa ter′nal
ped′i gree
e vap′o rate

ca′pa ble
im por′tant
pla′cate
her′ald ry
al′pha bet
or gan′ic
stu pid′i ty
sen′si tive
pla card′
gon do lier′
ex on′er ate
mim′ic ry

144—ANTONYMS

Antonyms are words directly opposed to each other in meaning. It is important for every one to be familiar with words of this kind, because a very effective method of defining terms is to tell what they do not denote nor signify. If a person has an understanding of antonyms, he will often be able to describe his ideas by means of contrasting terms or words that are opposite in meaning to those he would naturally select. In the exercises on antonyms, the correct spelling of the words should be learned first. Then attention should be paid to the contrasting meanings of the words.

true
un′true

sure
doubt′ful

new
old

o′pen
closed

a gree′
dif′fer

ad mire′
ab hor′

hoard
squan′der

cause
ef fect′

cleanse
pol lute′

calm
ag′i ta ted

gay
sad

a part′
to geth′er

near
dis′tant

sweet
sour

a like′
un like′

dry
moist

145—PLURALS

Write the plural form of each of the following words:

es′say
pul′ley
flour′ish
par′a graph
horse′man
em bar′go
cro′cus
chim′ney
mar′tyr
pro vi′so
nar cis′sus
Es′ki mo
nose′gay
isth′mus
spec′i men
phys i ol′o gy
mem o ran′dum
farm′house
del′i ca cy
con′so nant
ap pren′tice
ca the′dral
ed′dy
pha′e ton
pa′tron ess
sec′re ta ry
mas′sa cre
pa trol′man

146

flip′pant
blas pheme′
stam pede′
cav′al cade
mu ti neer′
re frig′er a tor
so ci ol′o gist
in can des′cent
wheel′bar row

mer′cer ized
ter res′tri al
u ni ver′si ty
pre ca′ri ous
be nev′o lence
an ti sep′tic
yacht′ing

spon ta′ne ous
in cor′po rate
Hu′gue not
ne ces′si tate
vac ci na′tion
an′thra cite
pu′ri fy ing

con grat′u late
mo nop′o lize
e lec′tri cute
in cen′di a ry
ag′ri cul ture
bi tu′mi nous
al′tru ism

147—CRIMES

rob′ber y
pi′ra cy
ex tor′tion
bur′gla ry
cheat′ing
for′ger y

man′slaugh ter
maim′ing
may′hem
coun′ter feit
mur′der
as sault′

big′a my
per′ju ry
trea′son
lar′ce ny
gam′ing
gam′bling

poi′son ing
bri′ber y
kid′nap ing
ab duc′tion
swin′dle
ar′son

148

churl′ish
a lac′ri ty
ac ces′si ble
prec′i pice
det′ri ment
ar te′sian
type′wri ter
ri′val ry

re cede′
stren′u ous
prox im′i ty
ir rel′e vant
thor′ough fare
quar′rel some
man′u script
tor′tured

en′vi ous
ex ten′sion
pur′pose
de sir′ous
ex pect′an cy
trus tees′
vir′tu ous
mo not′o ny

jeop′ard y
guid′ance
bou′le vard
fa tigue′
re liev′ing
e ma′ci a ted
fe lo′ni ous
ap pli′an ces

149—WORDS ENDING IN *sy* OR *cy*

noi′sy
flim′sy
clum′sy
her′e sy
ec′sta sy
jeal′ous y
em′bas sy

au′top sy
phan′ta sy
a pos′ta sy
proph′e sy
con′tro ver sy
hy poc′ri sy
min′strel sy

leg′a cy
in′fan cy
de′cen cy
se′cre cy
flu′en cy
va′can cy
ur′gen cy

in′ti ma cy
ac′cu ra cy
del′i ca cy
di plo′ma cy
bril′lian cy
com′pe ten cy
e mer′gen cy

150

rag′time
al′ka li
mas seur′
mas seuse′

lis′ter ine
mer′cer ize
lin ge rie′
mu′to scope

in terne′
ad′e noid
mon′o plane
eu gen′ics

mon′o graph
an es thet′ic
dir′i gi ble
del i ca tes′sen

Box′er
suede
Yid′dish
neu′tral
bi′o scope
so ror′i ty
vac′u um
neu rit′ic
lev′ee
drain′age
crem′a to ry
e lect′or ate
hyp o der′mic
or tho pe′dics
os′cil la tor
mon o lith′ic

151

ki net′ic
a′er a ted
a vi a′tion
a′er o gram
a′er o dome
gy′ro plane
a′er o plane
a′vi a tor
wire′less
vol′a tile
graph′o type
san i ta′tion
gym na′si um
con fet′ti
path o gen′ic
tut′ti-frut′ti
mim′e o graph
man′i cure
con ven′tion
pe tro′le um
pas teur i za′tion
ca nal i za′tion
re con cen tra′tion
o cean grey′hound
au to sug ges′tion
car bo hy′drate
in sti tu′tion al

152—WORDS SPELLED ALIKE BUT PRONOUNCED DIFFERENTLY

Another class of words that present a little difficulty and that should, therefore, be studied thoughtfully are words that are spelled alike but that are pronounced differently and have different meanings. The difference in the pronunciation of these words is generally due to the placing of the accent. For instance, when the accent is placed on the first syllable of the word *con′vert*, this word refers to a person that has been converted from one belief or opinion to another. The placing of the accent on the last syllable of this word changes it to *convert′*, which means to change into another state. The following list contains a number of words of this kind with their meanings.

1 con′tract, *an agreement*
con tract′, *to draw together*

2 es′cort, *a guard*
es cort′, *to accompany*

3 des′ert, *a barren waste*
de sert′, *merit; to forsake*

4 ref′use, *worthless remains*
re fuse′, *to reject; to decline to do*

5 min′ute, *sixty seconds*
mi nute′, *very small; very exact*

6 ob′ject, *purpose; thing*
ob ject′, *to oppose*

7 reb′el, *one who revolts*
re bel′, *to rise up against*

8 in′va lid, *a sick person*
in val′id, *of no force*

Write the following sentences, and fill each blank space with a word selected from the preceding list. Mark the accent of this word. Thus, *We drew up a —— for two years. We drew up a con′tract for two years.*

(*a*) Why did he —— his family?
(*b*) Why do you —— to his company?
(*c*) The —— is growing weaker every day.
(*d*) Throw the —— in the garbage can.
(*e*) I shall be ready in five ——s.
(*f*) The boys frequently —— against their teacher.
(*g*) A —— report was made.
(*h*) We had an —— of fifty soldiers.
(*i*) The will is —— because he failed to sign it.
(*j*) Cold will cause iron to ——
(*k*) We went so far in the —— that our guide ——d to go any farther.

153—ĩ AS IN *sir*

thĩr′ty
squir′rel
stir′rup
cir′cus
vĩr′tue
mirth′ful
irk′some
sir′up
cĩr′cu lar
thir teenth′
birth′day
fir′ma ment
black′bĩrd
ta′pir
con firm′
ex′tir pate

154—û AS IN *urn*

cûr′few
fur′ther
ur′chin
pur′chase
mûr′mûr
nurs′ling
fur′tive
bur′den
tûr′moil
jour′nal
sur′ger y
ex cur′sion
cûr′ren cy
sur′name
hur′dle
dis burse′

155—THE FARM

barn
sta′ble
pig′pen
pig′sty
sic′kle
man′ger
bar′row
hay′rake
hay′mow
car′riage
gar′den
en′si lage
chick′ens
farm′house
mar′tin gale
wind′row
har′row
reap′er
bri′ers
bri′dle
hay′cock
har′ness
hames
crup′per

156—ANTONYMS

rare
com′mon
ex on′er ate
con demn′
fail′ure
suc cess′
gen′er ous
par si mo′ni ous

o bey′
dis o bey′

hon′es ty
du plic′i ty

ab′sent
pres′ent

ac′tive
in′do lent

a′ged
youth′ful

guilt′y
in′no cent

de stroy′
con struct′

em′i grate
im′mi grate

ven′e rate
dis hon′or

in te′ri or
ex te′ri or

stub′born
yield′ing

va′cant
in hab′it ed

157—ARITHMETIC

num′ber
con′crete
Ar′a bic
fig′ures
ci′pher
per cent′
no ta′tion
ad di′tion
ra′ti o
root
rad′i cal

nu mer a′tion
sub trac′tion
di vi′sion
dif′fer ence
e qua′tion
min′u end
sub′tra hend
re main′der
bal′ance
in vo lu′tion
ev o lu′tion

math e mat′ic al
di vi′sor
quo′tient
prod′uct
mul ti pli ca′tion
mul′ti pli er
mul′ti pli cand
e qual′i ty
e quiv′a lent
di ag′o nal
per im′e ter

158—SYNONYMS

ab′so lute
des pot′ic
ar′bi tra ry
ty ran′nic al

busi′ness
oc cu pa′tion
pro fes′sion
vo ca′tion

ca tas′tro phe
ca lam′i ty
dis as′ter
mis for′tune

break
de stroy′
frac′ture
de mol′ish

ease
fa cil′i ty
read′i ness

con sole′
com′fort
en cour′age

em′blem
sym′bol
to′ken

teach′er
ed′u ca tor
in struct′or

159

kiln′-dried
in ci′sive
um brel′la
dys′en ter y
ag nos′tic
meer′schaum
mag′is trate
land′scape

fu′mi gate
a cous′tics
bev′er age
po lyg′a my
or ange ade′
nup′tials
dis′ci pline
port′li ness

black′mail
a cet′y lene
sanct′u a ry
chi rop′o dist
fla′grant
con ge′ni al
brick′kiln
al′tru ist

ban dan′na
pam′phlet
cat′er pil lar
chrys′a lis
con′fis cate
guar an tee′
in sur′er
O lym′pic

160—ä AS IN *arm*

em bälm′
gar′nish
par′don
laughed
har′di hood
mar′ble
calm′ly
där′ling
ar′sen ic
farm′yard
har′vest
ar′ti cle
lar′der
ar′mor er
em bär′go
car bon′ic
par′ti tive
ar′ter ies
par′ti ci ple
har mo′ni ous
car niv′o rous
mär′gin
sparse′ly
mar′ti net
fa′ther less
bar′be cue
far′sight ed
mar′tyr dom

161—WORDS LIABLE TO BE CONFOUNDED

in gen′ious, *skilful*
in gen′u ous, *honest*

jest′er, *one who jests*
ges′ture, *action*

lin′i ment, *liquid ointment*
lin′e a ment, *a feature*

lose, *to suffer loss*
loose, *to untie*

pas′tor, *a minister*
pas′ture, *a field for cattle*

pres′ence, *the state of being present*
pres′ents, *gifts*

pros′e cute, *to bring suit against*
per′se cute, *to vex or afflict*

prec′e dent, *an example*
pre ced′ence, *superiority*
pres′i dent, *chief magistrate*

par ti′tion, *division*
pe ti′tion, *a request*

rel′ic, *a memorial; a remnant*
rel′ict, *a widow*

stat′ue, *an image*
stat′ure, *height*
stat′ute, *a law*

sur′plus, *the remainder*
sur′plice, *clergyman's robe*

se′ries, *a succession*
se′ri ous, *solemn*

spe′cies, *a kind; a variety*
spe′cious, *plausible*

track, *a footprint; a beaten path*
tract, *a region*

ten′ure, *manner of holding land*
ten′or, *part in music*

ve rac′i ty, *truthfulness*
vo rac′i ty, *greediness*

which, *a pronoun*
witch, *a sorceress*

162

ab′sence
at′om
bar′gain
but′ton
cat′kin
lin′guist
cleat
clev′er
e clipse′
en cir′cle
glob′ule
griz′zly
haz′ard
hu′mor
jock′ey
la pel′
fag′ot
lag′gard
loz′enge
per spire′

glut'ton
eye'let
fa tigue'
mar'tyr
graph'ic al
naph'tha
fer'rule
poul'tice

163

change	change'a ble	man'age	man'age a ble
charge	charge'a ble	mar'riage	mar'riage a ble
trace	trace'a ble	for get'	for get'a ble
no'tice	no'tice a ble	pre fer'	pref'er a ble
ser'vice	ser'vice a ble	a vail'	a vail'a ble

164—*ch* UNMARKED, AS IN *child*

cham'ber
chap'el
dis patch'
chil'blains
char'i ty
chas'ti ty
chief'tain
en chant'
chal'ice
dis charge'
satch'el
chap'ter
re touch'
charm'ing
a chieve'
choc'o late
re trench'
chair'man
chal'lenge
chan'cel lor

165—HISTORY

char'ter
spear
wig'wam
wam'pum
moc'ca sin
tom'a hawk
squaw
pap poose'
war'rior
cal'u met
dis cov'er y
pi o neers'
scalped
ship'wreck
ex plore'
col'o ny
per'ma nent
nav'i ga tor
wil'der ness
suf'fer ing
set'tle ment
trea'ty
per'se cute
mas'sa cre

166—VEGETABLES

kale
beet
bean
cress
gar'lic
tur'nip
rhu'barb
pars'nip
cab'bage
mush'room
car'rot
let'tuce
pump'kin
spin'ach
to ma'to
cu'cum ber
as par'a gus
cau'li flow er
egg'plant
rad'ish

167

monk
mur'mur
mys'ter y
par'a ble
par'cel
mes'sage
lan'guage
make'shift
piv'ot
part'ner
phys'ic
pi'ous
pi'ra cy
pit'i ful
mal treat'
i'ci cle
pub'lic
rum'mage
skir'mish
sol'emn
spe'cie
stam'mer
tor'ture
ag'i tate
tas'sel
trel'lis
tus'sle
weap'on
wel'fare
syr'inge
car'a mel
nec'es sa ry

168

moun tain eer′
im par′tial
in de fen′si ble
dis po si′tion
ce ler′i ty
e nor′mous
re cip′i ent
ver mil′ion
pen′ni less
sum′moned
fea′si ble
ap pre hen′sion
el′o quent
ad van′ta ges
com bi′ning
sat is fac′to ry
sa lu′ta to ry
su per′flu ous
de signed′
mo not′on ous
op′po site
con cep′tion
hes′i tan cy
un en light′ened
com pel′ling
dis ar range′
a vow′al

169—SYNONYMS

mirth
gay′e ty
hi lar′i ty

soothe
al le′vi ate
mit′i gate

cha grin′
vex a′tion
an noy′ance

talk′a tive
lo qua′cious
gar′ru lous

ban′quet
fes′ti val
ca rous′al

em′i nent
re nowned′
no to′ri ous

ad′age
prov′erb
max′im

wa′ver
vac′il late
fluct′u ate

170—ī LONG, AS IN *ice*

ī′dle
spi′der
pri′or
rip′en
wri′ter
re quire′
ī′dol īze
fright′en
brib′er y
guile′less
ho ri′zon
re fined′
dī′a logue
de fi′ance
al might′y
hi′ber nate
con dign′
po lite′ness
no to rī′e ty
ac cli′mate
grind′stone
por′cu pine
sym′pa thize
tri umph′ant

171

strict
is′su ing
di vulge′
in quir′y
in cense′
get′ting
mag′net
phase
fer′ry
ear′li est
com pels′
im mense′
mod′ern
dis pel′
bulge
re ly′
de nied′
se dan′
thread
for′feit
im peach′
fierce
con′sul
cres′cent
heav′y
failed
fe′line
peo′ple
beg′gar
mar′ket
vil′lage
auc′tion
por′tion
con tain′
de vel′op
road′ster

172—RELATING TO TIME

hour
year
week
month
fall
spring
sec′ond
min′ute
in′stant
au′tumn
win′ter
sum′mer
fore noon′
aft er noon′
e′ven ing
dec′ade
ep′och
sea′son
year′ly
an′nu al
cen′tu ry
fort′night
bi en′ni al
for ev′er
al′ways
im mor′tal
tem′po ra ry
e ter′ni ty
an tique′
per en′ni al
per′ma nent
ha bit′u al
cal′en dar
al′ma nac
per pet′u al
mo′men ta ry

173

dy′nas ty
cat′a combs
ra vine′
rou tine′
pau′ci ty
mi′cro scope
ni′tro gen
clock′work
cap′il la ry
es pe′cial ly
in dis creet′
com′pe tent
ha rangue′
phy sique′
venge′ance
rai′ment
il lic′it
re scind′
Af′ghan
co logne′
con′duit
lam poon′
or′ches tra
cor′ri dor
stat′u to ry
pal′a din
u ten′sil
vi bra′tion
ap par′el
bric′-a-brac
gel′a tine
o le o mar′ga rine

174—WORDS SPELLED ALIKE BUT PRONOUNCED DIFFERENTLY

1 prog′ress, *advance*
pro gress′, *to go forward; to advance*

2 pres′ent, *a gift*
pre sent′, *to give; to introduce*

3 sub′ject, *a topic*
sub ject′, *to bring under control*

4 rec′ord, *a register*
re cord′, *to register; to make note of*

5 prod′uce, *what is yielded*
pro duce′, *to yield; to bring forth*

6 in′sult, *an affront*
in sult′, *to offer abuse*

7 con′flict, *a struggle*
con flict′, *to be opposed*

8 per′mit, *leave; warrant*
per mit′, *to allow*

9 ex′tract, *what is drawn from*
ex tract′, *to draw from*

10 con′duct, *behavior*
con duct′, *to lead*

Write each of the following sentences, and in place of the number enclosed in marks of parenthesis write the word that should be used. Select this word from the group having the same number as the number enclosed. Mark the accent of each word inserted.

(*a*) Will you allow me to (2) my friend?
(*b*) You may use the vanilla (9) for your cake.
(*c*) The boy's (10) caused his mother to punish him.
(*d*) She resented the (6).
(*e*) You should (3) it to great pressure.
(*f*) If I allow my pleasures to (7) with my studies, my (1) in that (3) will not be very rapid.
(*g*) The (5) from their farm sells very readily.
(*h*) These phonograph (4)s were a (2) from my father.
(*i*) You must obtain a (8) from the superintendent before I can (10) you through the building.
(*j*) The Civil War was a long, bloody (7).
(*k*) They told him to (4) the votes.
(*l*) That field will (5) a good crop of wheat if you will (8) me to use the proper fertilizers.
(*m*) The dentist will (9) the tooth for you.
(*n*) If you study, you will (1) in your lessons.
(*o*) Why did you (6) him in such a manner?

175—ȧ AS IN *ask*

cȧ reer′
graft′er
ma chine′
draft′ing
cȧ ress′
in′fa my
co ro′na
ten′a ble
blȧst′ed
dis a gree′
rel′a tive
Ma don′na
cig ȧ rette′
im ma ture′
in′fa mous
im′i ta ble

176—â AS IN *care*

fâir′y
gar′ish
par′ents
hare′bell
scâre′crow
de clare′
hair′brush
bare′ly
char′i ly
fare well′
dare′-dev il
scar′ci ty
down′ stâirs
deb o nair′
rare′ly
har′um-scar′um

177—OCCUPATIONS

la′bor er
in vent′or
phy si′cian
su per vi′sor
sur′geon
man′a ger
po lice′man
fire′man
ca′ter er
ma chin′ist
e lec tri′cian
ar′chi tect
ac count′ant
book′keep er
pro fess′or
com pos′i tor
sur vey′or
auc tion eer′
gau′ger
pho tog′ra pher

178—BUILDINGS

manse
ware′house
fac′tor ies
a cad′e my
boat′house
a re′na
pris′on

tow′er
post′ of fice
a sy′lum
man′sion
sem′i na ry
par′son age
pal′ace

light′house
home′stead
saw′mill
dwell′ing
brew′er ies
li′bra ries
rec′to ry

dor′mi to ry
op′e ra house
con serv′a to ry
kin′der gar ten
mon′as ter y
cot′tage
ab′bey

LESSON 5

179—SYNONYMS

grat′i tude
thank′ful ness

cour′te sy
po lite′ness

ob nox′ious
o′di ous

friend′ly
cor′dial

ri′val ry
com pe ti′tion

down′cast
de ject′ed

com pas′sion
sym′pa thy

mod′es ty
dif′fi dence

grate′ful
thank′ful

se clu′sion
pri′va cy

re sent′ment
an i mos′i ty

de cide′
de ter′mine

180—GEOGRAPHICAL NAMES

Mis sou′ri
Ten nes see′
Ha van′a
Mil wau′kee
Ma ni′la

Atch′i son
Del′a ware
Nin′e veh
Mis sis sip′pi
Eu ro pe′an

Great Brit′ain
Chris′ten dom
Sac ra men′to
Knox′ville
Cal i for′ni a

Cay enne′
Ra cine′
Guin′ea
A ra guay′
Vin cennes′

181

rough′shod
fals′i ty
brev′i ty
stu pid′i ty
ma lig′ni ty
fer til′i ty
pen′u ry
de cis′ion

scrim′mage
ve′he mence
fal′la cy
ac tiv′i ty
mel′o dy
bra′ver y
loy′al ty
fal la′cious

vol′u ble
rec′om pense
ob scen′i ty
se ren′i ty
ob liq′ui ty
gen til′i ty
an tiq′ui ty
su prem′a cy

182

re call′
in′come
op′tion
fran′chise
dead′lock
tar′iff
ses′sion
suf′frage
e co nom′ic
rev′e nue
an ti trust′
a mend′ment
ar′bi trate
op po′nent
suf′fra gist
suf fra gette′
states′man ship
re con struc′tion
pro gres′sive
po lit′i cal
ad min is tra′tion
pro tect′ive
ref er en′dum
in i′ti a tive

183

in′dus try
mil′i tant
de ben′ture
in vest′ment
a bate′ment
syn′di cate
ex emp′tion
col′leagues
mon op′o ly
con ces′sion
quo ta′tion
re or′gan ize
ag gres′sive
ar′bi tra ry
in her′it ance
ex pend′i ture
dev as ta′tion
part′ner ship
na′tion al ize
com mer′cial ism
pro tect′or ate
ex pan′sion ist
ex pan′sion ism
com pe ti′ tion
ex ploi ta′tion
mil′i ta rism
nat′u ral ize

184

air′ship
bat′tle ship
mon′o rail
li′ner
snap′shot
hook′worm
vaude′ville
put′ter
tung′sten
dry′ goods
Wels′bach
Roent′gen
ra′di um
Pas′teur
so ro′sis
Ca mor′ra
raf′fi a
pa ja′mas
mac ad′am
ter′mi nal
sub ma rine′
Ber til lon′
cra ven ette′
Fil i pi′no
Mar co′ni
mar co′ni gram
an ti tox′ic
os te op′a thist
os te o path′ic
ro tun′da

185

Jes′u it
in′fi del
kha′ki
i′ro ny
sar cas′tic
fa nat′ic
hand′i cap
sweet′heart
ha′rem
au′di ence
non cha lance′
gai′e ty
mau so le′um
men ag′e rie
am bas′sa dor
rat′tle snake
en cy clo pe′di a
vi cis′si tude
ar ti fi′cial
in ci den′tal
sou brette′
con de scend′
in an′i mate
pe cu′ni a ry
par ti al′i ty
fil′i bus ter
fu′ri ous ly
in au′gu rate

186

nui′sance
in her′ent
com mit′ted
dis hon′or
em bar′rass ment
jour′nal ism
shunned
di vulg′es
pro hib′it ive
un rea′son a ble
in ter vene′
de liv′er ance
ob′so lete
sub merged′
dis sat′is fied
con ceit′ed
mem′bra nous
blam′a ble
hyp′o crite
in tox′i cat ing
in com pat′i ble
er ro′ne ous
mel′an chol y
dis al low′
vy′ing
mys′tic al
im′bec ile

187—*n* EQUIVALENT TO *ng*

wrin′kle
han′ker
an′chor
Yan′kee
an′kle
con′cord
cinc′ture
crank′y
drink′er
func′tion
drunk′ard
con′clave
bank′rupt
con′course
punc′tu ate
ran′cor ous
sanc′tion
don′key
con′cu bine
an′chor age
crank′i ness
ban′quet
sanc′ti fy
punc tu al′i ty
drink′a ble
hand′ker chief
dis junc′tive
pro pin′qui ty

188—WORDS ENDING IN *cious* OR *tious*

te na′cious
pre co′cious
fe ro′cious
sa ga′cious
au da′cious
vo ra′cious
a tro′cious
aus pi′cious
per ni′cious
ca pri′cious
de li′cious
av a ri′cious
pug na′cious
lus′cious
of fi′cious
vi va′cious
vex a′tious
am bi′tious
pro pi′tious
in fec′tious
ex pe di′tious
nu tri′tious
fa ce′tious
os ten ta′tious
sen ten′tious
con ten′tious
fic ti′tious
fac ti′tious
se di′tious
con sci en′tious
fla gi′tious
fac′tious

189—ANTONYMS

cer′tain
un cer′tain

cor rect′
de fect′ive

com plete′
in′com plete

pub′lic
pri′vate

per′fect
im per′fect

pos′i tive
hes i ta′ting

un like′
un′i form

vig′i lant
neg′li gent

learn′ed
ig′no rant

fru gal′i ty
lib er al′i ty

har′mo ny
dis′cord

ab surd′
sen′si ble

in′dus try
in′do lence

gar′ru lous
ret′i cent

ac quaint′ed
un ac quaint′ed

dain′ty
clum′sy

190—HOMONYMS

Write the following sentences, selecting from the homonyms enclosed in marks of parenthesis the one that makes sense in the sentence in which it occurs. If you are uncertain about the meaning of any of these homonyms, look it up in a dictionary.

(*a*) Better alone than in (bad, bade) company.
(*b*) Time and (tied, tide) (weight, wait) for (no, know) man.
(*c*) Where (there's, theirs) a will, (there's, theirs) always a (weigh, way).
(*d*) Then he said, "Good (night, knight)," and with muffled (ore, oar, o'er) silently (rode, road, rowed) to the Charlestown shore.—*Longfellow.*
(*e*) Will you help me (altar, alter) my (surge, serge) dress?
(*f*) The men will (sheer, shear) the sheep.
(*g*) He bought a slice of sirloin (stake, steak).
(*h*) The dependent (claws, clause) in that sentence is used as an adjective.
(*i*) The man who walked (through, threw) the (gait, gate) had a very peculiar (gait, gate).
(*j*) Do you intend to (sew, sow, so) the grain today?
(*k*) This medicine will (lesson, lessen) the pain.
(*l*) We went on a visit to the (capitol, capital) of the United States.
(*m*) The (cellar, seller) of the house is made of concrete.
(*n*) This (suite, sweet) is ten dollars per day.
(*o*) These men were very (rood, rude) to that woman.

191—*ie* OR *ei*

bier
tier
mien
wield
lien
niece
siege
frieze

ag grieve′
re lief′
shield
re trieve′
re prieve′
mis′chief
sor′tie
ker′chief

ceil′ing
de ceit′
de ceive′
con ceive′
con ceit′
re ceipt′
re ceive′
per ceive′

rein
reign
weigh
skein
hei′nous
o bei′sance
in veigh′
neigh′bor

192—WORDS BEGINNING WITH *per* OR *pur*

per form′
per plex′
per suade′

per se vere′
per pet′u al
per′co late

purge
pur′port
pur′pose

pur sue′
pur vey′
purs′er

per′jure
per verse′
per vade′
per′fi dy
per di′tion
per son′i fy
per ox′ide
per′emp to ry
per im′eter
per mis′si ble
per′ma nent
per fo ra′tion
per cep′ti ble
per′se cute
pur suit′
pur vey′or
pur′ga to ry
pur′chase
pur loin′
pur′ple
pur′view
pur′lieu
pur su′a ble
pur′ga tive
pur su′ance
pur′pose ly
purs′lain
pur′blind

193—TRANSPORTATION

car′goes
weight
route
leak′age
ship′ping
ton′nage
ex press′
li′cense
trans ferred′
vi′ce ver′sa
in stal′ment
a mount′
ti′tle
ship′per
con sign ee′
re mit′tance
trans mit′ted
leas′a ble
du′ti a ble
break′age
ship′ment
des ti na′tion
ex or′bi tant
in′ter state
con sign′ment
val u a′tion
com mer′cial
rev′e nue

194—ş AS IN *has*

pleaş′ant
gos′ling
dis cern′
clum′sy
pro pose′
mus′lin
de vise′
na′şal
vis′i ble
po si′tion
res′i dent
phys′ic al
res′o nant
u′su ry
phy şique′
gas′e ous
ob serv′ant
com′pro mise
re sist′i ble
dis cern′ment
ad he′sion
miş′er a ble
de ser′tion
ar′ti san
fea′si ble
in quis′i tive
meas′ure ment
com′plai sant

195—WORDS ENDING IN *ence* OR *ance*

ev′i dence
pa′tience
con′fi dence
ex ist′ence
de pend′ence
con′fer ence
pre ced′ence
ref′er ence
in tel′li gence
dif′fer ence
im′pu dence
con do′lence
ret′i cence
con cur′rence
dil′i gence
re cur′rence
an noy′ance
a void′ance
ap pear′ance
de fi′ance
re sist′ance
a bun′dance
pet′u lance
con vey′ance
ac quaint′ance
or′di nance
tem′per ance
ob serv′ance
con triv′ance
at tend′ance
per se ver′ance
dis turb′ance

196—PERTAINING TO THE SICK

lin′i ment
lar′ynx
vic′tim
cur′a tive
ail′ment
stim′u lus
phys′ic al ly
fa tigu′ing
con ta′gious
al co hol′ic
ben e fi′cial
con va les′cence

nurs′ing
cleaned
liq′ue fy
liq′uor
suf′fer er
pes′ti lence
par′a lyzed
fe′ver ish
symp′toms
hem′or rhage
heart′rend ing
quar an tine′

197—*ie* OR *ei*

liege
a′lien
a dieu′
cli′ent
pa′tient
griev′ous
spe′cie
re trieve′
fiend′ish
be siege′
bre vier′
friend′ship
mes′sieurs
e mol′lient
chif fo nier′
re view′er
seize
feint
deign
sheik
weird
ei′der
eis tedd′fod
sei′zure
lei′sure
for′eign
heif′er
for′feit
en vei′gle
heir′loom
eight′een
re veil′le

198—WORDS ENDING IN *able* OR *ible*

re ceiv′a ble
hos′pi ta ble
re spect′a ble
im mov′a ble
laud′a ble
laugh′a ble
com′pa ra ble
char′i ta ble
trans fer′a ble
pal′a ta ble
in con sol′a ble
in dis pen′sa ble
im prov′a ble
ter′mi na ble
el′i gi ble
dis cern′i ble
re vers′i ble
di gest′i ble
plau′si ble
in flex′i ble
ed′i ble
gul′li ble
in vin′ci ble
in aud′i ble
in fal′li ble
col lect′i ble
sus cep′ti ble
ir re sist′i ble

199

Chris′to pher
Char′lotte
Dan′iel
I tal′ian
Brit′ish
Ben′ja min
An nap′o lis
Chris′tian ize
Eng′land
Phil′ip pine
Cu′ban
Mar′gar et
Han′ni bal
Ken ne bec′
Ni ca ra′gua
Cin cin na′ti
Con nect′i cut
Min ne so′ta
Min ne ap′o lis
Nar ra gan′ sett

200—ẽ AS IN *her*

vẽr′min
im merse′
serv′ant
ver′dict
mer′maid
com′merce
fẽr′vor
per spire′
con verge′
serf′dom
a ver′sion
nerv′ous
per chance′
per suade′
search′ing
pris′on er
per′vi ous
mer′cu ry
cẽr tif′i cate
her′mit age
mi′ser ly
fer′ti lize
der′vish
mer′chan dise

201—WORDS LIABLE TO BE CONFOUNDED

ac cede′, *to agree to*
ex ceed′, *to go beyond*
e lic′it, *to draw out*
il lic′it, *unlawful*

af fect′, *to act upon*
ef fect′, *to accomplish*

ad di′tion, *process of adding*
e di′tion, *publication*

as say′, *to test metals*
es say′, *to attempt*

bal′lad, *a song*
bal′lot, *a vote*

close, *to shut*
clothes, *articles of dress*

cen′tu ry, *hundred years*
sen′try, *a sentinel*

cel′e ry, *a vegetable*
sal′a ry, *wages*

de scent′, *a going down*
dis sent′, *to disagree*

em′i nent, *distinguished*
im′mi nent, *threatening*

e lude′, *to escape*
al lude′, *to refer to*

e rup′tion, *a bursting forth*
ir rup′tion, *an invasion*

em′i grate, *to leave*
im′mi grate, *to move into*

ex′er cise, *to use*
ex′or cise, *to drive away*

gla′cier, *an ice field*
gla′zier, *a glass setter*

go ril′la, *a savage animal*
guer ril′la, *an irregular soldier*

in cite′, *to stir up*
in′sight, *deep view*

202—WORDS ENDING IN *ing*

Many persons fail to sound the *g* of words ending in *ing*. For this reason, each person who desires to enunciate his words clearly should watch his pronunciation of words of this kind. Each word in the following list should be pronounced aloud several times.

play′ing
buy′ing
run′ning
ring′ing
dust′ing
box′ing
call′ing
catch′ing
cheat′ing
go′ing
com′ing
trot′ting
spell′ing
read′ing
writ′ing
draw′ing
drink′ing
hur′dling
fall′ing
eat′ing
fly′ing
faint′ing
hur′ry ing
du′el ing

203

pueb′lo
nu′cle us
ful′crum
cu pid′i ty
mis no′mer
sta tis′tics
per′ish a ble
con cert′ed
sym bol′ic
pi a no′la
in vi′o late
fel′low ship
prob′i ty
Lab′ra dor
ad va lo′rem

ap′a thy
ar′dent ly
co los′sal
ig′no min y
poign′ant
latch′key
tap′es try
in trigue′
fab′u lous
out′raged
a troc′i ties
hy′gi ene
self-re li′ant
nul′li fy
bel lig′er ents

204

pe des′tri ans
des pair′ing ly
coun′te nance
le git′i mate ly
vo lu′mi nous
per func′to ry
pat ron i′zing ly
an ni ver′sa ry
in cur′a ble
for ti fi ca′tions
pleas′ance
wist′ful ly
res′o nance
wan′ton ly
re por to′ri al
prol e ta′ri at
tu ber′cu lar
pil′grim age
be nig′nant
in au′gu ral
pro mul′gate
am′i ca bly
an nex a′tion
ma lev′o lent ly

205

sta bil′i ty
ma rooned′
grav′i ty
com mut′er
sal′a man der
im prov′i dent
ar ti fi′cial
col lab′o rate
ap pa ri′tions
ter′ri fy
sub ur′ban
con stit′u en cy
in sur rec′tion
af fil i a′tion
lu′cra tive
pre lim′i na ry
dom i na′tion
stu pen′dous
cen′te na ry
com po′nent
in dus′tri al
spec u la′tion
de sir a bil′i ty
in′tri ca cies
wain′scot ing
dom i neer′ing
un re mit′ting
col lat′er al
par′ti san
mon′e ta ry

206

i den′ti fied
in ter ven′tion
ded′i ca ted
Chris tian′i ty
Mar′di gras
mel o′di ous
ex hi bi′tion
noc tur′nal
et′i quette
sur viv′al
fla min′go
som bre′ro
rough′ rid er
vol′un ta ry
fi na′le
im peach′ment
in ves ti ga′tion
ca tas′tro phes
cas′se role
char′la tan
con tem′po ra ry
cen′sor ship
ne go ti a′tion
sports′man ship
self-con trol′
im pan′eled
or′tho dox
mel′an chol y
se duc′tive ness
re ceiv′er ship

207

main′te nance
per vade′
in cu ba′tion
dis tin′guish
con trol′
stead′i er
tend′en cy
mon op′o lies
con cede′
in ter cede′
ma te′ri al ly
stud′y ing
priv′i lege
ac cru′ing
pe′ri od
con ferred′
al′i mo ny
leath′er
sup ple men′ta ry
com mo′di ous
nar′ra tive
im′ple ment
hy drau′lic
ad journ′ment

208—WORDS PERTAINING TO NAVIGATION

buoy′an cy
pas′sage
rud′der
ca nal′
wa′ter
pi′lot
is′land er
im per′iled
aq′ue duct
ex er′tion
ob′sta cles
at′mos phere
pro pel′ler
com′mo dore
source
pro pel′
trav′el ing
voy′age
helms′man
sea′far ing
nau′tic al
re gat′ta
cruis′er
o ce an′ic
wa′ter course
har′bor
mar′i time
nav i ga′tion

209

con sti tu′tion
per mis′si ble
re spon si bil′i ty
re cur′rence
ef fi′cien cy
con sist′en cy
suf fi′cient
rar′e fy
right′eous
pre ten′tious
in cip′i ent
im′i tate
eq′ui ty
e quipped′
ac quire′
cab′i net
en joined′
pre pared′
es sen′tial
as cend′
pro cure′
com mit′
de pend′ent
plat′i tude

210

cou ra′geous
im pris′oned
de fen′sive
con′quer or
em′bry o
sports′man
keen′ness
ex pe′ri ence
vig′or ous
tech nique′
pre par′a to ry
skilled
non-re sist′ant
dis′tance
ex hib′it
a breast′
ex ci′ting
rhyth′mic al
out stripped′
hand′i capped
ar range′ment
ex er′tion
non pa reil′
vir′tue
in ves′ti gate
vin′di cate
ful fil′ment
pro ce′dure

211—NOTED AUTHORS—BRITISH

Ba′con
Shake′speare
Jon′son
Mil′ton
Dry′den
Swift
Ad′di son
Gold′smith
Burke
Cow′per
Gib′bon
Burns
Lan′dor
Rus′kin
Scott
Words′worth
Cole′ridge
Lamb
By′ron
Moore
Car lyle′
Brown′ing
Thack′er ay
Ma cau′lay
Tyn′dall
Hux′ley
Dar′win
Ste′ven son

Pope
John′son
De Foe′
Shel′ley
Keats
Ar′nold
Sou′they
Bul′wer
Dick′ens
George El′i ot
Ten′ny son
Kings′ley

212—NOTED AUTHORS—AMERICAN

Frank′lin
Poe
Ir′ving
Coop′er
Em′er son
Long′fel low
Whit′ti er
Holmes
Whit′man
Pres′cott
Gray
Low′ell
Drake
Hal′leck
Haw′thorne
Park′man
Dra′per
Bry′ant
Tho′reau
Wil′lis
Mitch′ell
Stowe
Cur′tis
Mot′ley
Cle′mens
Al′drich
Harte, Bret
Simms
Sted′man
Tay′lor, Bay′ard
Da′na
Bur′roughs
Whip′ple
Lan ier′
How′ells
Fiske

213—STATES, TERRITORIES, AND ISLAND POSSESSIONS WITH THEIR ABBREVIATIONS

The United States consists of forty-eight states, two territories, one district, and a number of detached possessions.

New England States

Maine	Me.	Mas sa chu′setts	Mass.
New Hamp′shire	N. H.	Rhode Is′land	R. I.
Ver mont′	Vt.	Con nect′i cut	Conn.

Middle Atlantic States

New York	N. Y.	Ma′ry land	Md.
New Jer′sey	N. J.	Vir gin′i a	Va.
Penn syl va′ni a	Pa.	West Vir gin′i a	W. Va.
Del′a ware	Del.	Dis′trict of Co lum′bi a	D. C.

Southern States

North Car o li′na	N. C.	Lou i si a′na	La.
South Car o li′na	S. C.	Tex′as	Tex.
Geor′gi a	Ga.	Ar′kan sas	Ark.
Flor′i da	Fla.	Ten nes see′	Tenn.
Al a ba′ma	Ala.	Ok la ho′ma	Okla.
Mis sis sip′pi	Miss.		

Central States

Ken tuck′y	Ky.	Ne bras′ka	Neb.
O hi′o	O.	Mich′i gan	Mich.

In di an′a	Ind.	Wis con′sin	Wis.
Il li nois′	Ill.	Min ne so′ta	Minn.
Mis sou′ri	Mo.	North Da ko′ta	N. Dak.
Kan′sas	Kan.	South Da ko′ta	S. Dak.
I′o wa	Ia.		

Rocky Mountain and Pacific States

Col o ra′do	Colo.	U′tah	U.
Mon ta′na	Mont.	Ar i zo′na	Ariz.
Wy o′ming	Wyo.	Cal i for′ni a	Cal.
New Mex′i co	N. Mex.	Or′e gon	Ore. *or* Oreg.
Ne va′da	Nev.	Wash′ing ton	Wash.
I′da ho	Ida.	A las′ka Ter′ri to ry	Alas.

Detached Possessions

Ha wai′ian Is′lands	H. I.	Guam
Por′to Ri′co	P. Rico *or* P. R.	Wake Is′land
Phil′ip pines	Phil.	Tu tu i′la
Ma nu′a		Pan a ma′ Zone

The names *Ohio*, *Utah*, *Idaho*, and *Alaska* are seldom abbreviated.

GENERAL INFORMATION

WORDS COMMONLY MISPRONOUNCED

214. Probably one of the best means we have of judging a person's education or ability is the language that he uses to express his thoughts. If he has only a limited number of words at his command and if he often pronounces some of these in a way that is not approved by authorities on the matter, we decide that his education has been neglected or that he is careless about his speech. A large vocabulary and an understanding of the correct pronunciation of words is of great importance, for it is necessary not only in social life, but also in business life. Every person should therefore be on the alert at all times and endeavor to acquire a large store of words, as the lack of a good vocabulary is likely to retard his progress in many lines.

The following suggestions will doubtless be found helpful, and careful attention should be given to them:

1. *Always note a new word;* that is, a word with which you are not familiar. Frequently you will meet with new words in books, magazines, lectures, sermons, and the conversation of friends. Words of this kind, whose meanings you cannot determine from the way in which they are used and about whose pronunciation you are in doubt, should be looked up in a dictionary. The uses, the meanings, and the illustrations given there should be studied until the words are so thoroughly understood that they can be used correctly.

2. *Seek good society.* Nothing shows so quickly in a person's speech as the kind of associates he has. The language of cultured, refined persons is more likely to contain words and expressions that should be added to one's vocabulary than the language of ignorant persons. Try to profit by association with persons of education and culture.

3. *Use a dictionary constantly.* The habit of using a dictionary constantly is an excellent one to acquire and cannot be too strongly recommended. If a dictionary is consulted for every new word that you meet with, it will not be long before many additions will be made to your word store. In looking up a word, note carefully its correct spelling and pronunciation, and study its definitions and the illustrations given for its use. Then pronounce the word aloud and try to use it in sentences of your own construction. When you can use a word and pronounce it in the proper way, it is yours for all time.

4. *Note how others pronounce words.* If at any time you hear some one pronounce a word differently from the way in which you pronounce it, do not fail to find out which pronunciation is correct. If you follow this plan, you will be surprised many times to note just how the word should be pronounced.

215. List of Words Commonly Mispronounced.—So many errors are made in the pronunciation of words that it seems advisable to give the following list, which is arranged alphabetically and contains a great many words that are commonly mispronounced. It will be a good plan for you to pronounce these words aloud until you are sure that you know exactly how they should be given. Then you will not be likely to forget the proper pronunciation and you will be able to employ the words whenever you need them. Reference to the table of diacritical marks in Art. 4 should be made in case of doubt about any of the marks used in this list, as all of these marks are explained in that article.

A

abdomen (ăb dō′mĕn)
abstract, *adj.* (ăb′străkt)
abstract, *verb* (ăb străkt′)
accent, *noun* (ăk′sĕnt)
accent, *verb* (ăk sĕnt′)
accept (ăk sĕpt′)
accessory (ăk sĕs′sō rĭ)
acclimate (ăk klī′māt)
acclimatize (ăk klī′mȧ tīz)
accompaniment (ăk kŭm′pȧ nĭ mĕnt)
accurate (ăk′kū rāt)
aconite (ăk′ō nīt)
acorn (ā′kŭrn)
acoustics (ȧ ko͞os′tĭks)
acumen (ȧ kū′mĕn)
address (ăd drĕs′)
adept (ȧ dĕpt′)
admirable (ăd′mĭ rȧ bl)
adult (ȧ dŭlt′)
ad valorem (ăd vȧ lō′rĕm)
advance (ăd vȧns′)
adverse (ăd′vẽrs)
aerial (ā ē′rĭ ȧl)
aeriform (ā′ẽr ĭ fôrm)
aerolite (ā′ẽr ō līt)
aeronaut (ā′ẽr ō nôt)
aeroplane (ā′ẽr ō plān)
against (ȧ gĕnst′)
aged, *adj.* (ā′jĕd)
aged, *verb* (ājd)
agile (ăj′ĭl)
ailment (āl′mĕnt)
albumen (ăl bū′mĕn)
algebra (ăl′jē brȧ)
alias (ā′lĭ ăs)
alien (āl′yĕn)
allopathic (ăl lō păth′ĭk)
allopathist (ăl lŏp′ȧ thĭst)
allopathy (ăl lŏp′ȧ thĭ)
ally (ăl lī′)
almond (ä′mŭnd)
alpaca (ăl păk′ȧ)
altercate (ăl′tẽr kāt)
alternate, *noun* and *adj.* (ăl tẽr′nāt)
alternate, *verb* (ăl′tẽr nāt *or* ăl tẽr′nāt)
amateur (ăm ȧ tẽr′ *or* ăm′ȧ tūr)
amenable (ȧ mē′nȧ bl)
amiable (ā′mĭ ȧ bl)
amicable (ăm′ĭ kȧ bl)
anchovy (ăn chō′vy)
ancient (ān′shĕnt)
annex, *noun* (ăn nĕks′ *or* ăn′nĕks)
annex, *verb* (ăn nĕks′)
antarctic (ănt ärk′tĭk)

aperture (ăp′ẽr tūr)
apparatus (ăp pȧ rā′tŭs)
apricot (ā′prĭ kŏt)
aqueous (ā′kwē ŭs)
Arab (ăr′ăb)
Arabic (ăr′ȧ bĭk)
archipelago (är kĭ pĕl′ȧ gō)
architect (är′kĭ tĕkt)
arctic (ärk′tĭk)
area (ā′rē ȧ)
artiste (är tēst′)
Asia (ā′shĭ ȧ *or* ā′shȧ)
asphalt (ăs′fălt *or* ăs fălt′)
aspirant (ăs pīr′ănt)
associate (ăs sō′shĭ āt)
attache (ȧt tȧ shā′)
automobile (au tō mō′bĭl)
avenue (ăv′ē nū)
aviary (ā′vĭ ā rĭ)

B

barrel (băr′rĕl)
bath (bȧth)
bayou (bī′o͞o)
bestial (bĕs′chȧl)
bicycle (bī′sĭ kl)
bijou (bē zho͞o′)
biography (bī ŏg′rȧ fĭ)
bitumen (bĭ tū′mĕn)
blackguard (blăg′gärd)
bouquet (bo͞o kā′)
bravado (brȧ vā′dō)
brooch (brōch)
Buddha (bo͝od′dȧ)
burlesque (bûr lĕsk′)

C

café (kȧ fā′)
calf (käf)
canine (kȧ nīn′)
cant (kănt)
can't (kȧnt)
carbine (kär′bīn)
Carnegie (kär nĕg′ĭ)
casualty (kăzh′ū ạl tĭ)
catholicism (kȧ thŏl′ĭ sĭzm)
centenary (sĕn′tē nā rĭ)
chasten (chā′sn)
chauffeur (shō fẽr′)
chef (shĕf)
cheviot (chĕv′ĭ ŭt, *not* shĕv′ĭ ŭt)
chlorine (klō′rĭn *or* klō′rēn)
chorist (kō′rĭst)
chorister (kŏr′ĭs tẽr)
christen (krĭs′n)
civil (sĭv′ĭl *not* sĭv′ l)
cleanly, *adj*. (klĕn′lĭ)
cleanly, *adv*. (klēn′lĭ)
clematis (klĕm′ȧ tĭs)
clientele (klī ĕn tēl′ *or* klī′ĕn tēl)
clique (klēk)
coadjutor (kō ăđ jū′tẽr)
cocaine (kō′kȧ ĭn, *not* kō kān′)
combatant (kŏm′băt ȧnt)
combative (kŏm′băt ĭv *or* kŏm-băt′ĭv)
comely (kŭm′lĭ)
comparable (kŏm′pȧ rȧ bl)
concrete, *noun* or *adj*. (kŏn′krēt)
concrete, *verb* (kŏn krēt′)
condolence (kŏn dō′lens)
connoisseur (kŏn nĭs sûr′ *or* kŏn nĭs sūr′)
contour (kŏn to͞or′)
contract, *noun* (kŏn′trăkt)
contract, *verb* (kŏn trăkt′)

contrary (kŏn′trȧ rĭ *or* kŏn′trā rĭ)

contrast, *noun* (kŏn′trȧst)

contrast, *verb* (kŏn trȧst′)

contrite (kŏn′trīt)

contumely (kŏn′tū mē lĭ)

conversant (kŏn′vẽr sȧnt)

converse, *noun* or *adj.* (kŏn′vẽrs)

converse, *verb* (kŏn vẽrs′)

convert, *noun* (kŏn′vẽrt)

convert, *verb* (kŏn vẽrt′)

coquetry (kō kĕt′rĭ *or* kō′kĕt rĭ)

coquette (kō ket′)

coquettish (kō kĕt′tĭsh)

coral (kŏr′ȧl)

cordiality (kôr jăl′ĭ tĭ *or* kôr dĭ ăl′ĭ tĭ)

corps (kōr)

coterie (kō te rē′)

courteous (kûr′tē ŭs)

courtier (kōrt′yẽr)

coyote (kī′ō tē *or* kī′ōt)

creek (krēk)

crematory (krĕm′ȧ tō ri *not* krēm′ȧ tō rĭ)

curator (kū rā′tẽr)

cyclamen (sĭk′lȧ mĕn)

D

data (dā′tȧ)

débris (dā brē′)

decade (dĕk′ād)

decadence (dē kā′dens)

decorous (dē kō′rŭs)

decorum (dē kō′rŭm)

defalcate (dē făl′kāt)

deficit (dĕf′ĭ sĭt)

deposition (dĕp ō zĭsh′ŭn)

desuetude (dĕs′wē tūd)

desultory (dĕs′ŭl tō rĭ)

digression (dĭ grĕsh′ŭn)

diploma (dĭ plō′mȧ)

direct (dĭ rĕkt′)

disputable (dĭs′pū tȧ bl)

disputant (dĭs′pū tȧnt)

docile (dŏs′ĭl *or* dō′sĭl)

drought (drowt)

drouth (drouth)

drowned (drownd)

dyspepsia (dĭs pĕp′sĭ ȧ *or* dĭs pĕp′shȧ)

E

eczema (ĕk′zē mȧ)

egotism (ē′gō tĭzm *or* ĕg′ō tĭzm)

egotist (ē′gō tĭst)

embalm (ĕm bäm′)

encore (äng kōr′)

enema (ĕn′ē mȧ *or* ē nē′mȧ)

engine (ĕn′jĭn)

enquiry (ĕn quīr′ĭ)

epistle (ē pĭs′l)

equable (ē′kwȧ bl)

excise (ĕk sīz′)

excretory (ĕks′krē tō rĭ *or* ĕks krē′tō rĭ)

exemplary (ĕgz′ĕm plā rĭ)

exhilarate (ĕgz ĭl′ȧ rāt *or* ĕks hĭl′ȧ rāt)

exhort (ĕgz ôrt′ *or* ĕks hôrt′)

expletive (ĕks′plē tĭv)

explicable (ĕks′plĭ kȧ bl)

exponent (ĕks pō′nent)

exquisite (ĕks′kwĭ zĭt)

extant (ĕks′tănt)

extol (ĕks tŏl′)

extraordinary (ĕks trôr′dĭ nā rĭ *or* ĕks trȧ ôr′dĭ nā rĭ)

F

favorite (fā′vẽr ĭt)
February (fĕb′rōō ā rĭ)
feminine (fĕm′ĭ nĭn)
figure (fĭg′ūr)
finance (fĭ năns′)
finances (fĭ năn′sĕz)
financial (fĭ năn′shȧl)
financier (fĭn ăn sēr′)
finesse (fē nĕs′ *or* fĭ-nĕs′)
floral (flō′rȧl)
florid (flŏr′ĭd)
florist (flō′rĭst *or* flŏr′ĭst)
fountain (foun′tĭn)
fracas (frā′kȧs)
fragile (frăj′ĭl)
frontier (frŏn′tēr)

G

gallant, *adj.* (găl′lȧnt *or* găl-lănt′)
gallant, *noun* (găl lănt′ *or* găl′-lănt)
gallant, *verb* (găl lănt′)
genealogy (jĕn ē ăl′ō jy)
genuine (jĕn′ū ĭn)
geography (jē ŏg′rȧ fĭ)
gladiolus (glȧ dī′ō lŭs *or* glăd-ĭ ō′lŭs)
glamour (glăm′ẽr *or* glā′mōōr)
glisten (glĭs′n)
gondola (gŏn′dō lȧ)
government (gŭv′ẽrn mĕnt)
granary (grăn′ȧ rĭ)
gratis (grā′tĭs)
grievous (grēv′ŭs)
grimace (grĭ mās′)
gristle (grĭs′l)

H

handkerchief (hăng′kẽr chĭf)
hearth (härth)
heinous (hā′nŭs)
herculean (hẽr kū′lē ȧn)
heroic (hē rō′ĭk)
heroine (hĕr′ō ĭn)
heroism (hĕr′ō ĭsm)
hiccough (hĭk′ŭp)
hickory (hĭk′ō rĭ *not* hĭk′rĭ)
history (hĭs′tō-rĭ *not* hĭs′trĭ)
homeopathist (hō mē ŏp′ȧ-thĭst)
horizon (hō rī′zŭn)
hover (hŭv′ẽr)
hydrangea (hī drăn′jē ȧ)
hygiene (hī′jĭ ēn *or* hī′jēn)
hygienic (hī jĭ ĕn′ĭk)
hymeneal (hī mē nē′ȧl)
hypocrisy (hĭ pŏk′rĭ sĭ)
hypodermic (hĭp ō dẽr′mĭk)

I

idea (ī dē′ȧ)
ignition (ĭg nĭsh′ŭn)
ignoramus (ĭg nō rā′mŭs)
illustrate (ĭl lŭs′trāt)
imbecile (ĭm′bē sĭl *or* ĭm′bē sēl)
impious (ĭm′pĭ ŭs)
impotent (ĭm′pō tĕnt)
incendiary (ĭn sĕn′dĭ ā rĭ)
incense, *noun* (ĭn′sĕns)
incense, *verb* (ĭn sĕns′)
incomparable (ĭn kŏm′pȧ rȧ bl)
incongruous (ĭn cŏn′grōō ŭs)
increase, *noun* (ĭn′krēs *or* ĭn-krēs′)
increase, *verb* (ĭn krēs′)

indecorous (ĭn dē kō′rŭs *or* ĭn-dĕk′ō rŭs)
indict (ĭn dīt′)
indictment (ĭn dīt′mĕnt)
indigent (ĭn′dĭ jĕnt)
indisputable (ĭn dĭs′pū tȧ bl)
industry (ĭn′dŭs trĭ)
infamous (ĭn′fȧ mŭs)
infinite (ĭn′fĭ nĭt)
influence (ĭn′flū ens)
inquiry (ĭn kwīr′ĭ)
integral (ĭn′tē grạl)
interest (ĭn′tẽr ĕst)
inventory (ĭn′vĕn tō rĭ)
irascible (ī răs′sĭ bl)
irreparable (ĭr rĕp′ȧ rȧ bl)
irrevocable (ĭr rĕv′ō kȧ bl)
isolation (ī sō lā′shŭn)
Italian (ĭ tăl′yȧn)

J

jugular (jū′gū lẽr)
juvenile (jū′vē nĭl)

L

learned, *adj*. (lẽrn′ĕd)
learned, *verb* (lẽrnd)
legislature (lĕj′ĭs lā tūr)
lenient (lē′nĭ ĕnt *or* lēn′yĕnt)
lichen (lī′kĕn)
licorice (lĭk′ō rĭs)
lien (lēn *or* lī′ĕn)
literature (lĭt′ẽr ȧ tūr)
ludicrous (lū′dĭ krŭs)
lyceum (lī sē′ŭm)

M

maintenance (mān′tē nȧns)
maniacal (mȧ nī′ȧ kȧl)
medicinal (mē dĭs′ĭ nȧl)
membranous (mĕm′brȧ nŭs)
memoir (mĕm′wŏr)
menu (mĕ nū′)
mercantile (mẽr′kăn tĭl)
metric (mĕt′rĭk)
mineralogy (mĭn ẽr ȧl′ō jĭ)
mischievous (mĭs′chē vŭs)
misconstrue (mĭs kŏn′strōō)
mistletoe (mĭz′l tō)
molecular (mō lĕk′ū lẽr)
morphine (môr′fĭn *or* môr′fēn)
mountainous (moun′tĭn ŭs)
museum (mū zē′ŭm)
mustache (mŭs tȧsh′)
mythology (mĭ thŏl′ō jĭ)

N

naiad (nā′yăd)
naphtha (năf′thȧ *or* năp′thȧ)
national (năsh′ŭn ȧl)
nectarine (nĕk′tẽr ĭn)
negligee (nĕg lĭ zhā′)

O

obeisance (ō bā′sȧns *or* ō bē′-sȧns)
obligatory (ŏb′lĭ gā tō rĭ)
oleomargarine (ō lē ō mär′gȧ-rĭn *or* ō lē ō mär′gȧ rēn)
onerous (ŏn′ẽr ŭs)
opponent (ŏp pō′nĕnt)
opportune (ŏp pŏr tūn′)
ordeal (ôr′dē ȧl)

P

parachute (păr′ȧ shōōt)
pathos (pā′thŏs)
patriot (pā′trĭ ŏt)
patriotism (pā′trĭ ŏt ĭzm)
patron (pā′trŭn)
pedagogy (pĕd′ȧ gō jy)
peony (pē′ō nĭ)

peremptory (pĕr′ĕmp tō rĭ)
petit (pĕt′y)
petite (pẽ tēt′)
pharyngitis (făr ĭn jī′tĭs)
pianist (pĭ ȧn′ĭst)
piano (pĭ ȧn′ō)
pianoforté (pĭ ȧn′ō fōr′tā)
pincers (pĭn′sẽrz)
piquancy (pē′kȧn sĭ *or* pĭk′ȧn sĭ)
piquant (pē′kȧnt *or* pĭk′ȧnt)
placable (plā′kȧ bl)
plebeian (plē bē′yȧn)
plethoric (plē thŏr′ĭk *or* plĕth′ō rĭk)
porpoise (pôr′pŭs)
precedence (prē cēd′ens)
precedent, *noun* (prĕs′ē dent)
precedent, *adj.* (prē sēd′ent)
premise, *noun* (prĕm′ĭs)
premise, *verb* (prē mīz′)
presage, *noun* (prē′sāj *or* prĕs′āj)
presage, *verb* (prē sāj′)
pretense (prē tĕns′)
process (prŏs′ĕs)
produce, *noun* (prŏd′ūs)
produce, *verb* (prō dūs′)
profile (prō′fīl *or* prō′fēl)
progress, *noun* (prŏg′rĕs)
progress, *verb* (prō grĕs′)
project, *noun* (prŏj′ĕkt)
project, *verb* (prō jĕkt′)
promulgate (prō mŭl′gāt)
proteid (prō′tē-ĭd)
protein (prō′tē ĭn)
protest, *noun* (prō′tĕst)
protest, *verb* (prō tĕst′)
puerile (pū′ẽr ĭl)
pumpkin (pŭmp′kĭn)

Q

quietus (kwī ē′tŭs)
quinine (kwī′nīn *or* kwĭ nīn′)

R

rapacious (rȧ pā′shŭs)
recess (rē sĕs′ *or* rē′sĕs)
recipe (rĕs′ĭ pē)
reciprocity (rĕs ĭ prŏs′ĭ tĭ)
recreate, *to amuse* (rĕk′rē āt)
recreate, *to create anew* (rē krē āt′)
referable (rĕf′ẽr ȧ bl)
reflex, *adj.* or *noun* (rē′flĕks)
reflex, *verb* (rē flĕks′)
refuse, *noun* (rĕf′ūs)
refuse, *verb* (rē fūz′)
reparable (rĕp′ȧ rȧ bl)
reptile (rĕp′tĭl)
reputable (rĕp′ū tȧ bl)
requiem (rē′kwĭ ĕm)
reservoir (rĕz′ẽr vwôr)
resource (rē sōrs′)
respite (rĕs′pĭt)
restaurant (rĕs′tō rȧnt)
résumé (rā zo͞o mā′)
robust (rō bŭst′)
roil (roil *not* rīl)
romance, *noun* or *verb* (rō-măns′)
roof (ro͞of *not* ro͝of)
route (ro͞ot *or* rout)

S

sacrifice (săk′rĭ fīs *or* săk′rĭ fīz)
sacrilege (săk′rĭ lĕj)
sagacious (sȧ gā′shŭs)
salmon (săm′ŭn)

scabies (skā′bĭ ēz)
seance (sā′äns)
semester (sē mĕs′tẽr)
senile (sē′nīl)
señor (sā nyōr′)
señora (sē nyō′rȧ)
sepulcher (sĕp′ŭl kẽr)
sergeant (sär′jĕnt *or* sẽr′jĕnt)
servile (sẽrv′ĭl)
short-lived (shôrt′ līvd)
sinecure (sī′nē kūr)
solace (sŏl′ās)
souvenir (so͞o vē nēr′ *or* so͞o′vē nēr)
specious (spē′shŭs)
status (stā′tŭs)
strata (strā′tȧ)
stratum (strā′tŭm)
suave (swāv *or* swäv)
suavity (swăv′ĭ tĭ)
subject, *noun* (sŭb′jĕkt)
subject, *verb* (sŭb jĕkt′)
subtle (sŭt′l)
subtlety (sŭt′l tĭ)
suburb (sŭb′ûrb *not* so͞o′bûrb)
suburban (sŭb ûrb′ȧn)
suffice (sŭf fīz′)
suite (swēt)
surveillance (sûr vāl′yȧns)
survey, *noun* (sûr′vā *or* sûr vā′)
survey, *verb* (sûr vā′)
Syracuse (sĭr′ȧ kūs)
syringe (sĭr′ĭnj)

T

tapestry (tăp′ĕs trĭ)
telegraphy (tē lĕg′rȧ fĭ)
tenacious (tē nā′shŭs)
tenet (tĕn′ĕt)
tepid (tĕp′ĭd)
tiny (tī′nĭ *not* tē′nĭ)
tonsillitis (tŏn sĭl lī′tĭs)
towards (tō′ẽrdz *or* tōrdz)
tremendous (trē mĕn′dŭs)
tribunal (trī bū′nȧl)
tribune (trĭb′ūn)
turquoise (tûr koiz′ *or* tûrkēz′)
twopence (tŭp′ĕns)
tympanum (tĭm′pȧ nŭm)
tyrannize (tĭr′ȧn nīz)
tyranny (tĭr′ȧ nĭ)

U

umbrella (ŭm brĕl′lȧ)
uncouth (ŭn ko͞oth′)
undersigned (ŭn dẽr sīnd′)

V

vaccine (văk′sĭn *or* văk′sēn)
vacuum (văk′ū ŭm)
vagary (vȧ gā′rĭ)
vagaries (vȧ gā′rĭz)
vaudeville (vōd′vĭl)
vehemence (vē′hē mens)
venison (vĕn′ĭ zn *or* vĕn′zn)
veracious (vē rā′shŭs)
verbatim (vẽr bā′tĭm)

W

weapon (wĕp′ŭn)
wistaria (wĭs tā′rĭ ȧ)
withe (wĭth; *th*, as in *thin*)
worsted (wûrst′ĕd)

Y

Yosemite (yō sĕm′ĭ tē)

Z

zoology (zō ŏl′ō jy)

CENSUS FACTS

216. The matter that follows is given merely for reference purposes. However, it contains much information of practical value, and should therefore receive careful attention. Table I gives the area and the population of the United States and its possessions according to the census of 1910, and following this table is a list of the largest cities of the various states of the United States, with the population of these cities according to the census of 1910. For convenience of reference, the names of the states are arranged alphabetically. The cities of each state are given in the order of their size.

TABLE I

AREA AND POPULATION OF UNITED STATES AND POSSESSIONS—CENSUS 1910

	Area in Square Miles	Population	Largest City	Population
United States........	3,025,600	91,716,009	New York	5,173,064
Philippine Islands....	115,026	7,635,426	Manila....	234,409
Hawaiian Islands....	6,449	191,909	Honolulu..	52,183
Tutuila and Islets....	77	7,251	Pago-Pago	
Guam..............	210	12,517	Agana.....	7,000
Panama Zone........	474	....	Panama...	35,500
Alaska..............	590,884	64,356	Fairbanks.	3,541
Porto Rico..........	3,606	1,118,012	San Juan..	48,716
Total.............	3,742,326	100,745,480		

LARGEST CITIES OF THE VARIOUS STATES, CENSUS OF 1910

Alabama		Arkansas	
Bir′ming ham......	132,685	Lit′tle Rock.......	45,941
Mo bile′ (bēl)......	51,521	Fort Smith........	23,975
Mont gom′er y.....	38,136	Pine Bluff.........	15,102
Sel′ma...........	13,649	Hot Springs........	14,434
An′nis ton.........	12,794	Ar gen′ta..........	11,138
Arizona		**California**	
Tuc son′(tū sŏn′)...	13,193	San Fran cis′co.....	416,912
Phœ′nix...........	11,134	Los An′ge les.......	319,198

Oak'land	150,174
Sac ra men'to	44,696
Berke'ley	40,434
San Di e'go (dē ā'gō)	39,578
Pas a de'na	30,291
San Jo sé' (hō zā')	28,946

COLORADO

Den'ver	213,381
Pueb'lo (pwĕb'lō)	44,395
Col o ra'do Springs	29,078
Trin i dad'	10,204
Boul'der (bōl')	9,539

CONNECTICUT

New Ha'ven	133,605
Bridge'port	102,054
Hart'ford	98,915
Wa'ter bu ry	73,141
New Brit'ain	43,916
Mer'i den	32,066
Stam'ford	28,836
Nor'wich (nŏr'ĭj)	28,219
Nor'walk (nôr'wôk)	24,211
Dan'bu ry	23,502
Mid'dle town	20,749

DELAWARE

Wil'ming ton	87,411

DISTRICT OF COLUMBIA

Wash'ing ton	331,069

FLORIDA

Jack'son ville	57,699
Tam'pa	37,782
Pen sa co'la	22,982
Key West	19,945

GEORGIA

At lan'ta	154,839
Sa van'nah	65,064
Au gus'ta	41,040
Ma'con	40,665
Co lum'bus	20,554
Ath'ens	14,913
Way'cross	14,485

IDAHO

Boi'se (boi'zā)	17,358

ILLINOIS

Chi ca'go (shĭ kô'gō)	2,185,283
Pe o'ri a	66,950
East St. Lou'is	58,547
Spring'field	51,678
Rock'ford	45,401
Quin'cy	36,587
Jo'li et	34,670
De ca'tur	31,140
Au ro'ra	29,807
Dan'ville	27,871
El'gin	25,976
Bloom'ing ton	25,768

INDIANA

In di an ap'o lis	233,650
Ev'ans ville	69,647
Fort Wayne	63,933
Ter're Haute (tĕr'e hōt)	58,157
South Bend	53,684
Mun'cie	24,005
An'der son	22,476
Rich'mond	22,324
Ham'mond	20,925
New Al'ba ny	20,629
La fay ette'	20,081

IOWA

Des Moines (de moin')	86,368
Sioux Cit'y (sōō)	47,828
Dav'en port	43,028

Du buque′ (do͞o būk′)	38,494
Ce′dar Rap′ids	32,811
Coun′cil Bluffs	29,292
Wa′ter loo	26,693
Clin′ton	25,577
Bur′ling ton	24,324

KANSAS

Kan′sas Cit′y	82,331
Wich′i ta	52,450
To pe′ka	43,684
Leav′en worth	19,363

KENTUCKY

Lou′is ville	223,968
Cov′ing ton	53,270
Lex′ing ton	35,099
New′port	30,309
Pa du′cah	22,760

LOUISIANA

New Or′le ans	339,075
Shreve′port	28,015
Bat′on Rouge (băt′ŭn ro͞ozh)	14,897

MAINE

Port′land	58,571
Lew′is ton	26,247
Ban′gor	24,803
Bid′de ford	17,079
Au′burn	15,064

MARYLAND

Bal′ti more	558,485
Cum′ber land	21,839
Ha′gers town	16,507

MASSACHUSETTS

Bos′ton	670,585
Worces′ter (wo͝os′tẽr)	145,986
Fall Riv′er	119,295
Low′ell	106,294
Cam′bridge (kām′brĭj)	104,839
New Bed′ford	96,652
Lynn	89,336
Spring′field	88,926
Law′rence	85,892
Som′er ville	77,236
Hol′yoke	57,730
Brock′ton	56,878
Mal′den	44,404
Hav′er hill	44,115
New′ton	39,806
Fitch′burg	37,826
Taun′ton	34,259
Ev′er ett	33,484
Quin′cy	32,642
Chel′sea	32,452
Pitts′field	32,121
Wal′tham	27,834
Brook′line	27,792
Chic′o pee	25,401
Glouces′ter (glŏs′ter)	24,398
Med′ford	23,150

MICHIGAN

De troit′	465,766
Grand Rap′ids	112,571
Sag′i naw	50,510
Bay Cit′y	45,166
Kal a ma zoo′	39,437
Flint	38,550
Jack′son	31,433
Lan′sing	31,229
Bat′tle Creek	25,267
Mus ke′gon	24,062

MINNESOTA

Min ne ap'o lis	301,408
St. Paul	214,744
Du luth'	78,446
Wi no'na	18,583

MISSISSIPPI

Me rid'i an	23,285
Jack'son	21,262
Vicks'burg	20,814

MISSOURI

St. Lou'is	687,029
Kan'sas Cit'y	248,381
St. Jo'seph	77,403
Spring'field	35,201
Jop'lin	32,073

MONTANA

Butte (būt)	39,165
Great Falls	13,948
Mis sou'la	12,869
Hel'e na	12,515

NEBRASKA

O'ma ha (ō'mȧ hô)	124,096
Lin'coln	43,973
South O'ma ha	26,259

NEVADA

Re'no	10,867

NEW HAMPSHIRE

Man'ches ter	70,063
Nash'u a	26,005
Con'cord	21,497

NEW JERSEY

New'ark	347,469
Jer'sey Cit'y	267,779
Pat'er son	125,600
Tren'ton	96,815
Cam'den	94,538
E liz'a beth	73,409
Ho'bo ken	70,324
Ba yonne'	55,545
Pas sa'ic	54,773
At lan'tic Cit'y	46,150
West Ho'bo ken	35,403
East Or'ange	34,371
Perth Am'boy	32,121
Or'ange	29,630
New Bruns'wick	23,388
Mont clair'	21,550
U'nion	21,023

NEW MEXICO

Al bu quer'que (äl-boo̅ kĕr'kā *or* ăl'-boo̅ kẽrk)	11,020

NEW YORK

New York	4,766,883
Buf'fa lo	423,715
Roch'es ter	218,149
Syr'a cuse	137,249
Al'ba ny	100,253
Yon'kers	79,803
Troy	76,813
U'ti ca	74,419
Sche nec'ta dy	72,826
Bing'ham ton	48,443
El mi'ra	37,176
Au'burn	34,668
James'town	31,297
Am'ster dam	31,267
Mount Ver'non	30,919
Ni ag'a ra Falls	30,445
New Ro chelle'	28,867
Pough keep'sie (pō-kĭp'sĭ)	27,936
New'burgh	27,805
Wa'ter town	26,730
King'ston	25,908

Co hoes′ (kō hōz′)	24,709
Os we′go	23,368
Glov′ers ville	20,642
Rome	20,497

NORTH CAROLINA

Char′lotte	34,014
Wil′ming ton	25,748
Ra′leigh	19,218
Ashe′ville	18,762
Dur′ham	18,241
Win′ston	17,167

NORTH DAKOTA

Far′go	14,331
Grand Forks	12,478

OHIO

Cleve′land	560,663
Cin cin na′ti	363,591
Co lum′bus	181,511
To le′do	168,497
Day′ton	116,577
Youngs′town	79,066
Ak′ron	69,067
Can′ton	50,217
Spring′field	46,921
Ham′il ton	35,279
Li′ma (lī′mȧ)	30,508
Lo rain′	28,883
Zanes′ville	28,026
New′ark	25,404
Ports′mouth	23,481
Steu′ben ville	22,391
Mans′field	20,768
East Liv′er pool	20,387

OKLAHOMA

Ok la ho′ma Cit′y	64,205
Mus co′gee (mŭs-kō′gē)	25,278

OREGON

Port′land	207,214
Sa′lem	14,094

PENNSYLVANIA

Phil a del′phi a	1,549,008
Pitts′burgh	533,905
Scran′ton	129,867
Read′ing (rĕd)	96,071
Wilkes′-Bar re	67,105
E′rie	66,525
Har′ris burg	64,186
Johns′town	55,482
Al too′na	55,127
Al′len town	51,913
York	44,750
Lan′cas ter	44,227
Mc Kees′port	42,694
Ches′ter	38,537
New′Cas tle	36,280
Wil′liams port	31,860
Eas′ton	28,523
Nor′ris town	27,875
Shen an do′ah	25,744
Ha′zle ton	25,452

RHODE ISLAND

Prov′i dence	224,326
Paw tuck′et	51,622
Woon sock′et	38,125
New′port	27,149
War′wick (wŏr′ĭk)	26,629
Cen′tral Falls	22,754
Cran′ston	21,107

SOUTH CAROLINA

Charles′ton	58,833
Co lum′bi a	26,319

South Dakota

Sioux Falls (so͞o)	14,094
Ab er deen′ (ăb ẽr-dēn′)	10,753

Tennessee

Mem′phis	131,105
Nash′ville	110,364
Chat ta noo′ga	44,604
Knox′ville	36,346
Jack′son	15,779

Texas

San An to′ni o	96,614
Dal′las	92,104
Hous′ton	78,800
Fort Worth	73,312
El Pa′so	39,279
Gal′ves ton	36,981
Aus′tin	29,800
Wa′co (wā′cō)	26,425
Beau mont′	20,640

Utah

Salt Lake Cit′y	92,777
Og′den	25,580

Vermont

Bur′ling ton	20,468
Rut′land	13,546

Virginia

Rich′mond	127,628
Nor′folk	67,452
Ro′an oke	34,874
Ports′mouth	33,130
Lynch′burg	29,494
Pe′ters burg	24,127
New′port News	20,205
Dan′ville	19,020
Al ex an′dri a	15,329

Washington

Se at′tle (sē at′l)	237,194
Spo kane′ (spō kăn′)	104,402
Ta co′ma	83,743
Ev′er ett	24,814
Bel′ling ham	24,298
Wal′la Wal′la (wŏl′-ȧ wŏl′ȧ)	19,364
North Yak′i ma	14,082

West Virginia

Wheel′ing	41,641
Hunt′ing ton	31,161
Charles′ton	22,996
Par′kers burg	17,842

Wisconsin

Mil wau′kee	373,857
Su pe′ri or	40,384
Ra cine′ (rȧ sēn′)	38,002
Osh′kosh	33,062
La Crosse′	30,417
She boy′gan	26,398
Mad′i son	25,531
Green Bay	25,236
Ke no′sha (kē nō′-shȧ)	21,371
Fond du Lac′ (fŏn-du lăk′)	18,797
Eau Claire (ō klâr)	18,310
Ap′ple ton	16,773
Wau′sau (wō′sō)	16,560
Be loit′	15,125
Chey enne′ (shī ĕn′)	11,320

LATIN WORDS AND PHRASES

217. The following Latin expressions occur frequently in books and conversation. They are given here for reference.

A

A fortiori, *with stronger reason*
A posteriori, *from effect to cause*
A priori, *from cause to effect*
Ab initio, *from the beginning*
Ad infinitum, *to infinity*
Ad interim, *in the meantime*
Ad libitum, *at pleasure*
Ad nauseam, *to disgust*
Ad valorem, *according to value*
Alias, *other; another name*
Alibi, *elsewhere*
Alma mater, *a fostering mother*
Ante bellum, *before the war*
Ante meridiem, *before midday*
Aqua vitæ, *water of life; brandy*
Argumentum ad hominem, *an argument that gets its force from the conduct or principles of the man to whom it is applied*

B

Bona fide, *in good faith*

C

Casus belli, *a cause for war*
Cui bono? *of what use is it?*

D

De facto, *in reality*
De jure, *by right*
De novo, *anew*
De profundis, *out of the depths*
Deo volente, *God being willing*
Dirigo, *I direct*
Disjecta membra, *scattered remains*

E

Ecce homo, *behold the man*
E pluribus unum, *one out of many*
Ex cathedra, *from the chair; with authority*
Excelsior, *higher*
Exeunt, *they go out*
Exeunt omnes, *all go out*
Ex officio, *by virtue of office*
Ex parte, *on one side*
Ex post facto, *after the act*

F

Facile princeps, *easily the first or chief*
Festina lente, *make haste slowly*
Fiat justitia, ruat cælum, *let justice be done though the heavens fall*
Fidus Achates, *a true friend*
Fortiter et recte, *bravely and rightly*
Fortiter in re, *strong in action*

G

Gloria in excelsis, *glory in the highest*

H

Humanum est errare, *it is human to err*

I

In hoc signo vinces, *in this sign thou shalt conquer*
In loco parentis, *in place of parent*
In medias res, *in the midst of affairs*
In memoriam, *in memory*
In nubibus, *in the clouds*
In situ, *in its original place*
In statu quo, *in its former state*
In terrorem, *as a warning*
In toto, *in the whole*
In transitu, *during passage*
Inter nos, *between ourselves*
Ipsi dixit, *he himself said it*
Ipso facto, *in the fact itself*

L

Labor omnia vincit, *labor conquers everything*
Lapsus calami, *a slip of the pen*
Lapsus linguæ, *a slip of the tongue*
Lapsus memoræ, *a slip of the memory*
Laus Deo, *praise to God*
Lex non scripta, *the unwritten law*
Lex talionis, *the law of retaliation*
Locum tenens, *holding the place; a deputy*
Lusus naturæ, *a freak of nature*

M

Magnum opus, *a great work*
Memento mori, *remember death*
Mens sana in corpore sano, *a sound mind in a sound body*
Meum et tuum, *mine and thine*
Mirabile dictu, *wonderful to relate*
Modus operandi, *manner of operating*
Multum in parvo, *much in little*

N

Ne plus ultra, *no further; the limit of achievement*
Nolens volens, *unwilling* (*or*) *willing, willy-nilly*
Nolle prosequi, *unwilling to proceed*
Non compos mentis, *not sound of mind*
Non est inventus, *he was not found*
Non sequitur, *it does not follow*
Non sibi, sed omnibus, *not for himself, but for all*
Nota bene, *mark well*

O

Ora pro nobis, *pray for us*

P

Pari passu, *with equal pace*
Particeps criminis, *an accomplice*
Paterfamilias, *father of a family*
Pater noster, *Our Father*
Per annum, *by the year*
Per capita, *by the head*
Per centum, *by the hundred*
Per diem, *by the day*
Per se, *by itself*
Pons asinorum, *asses' bridge*

Post mortem, *after death*
Pro bono publico, *for the public good*
Pro et con, *for and against*
Pro rata, *in proportion*
Pro tempore, *for the time*

Q

Quantum sufficit, *a sufficient quantity*
Quid pro quo, *an equivalent*

R

Rara avis, *a rare bird*
Reductio ad absurdum, *reduction to absurdity*
Requiescat in pace, *may he* (*or she*) *rest in peace*

S

Sanctum sanctorum, *holy of holies*
Semper idem, *always the same*
Sic semper tyrannis, *thus always to tyrants*
Sine die, *without* (*appointing*) *a day*
Sine qua non, (*without which not*) *an indispensable condition*
Stet, *let it stand; do not change it*
Suaviter in modo, *gentle in manner*
Sub rosa, *under the rose; privately*
Sui generis, *of its own kind*
Summum bonum, *the chief good*

T

Terra firma, *the solid earth*
Terra incognita, *an unknown country*

U

Ultima Thule, *the farthest limit*

V

Vade mecum, *go with me; a manual or hand book*
Veni, vide, vici, *I came, I saw, I conquered*
Verbatim et literatum, *word for word and letter for letter*
Via, *by the way of*
Vice versa, *conversely; the order reversed*
Vi et armis, *by force and arms; by main force*
Viva voce, *by living voice*
Vivere est cogitare, *to live is to think*
Vox populi, vox Dei, *the voice of the people is the voice of God*

COMMON ABBREVIATIONS

218. The following list contains many of the principal abbreviations and contractions used by English-speaking people.

A

A. America; American
A. B. Bachelor of Arts
abbr. *or* abbrev. Abbreviation
acc. *or* acct. Account
A. D. (*Anno Domini*) In the year of our Lord
adj. Adjective
adm. Admiral; admiralty
admr. Administrator
admx. Administratrix
adv. Adverb; advertisement
Agt. Agent
alg. Algebra
A. M. Master of Arts; before noon
amt. Amount
Anon. Anonymous
ans. Answer
arith. Arithmetic
Asst. Assistant
Att. *or* Atty. Attorney
Atty.-Gen. Attorney-general
Ave. *or* Av. Avenue
avoir. *or* avdp. Avoirdupois

B

B. A. British America; bachelor of arts
bal. Balance
Bat. *or* Batt. Battery; battalion
bbl. Barrel. bbls. Barrels
B. C. Before Christ; British Columbia
bdl. Bundle
B. I. British India
Bib. Bible; Biblical
biog. Biography
B. L. Bachelor of Laws; Bill of lading
bldg. Building
B. L. E. Brotherhood of Locomotive Engineers
B. M. Bachelor of Medicine
B. M. *or* B. Mus. Bachelor of Music
b. p. Bills payable
b. rec. Bills receivable
Brig. Brigade; brigadier
Brig. Gen. Brigadier General
Brit. Britain; British
Bro. Brother. Bros. Brothers
B. S. Bachelor of Surgery; Bachelor of Science
b. s. Bill of sale
bu. *or* bush. Bushel
bx. Box. bxs. Boxes

C

C. Centigrade
cap. Capital
Capt. Captain
Cash. Cashier
C. B. Cape Breton
C. E. Civil Engineer
Chap. Chapter

chem.	Chemistry
Civ.	Civil
Co.	Company; County
C. O. D.	Cash, *or* collect, on delivery
Col.	Colonel
Cong.	Congress
conj.	Conjunction
Cor. Sec.	Corresponding Secretary
Cr.	Creditor; credit
C. S.	Civil Service
ct.	Cent
cwt.	Hundredweight

D

D. D.	Doctor of Divinity
D. D. S.	Doctor of Dental Surgery
deg.	Degree; degrees
Dem.	Democrat; Democratic
Dep.	Deputy
Dept.	Department
diam.	Diameter
Dict.	Dictionary
dis. *or* disct.	Discount
Dist.	District
Dist. Atty.	District Attorney
div.	Dividend; divisor; division
D. Lit.	Doctor of Literature
D. M.	Doctor of Music
do.	(*ditto*) The same
doz.	Dozen
Dr.	Doctor; debtor

E

E.	East
ed.	Edition; editor
e. g.	(*exempli gratia*) For example
E. I.	East Indies; East India
elec.	Electricity; electrical
E. M.	Mining Engineer
Emp.	Emperor; Empress
Encyc.	Encyclopedia
Eng.	England; English
Epis.	Episcopal
Esq. *or* Esqr.	Esquire
etc., &c.	(*Et cetera*) And others; and so forth
Ex.	Example
Exec. *or* Exr.	Executor

F

F. *or* Fahr.	Fahrenheit
f.	Farthing; fathom
fem.	Feminine
fig.	Figure; figures
Fin. Sec.	Financial Secretary
Fo. *or* Fol.	Folio
f. o. b.	Free on board
Fr.	France; French
ft.	Foot; feet; fort
fur.	Furlong

G

gal.	Gallon; gallons
G. A. R.	Grand Army of the Republic
G. B.	Great Britain
g. c. d.	Greatest common divisor
Gen.	General; Genesis
geog.	Geography
geol.	Geology
geom.	Geometry
Ger. *or* Germ.	Germany; German
Gov.	Governor; government

H

hhd. Hogshead
H. M. His, *or* Her, Majesty
Hon. Honorable; honorary
h. p. Horsepower
H. R. House of Representatives
hr. Hour
H. R. H. His, *or* Her, Royal Highness

I

ib. *or* ibed. (*ibidem*) In the same place
id. (*idem*) The same
i. e. (*id est*) That is
in. Inch; inches
inst. (*instant*) Of the present month
isl. Island
It. *or* Ital. Italian; Italic

J

jr. or jun. Junior

L

lat. Latitude
l. c. m. Least common multiple
£, lb., *or* ℔. Pound
LL. D. Doctor of Laws
Lieut. *or* Lt. Lieutenant
long. Longitude
Ltd. Limited

M

Maj. Major
Maj. Gen. Major General
Mas. Masculine
M. C. Member of Congress
M. D. Doctor of Medicine
mdse. Merchandise
mi. Mile
min. Minute; minutes
Mlle. Mademoiselle
Mme. Madame
M. P. Member of Parliament
Mr. Master; Mister
Mrs. Mistress
MS. Manuscript
MSS. Manuscripts
Mt. Mountain; mount
Mus. D. Doctor of Music

N

N. North
N. A. North America; North American
naut. Nautical
N. B. North British; New Brunswick
N. B. (*nota bene*) Mark well; take notice
N. E. Northeast; New England
N. F. Newfoundland
N. G. National Guard
No. Number

O

obs. Obsolete
oz. Ounce; ounces

P

p. Page
per cent. *or* % (*percentum*) By the hundred
Ph. D. Doctor of Philosophy
pk. Peck
pkg. Package
plu. Plural
P. M. (*post meridiem*) Afternoon; Postmaster

P. O.	Post-Office
pop.	Population
Poss.	Possessive
P. P. C. (*pour prendre congé*)	To take leave
prep.	Preposition.
Pres.	President
prob.	Problem
Prof.	Professor
pro tem. (*tempore*)	For the time being
prox. (*proximo*)	Of next month
P. S. (*Post scriptum*)	Postscript
Ps.	Psalm; Psalms
pt.	Pint
pwt.	Pennyweight

Q

qt.	Quart

R

℞.	Recipe
R. A.	Royal Academy; Rear Admiral
Rec'd *or* Recd.	Received
Rec. Sec.	Recording Secretary
Regt.	Regiment
Rep.	Representative; Republic
Retd.	Returned
Rev.	Reverend; Revelation
R. F. D.	Rural free delivery
Rom.	Roman; Romans
Rom. Cath.	Roman Catholic
rpt.	Report
R. R.	Railroad
R. S. V. P. (*répondez s'il vous plaît*)	Answer, if you please
Rt. Hon.	Right Honorable
Rw. *or* Ry.	Railway

S

S. A.	South America; South Africa
S. E.	Southeast
Sec.	Secretary
Sen.	Senate; senator; senior
Serg. *or* Serj.	Sergeant *or* Serjeant
Sing.	Singular
sq.	Square
Sr. *or* Sen.	Senior
St.	Saint; street
supp.	Supplement
Supt.	Superintendent
S. W.	Southwest

T

tr.	Transpose; translator
Treas.	Treasury; treasurer

U

ult. (*ultimo*)	Last; Of the last month
Univ.	University
U. S.	United States
U. S. A.	United States of America; United States Army
U. S. M.	United States Mail; United States Marine
U. S. M. A.	United States Military Academy
U. S. N.	United States Navy
U. S. N. A.	United States Naval Academy
U. S. S.	United States steamer

V

Vice-Pres. *or* V. P.	Vice-President
viz. (*videlicet*)	Namely; to wit
vol.	Volume; volumes
vs. (*versus*)	Against; In opposition
V. S.	Veterinary Surgeon

W

W. A.	West Africa; West Australia
w. c.	Without charge
W. C. T. U.	Women's Christian Temperance Union
wf. *or* w. f.	wrong font
W. I.	West Indian; West Indies
wk.	Week
wt.	Weight

X

Xmas.	Christmas

Y

yd.	Yard; yards
yr.	Year